Chapter 1

Dimensions
Stick to Your Own Kind!

Q 1 Given Coulomb's law of electrostatic attraction, determine on dimensional grounds whether a quoted expression $E_{\rm i} = 2\pi^2 m_{\rm e}^2 k_{\rm e}^2 e^4/h^2$ for the ionization energy of the hydrogen atom could be correct. If it could, obtain its value in eV; if not, suggest a corrected formula.

[Note: the Coulomb constant $k_{\rm e}$ can, alternatively, be written as $1/(4\pi\epsilon_0)$.]

Q 2* According to the Wiedemann–Franz law, under certain conditions the ratio of a metal's thermal conductivity κ to its electrical conductivity σ is proportional to its absolute temperature T. Determine the dimensions of the constant of proportionality and, bearing in mind the processes to which it relates, suggest how it might be expressed in terms of fundamental constants.

Q 3 Use the fact that the electrostatic force acting between two electric charges, q_1 and q_2, is given by $q_1 q_2/4\pi\epsilon_0 r^2$, where r is the distance between the charges and $\epsilon_0 = 8.8 \times 10^{-12}\,\mathrm{F\,m^{-1}}$, to express one farad in terms of the SI base units.

Q 4* The following is a student's proposed formula for the energy flux S (the magnitude of the so-called Poynting vector) associated with an electromagnetic wave in a vacuum, the electric field strength of the wave being E and the associated magnetic flux density being B:

$$S = \frac{1}{2}\left[\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} E^2 + \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} B^2\right].$$

Assuming the normal definition of electric field strength, together with

(i) Coulomb's law in the form $F = q_1 q_2/4\pi\epsilon_0 r^2$,
(ii) the result that the magnetic field B inside a long empty solenoid of n turns per unit length, and carrying a current I, is given by $B = \mu_0 nI$,

(iii) the observation that the force acting on a rod of length ℓ that carries a current I at right angles to a field of magnetic flux density B is $BI\ell$,

determine whether the student's formula could be correct and, if not, locate the error as closely as possible.

Q 5 If hens' eggs should be boiled for five minutes, for how long should an ostrich egg be boiled? If you have not previously studied heat conduction, see the introduction to Q 196 on page 68.

Q 6* The radiation emitted per unit time by unit area of a 'black body' at temperature T is σT^4, where σ is the Stefan constant. Express σ in terms of the fundamental constants c, e, h, k and m_e, and then use the actual values of the quantities involved to evaluate any unknown constant.

Q 7 Whilst they were dancing by the light of the full Moon, the Pussy-cat remarked to the Owl that it seemed almost as bright as day. The Owl first looked at the Moon, estimating that it subtends 9×10^{-3} rad, and then recalled from his pub-quiz days that its albedo is 7%. As they had only recently been married by the Turkey who lives on the hill, he thought it best not to spoil the moment by giving his own assessment of the actual situation. Which was what?

Q 8* Whether or not an interstellar gas cloud undergoes gravitational collapse is determined by the relationship between the scale-length of its density perturbations and a characteristic length λ. The value of λ depends only on some or all of the following: the gravitational constant, the Boltzmann constant, the density of the gas, and the speed of sound in it.

Investigate the relationship between λ and the given quantities, (a) to determine whether the scale-length of the perturbations has to be noticeably greater than, or noticeably less than, λ to initiate collapse, and (b) to estimate the mass of a star formed from an atomic hydrogen cloud of density 10^{-17} kg m^{-3} and temperature 10 K, values typical of a so-called 'dense' cloud in interstellar space.

Q 9 Three very different lengths that appear in quantum physics and cosmology are the Planck length ℓ_p, the Compton wavelength λ_m for a particle of mass m, and the Schwarzchild radius r_s. They are given by

$$\ell_p = \sqrt{\frac{hG}{2\pi c^n}}, \qquad \lambda_m = \frac{h}{mc}, \qquad r_s = \frac{2GM}{c^2}.$$

Here m and M are masses and c is the speed of light. Using only this information – that is, treating G and h as totally unknown quantities, in both magnitude and nature – deduce the value of n.

Q 10* The following table gives the observed values for the radius R of the fireball resulting from the explosion of an atomic bomb at sea-level, as a function of the time t since it was detonated.

$t\,(10^{-3}\,\mathrm{s})$	0.24	0.66	1.22	4.61	15.0	53.0
$R\,(\mathrm{m})$	19.9	31.9	41.0	67.3	106.5	175.0

Assuming that any numerical constants involved have values close to unity, estimate the energy E released in the explosion. (Ref. G.I. Taylor, *Proc. Roy. Soc. A*, **201**, p175, 1950)

Q 11 The relativistic Schrödinger equation for a spinless particle of mass m (the Klein–Gordon equation) is a (partial) differential equation describing how the relevant wave function, $\psi = \psi(x, y, z, t)$ develops in space and time. On one occasion, it was mistakenly written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{(mc^2)^2}{\hbar^2}\psi, \qquad (*)$$

where $\hbar = h/2\pi$. Locate the likely error and suggest how it arose.

Q 12* When an incompressible fluid of density ρ flows through a small hole of diameter d in a thin plane sheet, the volume flow rate R depends upon ρ, d, η the viscosity of the fluid, and p the pressure difference between the two sides of the sheet. Measurements of the flow rate are made using a particular fluid. What can now be predicted for a second fluid with twice the density of the first, but only one-third of its viscosity? The size of the hole remains unaltered, but the pressure difference may have to be adjusted.