Statistical Modelling by Exponential Families

This book is a readable, digestible introduction to exponential families, encompassing statistical models based on the most useful distributions in statistical theory, such as the normal, gamma, binomial, Poisson, and negative binomial. Strongly motivated by applications, it presents the essential theory and then demonstrates the theory's practical potential by connecting it with developments in areas such as item response analysis, social network models, conditional independence and latent variable structures, and point process models. Extensions to incomplete data models and generalized linear models are also included. In addition, the author gives a concise account of the philosophy of Per Martin-Löf in order to connect statistical modelling with ideas in statistical physics, such as Boltzmann's law. Written for graduate students and researchers with a background in basic statistical inference, the book includes a vast set of examples demonstrating models for applications and numerous exercises embedded within the text as well as at the ends of chapters.

ROLF SUNDBERG is Professor Emeritus of Statistical Science at Stockholm University. His work embraces both theoretical and applied statistics, including principles of statistics, exponential families, regression, chemometrics, stereology, survey sampling inference, molecular biology, and paleoclimatology. In 2003, he won (with M. Linder) the award for best theoretical paper in the *Journal of Chemometrics* for their work on multivariate calibration, and in 2017 he was named Statistician of the Year by the Swedish Statistical Society.

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To Margareta,
and to Per;
celebrating 50 years

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Preface

The theoretical importance of exponential families of distributions has long been evident. Every book on general statistical theory includes a few sections introducing exponential families. However, there are three gaps in the literature that I want to fill with the present text:

- It is the first book devoted wholly to exponential families and written at the Master's/PhD level. As a course book, it can stand by itself or supplement other literature on parametric statistical inference.
- It aims to demonstrate not only the essentials of the elegant and powerful general theory but also the extensive development of exponential and related models in a diverse range of applied disciplines.
- It gives an account of the half-century-old but still influential ideas of Per Martin-Löf about such models. Part of this innovative material has previously been available only in my handwritten lecture notes in Swedish from 1969–1970.

Two older books are devoted to the general theory of exponential families: Ole Barndorff-Nielsen's *Information and Exponential Families in Statistical Theory* (Barndorff-Nielsen, 1978) and Larry Brown's *Fundamentals of Statistical Exponential Families* (Brown, 1988). The present text is quite different in intention. The general presentation is less focused on mathematical rigour, and instead the text is much more directed toward applications (even though it does not include much real data). My ambition is not to present a self-contained stringent theory, which would have required more space and distracted the reader from my main messages. Thus I refer to other works for some proofs or details of proofs not short enough or instructive enough to be included.

Instead, in order to show the theory's potential, I make room to describe developments in numerous areas of applied modelling. The topics treated include incomplete data, generalized linear models, conditional independence and latent variable structures, social network models, models for item analysis and spatial point process models. The methodology of generalized linear models will take us a bit beyond exponential family theory. We will also be outside the scope of the standard exponential families when the parameter space or the data are incomplete, but not so far outside that the theory is not beneficial.

The inferential philosophy underlying this text is that of a frequentist statistician. In particular, conditional inference plays a substantial role in the text. This should not preclude potential interest from a Bayesian point of view, however. For both frequentists and Bayesians the likelihood is central, and it plays a dominant role in the present text.

This textbook contains more material than typically covered in a single course. My own proposed selection for a basic course on the topic would cover Chapters/Sections 1, 2.1, 3.1–6, 4.1, 5, 7.1–2, 8.1–3, and 9.1–3. The choice of additional material should be driven by the interests of students and/or the instructor. Instruction on exponential families can also be combined with other aspects of statistical theory. At Stockholm University, a theoretical graduate level course consists of exponential families jointly with more general theory of parametric inference.

As mentioned, the text is written with graduate students primarily in mind. It has a large number of examples and exercises, many of them appearing recurrently. They range from small instructional illustrations to application areas of independent interest and of great methodological or applied utility in their own right, represented by separate sections or chapters. This simplifies the choice of topics by taste. Chapter 2 introduces a large number of common and less common distributions and models. Most of these examples reappear in later chapters. Chapters 10–13 can be regarded as elaborated examples per se.

Exercises are typically found at the ends of sections, but there are also sections entirely devoted to exercises. Solutions to exercises are not included here, but can be found at www.cambridge.org/9781108701112.

As prerequisites, readers should have a general knowledge of the theory of statistics and experience with the use of statistical methods (the more, the better). Appendix A defines some statistical concepts and lists the principles assumed to be in the mind of the reader. Some particularly useful calculus and matrix algebra is found in the other short appendices.

Application of an exponential family model in practice typically requires a statistical computer package. I give some references to the huge, free set of **R** packages, but these references could be regarded as examples, because alternatives often exist in other environments, or even in **R** itself.

My original inspiration to study the theory and methodology of exponential families goes 50 years back in time, in particular to unpublished lectures given by Per Martin-Löf in 1969–1970 (Martin-Löf, 1970). While skipping some technicalities, I try to do justice to many of his original thoughts and deep results, in particular in Chapter 6. Per Martin-Löf's lecture notes were written in Swedish, which of course severely restricted their accessibility and influence outside the Scandinavian languages. Nevertheless, these notes have got about one hundred Google Scholar citations. The Swedish original is available on my department's website:
www.math.su.se/PML1970.

Implicit above is my deep gratitude to Per Martin-Löf for introducing me to the area, for suggesting a fruitful thesis topic and for his sharp mind challenging me to achieve a clear understanding myself.

As mentioned, earlier versions of this text have been used as course texts in my department during their development. I'm particularly grateful to Michael Höhle, who took over the course after me, and whose many questions and comments have led to important corrections and other improvements. I am also grateful to students commenting on previous versions, and to colleagues inside and outside Stockholm University for comments and stimulation.

I appreciate greatly the support of David Cox and Diana Gillooly, who have both been eager to see these notes published in the IMS Textbook series with Cambridge University Press. After many years the goal is now achieved, and I hope they like the result.

Finally, my deep thanks go to my wife Margareta, my wife since 50 years ago, without whose patience and support this project would not have come to completion.