

Contents

	<i>Preface</i>	page xiii
	<i>Nomenclature</i>	xvi
1	The Fundamental Classic Analysis of Edelbaum, Sackett and Malchow, with Additional Detailed Derivations and Extensions	1
	1.1 Introduction	1
	1.2 The Technique of Averaging	3
	1.3 Summary of the Mechanics of the Equinoctial Orbit Elements	4
	1.4 The Equations of Motion	8
	1.5 The Averaged State and Costate Differential Equations for the Thrust-Constrained Case $\hat{u} \perp \hat{r}$	10
	1.6 A Discussion of the Zero Roll and Zero Pitch Case and the Maximization of the Solar Panels' Power Output	13
	1.7 The Analysis of the Zero Roll Constraint with Otherwise Free Pitch and Free Yaw	16
	1.8 The Perturbations Due to the Oblateness of the Earth	25
	1.9 The Flux Model of SECKSPOT and the Augmented State and Adjoint Equations	27
	1.10 The Calculations of the Thrust Angle, the Panel Orientation Angle, the Sun Incidence Angle on the Panel, and the Three Sides of the Spacecraft Body, in SECKSPOT	31
	1.11 Examples of Minimum-Time Transfers from LEO to GEO	35
	1.12 A Discussion of the SECKSPOT Software Capabilities and Limitations	39
	References	49
2	The Analysis of the Six-Element Formulation	50
	2.1 Introduction	50
	2.2 The Edelbaum Low-Thrust Orbit Transfer Problem	50
	2.3 The Full Six-State Formulation Using Nonsingular Equinoctial Orbit Elements	66
	2.4 Orbit Transfer with Continuous Constant Acceleration	83
	References	101

3	Optimal Low-Thrust Rendezvous Using Equinoctial Orbit Elements	102
	3.1 Introduction to the Minimum-Time Rendezvous Problem	102
	3.2 The Differential Equations in Terms of the Equinoctial Elements	103
	3.3 The Euler–Lagrange Differential Equations	106
	3.4 Numerical Results	109
	3.5 Summary of the Minimum-Time Rendezvous Problem	112
	3.6 Minimum-Fuel Time-Fixed Rendezvous Using Constant Low-Thrust	113
	3.7 Introduction to the Minimum-Fuel Rendezvous Problem	114
	3.8 The Dynamic Equations for the Seven State Variables	117
	3.9 The Euler–Lagrange Differential Equations	119
	3.10 Examples	122
	3.11 Conclusion	125
	Appendix 3.1 The Partial Derivatives of the M Matrix	126
	References	133
4	Optimal Low-Thrust Transfer Using Variable Bounded Thrust	135
	4.1 Introduction	135
	4.2 The Optimization of the Thrust Magnitude	136
	4.3 A Simple Example of Rendezvous in Near-Circular Orbit	145
	4.4 Conclusion	150
	References	153
5	Minimum-Time Low-Thrust Rendezvous and Transfer Using Epoch Mean Longitude Formulation	154
	5.1 Introduction	154
	5.2 The Equations of Motion for the Epoch Mean Longitude Formulation	155
	5.3 The Variational Hamiltonian	160
	5.4 Canonical Transformations	162
	5.5 Boundary Conditions for Minimum-Time Rendezvous and Example of a Free–Free Minimum-Time Transfer	168
	5.6 Conclusion	170
	Appendix 5.1 The Nonzero Partial Derivatives of Matrix M	174
	References	178
6	Trajectory Optimization Using Eccentric Longitude Formulation	180
	6.1 Introduction	180
	6.2 Equations of Motion in Terms of the Eccentric Longitude	181
	6.3 Numerical Example	188
	6.4 Conclusion	191
	Appendix 6.1 The $\partial M/\partial \mathbf{z}$ Partial Derivatives	192
	Appendix 6.2 The $\partial M^F/\partial \mathbf{z}$ Partial Derivatives	197
	References	200

