
Contents

<i>Examples</i>	ix
<i>Preface</i>	xii
1 What Is an Exponential Family?	1
2 Examples of Exponential Families	6
2.1 Examples Important for the Sequel	6
2.2 Examples Less Important for the Sequel	15
2.3 Exercises	21
3 Regularity Conditions and Basic Properties	24
3.1 Regularity and Analytical Properties	24
3.2 Likelihood and Maximum Likelihood	31
3.3 Alternative Parameterizations	36
3.4 Solving Likelihood Equations Numerically	45
3.5 Conditional Inference for Canonical Parameter	46
3.6 Common Models as Examples	50
3.7 Completeness and Basu's Theorem	59
3.8 Mean Value Parameter and Cramér–Rao (In)equality	61
4 Asymptotic Properties of the MLE	64
4.1 Large Sample Asymptotics	64
4.2 Small Sample Refinement: Saddlepoint Approximations	70
5 Testing Model-Reducing Hypotheses	75
5.1 Exact Tests	76
5.2 Fisher's Exact Test for Independence, Homogeneity, Etc.	80
5.3 Further Remarks on Statistical Tests	84
5.4 Large Sample Approximation of the Exact Test	86
5.5 Asymptotically Equivalent Large Sample Tests	90
5.6 A Poisson Trick for Deriving Test Statistics	94

<i>Contents</i>		vii
6	Boltzmann's Law in Statistics	100
6.1	Microcanonical Distributions	100
6.2	Boltzmann's Law	102
6.3	Hypothesis Tests in a Microcanonical Setting	108
6.4	Statistical Reduncancy	109
6.5	A Modelling Exercise in the Light of Boltzmann's Law	114
7	Curved Exponential Families	118
7.1	Introductory Examples	118
7.2	Basic Theory for ML Estimation and Hypothesis Testing	124
7.3	Statistical Curvature	129
7.4	More on Multiple Roots	131
7.5	Conditional Inference in Curved Families	136
8	Extension to Incomplete Data	143
8.1	Examples	143
8.2	Basic Properties	147
8.3	The EM Algorithm	150
8.4	Large-Sample Tests	155
8.5	Incomplete Data from Curved Families	155
8.6	Blood Groups under Hardy–Weinberg Equilibrium	156
8.7	Hidden Markov Models	159
8.8	Gaussian Factor Analysis Models	161
9	Generalized Linear Models	164
9.1	Basic Examples and Basic Definition	164
9.2	Models without Dispersion Parameter	169
9.3	Models with Dispersion Parameter	175
9.4	Exponential Dispersion Models	181
9.5	Quasi-Likelihoods	183
9.6	GLMs versus Box–Cox Methodology	184
9.7	More Application Areas	186
10	Graphical Models for Conditional Independence Structures	191
10.1	Graphs for Conditional Independence	192
10.2	Graphical Gaussian Models	195
10.3	Graphical Models for Contingency Tables	201
10.4	Models for Mixed Discrete and Continuous Variates	205
11	Exponential Family Models for Social Networks	210
11.1	Social Networks	210
11.2	The First Model Stage: Bernoulli Graphs	211
11.3	Markov Random Graphs	212

viii	<i>Contents</i>	
11.4	Illustrative Toy Example, $n = 5$	218
11.5	Beyond Markov Models: General ERGM Type	225
12	Rasch Models for Item Response and Related Models	228
12.1	The Joint Model	229
12.2	The Conditional Model	231
12.3	Testing the Conditional Rasch Model Fit	234
12.4	Rasch Model Conditional Analysis by Log-Linear Models	239
12.5	Rasch Models for Polytomous Response	240
12.6	Factor Analysis Models for Binary Data	241
12.7	Models for Rank Data	243
13	Models for Processes in Space or Time	246
13.1	Models for Spatial Point Processes	246
13.2	Time Series Models	254
14	More Modelling Exercises	258
14.1	Genotypes under Hardy–Weinberg Equilibrium	258
14.2	Model for Controlled Multivariate Calibration	259
14.3	Refindings of Ringed Birds	259
14.4	Statistical Basis for Positron Emission Tomography	262
<i>Appendix A</i>	Statistical Concepts and Principles	265
<i>Appendix B</i>	Useful Mathematics	268
B.1	Some Useful Matrix Results	268
B.2	Some Useful Calculus Results	269
	<i>Bibliography</i>	271
	<i>Index</i>	278