7	Low-Thrust Trajectory Optimization Based on Epoch Eccentric Longitude Formulation	202
	7.1 Introduction	202
	7.2 System and Adjoint Differential Equations for the Epoch Eccentric Longitude Formulation	204
	7.3 Transversality Condition for Minimum-Time Rendezvous and Examples of Free-Free Minimum-Time Transfer	213
	7.4 Conclusion	220
	Appendix 7.1 Partial Derivatives of M^{F_0} Matrix with Respect to Orbit Elements	220
	References	225
8	Mechanics of Trajectory Optimization Using Nonsingular Variational Equations in Polar Coordinates	227
	8.1 Introduction	227
	8.2 Dynamic and Adjoint Differential Equations in Polar Coordinates	228
	8.3 Numerical Example	236
	8.4 Conclusion	237
	Appendix 8.1 The B Matrix and its Partial Derivatives	237
	References	242
9	Trajectory Optimization Using Nonsingular Orbital Elements and True Longitude	243
	9.1 Introduction	243
	9.2 Equations of Motion with the True Longitude as the Sixth State Variable	244
	9.3 Numerical Results	250
	9.4 Conclusion	254
	Appendix 9.1 The B^L Matrix and its Partial Derivatives	255
	References	259
10	The Treatment of the Earth Oblateness Effect in Trajectory Optimization in Equinoctial Coordinates	260
	10.1 Introduction	260
	10.2 Resolution of the J_2 Acceleration in Terms of the Equinoctial Elements	261
	10.3 Minimum-Time Transfer Around the Oblate Earth	265
	10.4 The Thrust and J_2 -Perturbed Averaged Dynamic and Adjoint System of Equations	269
	10.5 Numerical Example	274
	10.6 Conclusion	278
	References	280
11	Minimum-Time Constant Acceleration Orbit Transfer with First-Order Oblateness Effect	281
	11.1 Introduction	281
	11.2 The Analysis of the Second Zonal Perturbation Effect in Minimum-Time Low-Thrust Transfers	282

x	Contents	
	11.3 The Averaged Rates of the Elements Due to J_2 and Their Partial Derivatives	295
	11.4 Conclusion	302
	References	303
12	The Streamlined and Complete Set of the Nonsingular J_2-Perturbed Dynamic and Adjoint Equations for Trajectory Optimization in Terms of Eccentric Longitude	304
	12.1 Introduction	304
	12.2 System and Adjoint Differential Equations in Terms of the Eccentric Longitude	306
	12.3 Accounting of the J_2 Perturbation	319
	12.4 Conclusion	325
	References	326
13	The Inclusion of the Higher-Order Harmonics in the Modeling of Optimal Low-Thrust Orbit Transfer	328
	13.1 Introduction	328
	13.2 Zonal Harmonics Perturbation Acceleration Components in the Euler–Hill Frame	330
	13.3 The Treatment of the J_3, J_4 Perturbations within the Eccentric Longitude Formulation	332
	13.4 The Treatment of the J_3, J_4 Perturbations within the True Longitude Formulation	345
	13.5 Numerical Results	353
	13.6 Conclusion	357
	Appendix 13.1 Transformation of the J_3, J_4 Inertial Accelerations to the Rotating Frame	358
	References	359
14	Analytic Expansions of Luni-Solar Gravity Perturbations Along Rotating Axes for Trajectory Optimization: The Dynamic System	361
	14.1 Introduction	361
	14.2 Solar Gravity Perturbation Acceleration Components in the Euler–Hill Frame	362
	14.3 Lunar Gravity Perturbation Acceleration Components in the Euler–Hill Frame	370
	14.4 The Use of de Pontécoulant’s Lunar Theory	374
	14.5 Numerical Results	377
	14.6 Conclusion	383
	References	389
15	Analytic Expansions of Luni-Solar Gravity Perturbations Along Rotating Axes for Trajectory Optimization: The Multipliers System and Simulations	391
	15.1 Introduction	391
	15.2 The Hamiltonian and Euler–Lagrange Equations	392

	15.3 The $\partial \mathbf{f}_\odot / \partial \mathbf{z}$ Partial Derivatives for the Solar Gravity	399
	15.4 The $\partial \mathbf{f}_\text{D} / \partial \mathbf{z}$ Partial Derivatives for the Lunar Gravity	401
	15.5 Transfer Simulations	404
	15.6 Conclusion	410
	Appendix 15.1 The Partial Derivatives of f_r^s , f_θ^s and f_h^s	410
	References	423
16	Fourth-Order Expansions of the Luni-Solar Gravity Perturbations along Rotating Axes for Trajectory Optimization	424
	16.1 Introduction	424
	16.2 The Extension to Fourth Order of the Luni-Solar Gravity Perturbation Accelerations	425
	16.3 Transfer Examples	431
	16.4 Conclusion	446
	Appendix 16.1 The Partial Derivatives of $(f_r^s)_4$, $(f_\theta^s)_4$ and $(f_h^s)_4$	446
	References	457
	<i>Index</i>	458