Applied Nonsingular Astrodynamics

Optimal Low-Thrust Orbit Transfer

This essential book describes the mathematical formulations and subsequent computer simulations required to accurately project the trajectory of spacecraft and rockets in space, using the formalism of optimal control for minimum-time transfer in general elliptic orbit. The material will aid research students in aerospace engineering, as well as practitioners in the field of spaceflight dynamics, in developing simulation software to carry out trade studies useful in vehicle and mission design. It will also teach them to develop flight software for operational applications in autonomous mode, to actually transfer space vehicles from one orbit to another. The hands-on real-life applications discussed will give all readers a clear understanding of the mathematics of orbit transfer, allow them to develop their own operational software to fly actual missions, and use the contents as a research tool to carry out even more complex analyses.

Jean Albert Kéchichian is a retired Engineering Specialist from *The Aerospace Corporation*. His career has included senior level engineering positions at *NASA's Jet Propulsion Laboratory* and at *Ford Aerospace*. His main areas of contribution are in spaceflight guidance and navigation. He is a Fellow of *The American Astronautical Society*, and his work has regularly appeared in *Acta Astronautica, Journal of Guidance Control and Dynamics, Journal of the Astronautical Sciences*, and *Journal of Spacecraft and Rockets*. He holds Degrees in Aeronautical and Mechanical Engineering from *l'Université de Liège, UC Berkeley*, and a PhD in Aeronautics and Astronautics from *Stanford University*.

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Applied Nonsingular Astrodynamics

Optimal Low-Thrust Orbit Transfer

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108472364
DOI: 10.1017/9781108560061

First published 2018

Printed in the United States of America by Sheridan Books, Inc.

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Kechichian, Jean Albert, 1945– author.
Title: Applied nonsingular astrodynamics optimal low-thrust orbit transfer /
Jean Albert Kechichian (The Aerospace Corporation (retired)).
Description: Cambridge : Cambridge University Press, [2018] | Series:
Cambridge aerospace series | Includes bibliographical references and index.
Identifiers: LCCN 2018015860 | ISBN 9781108472364 (alk. paper)
Subjects: LCSH: Orbital transfer (Space flight) | Trajectory optimization. | Space trajectories.
Classification: LCC TL1075 .K43 2018 | DDC 629.4/113–dc23
LC record available at https://lccn.loc.gov/2018015860

ISBN 978-1-108-47236-4 Hardback

Cambridge University Press
978-1-108-47236-4 — Applied Nonsingular Astrodynamics
Jean Albert Kéchichian
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To the memory of my beloved parents
Albert Kéchichian and Joséphine Séraydarian
blessed with extraordinary wisdom, and who placed selflessly my siblings and me at the gravitational center of their lives.

Contents

Preface

The objective of this comprehensive book is to lay out the mathematics of optimal transfer and rendezvous between any two general elliptic orbits, based on the use of nonsingular orbital elements in the form of the equinoctial elements, that lead to the casting of the system differential equations and their adjoints in a form that is free of singularities at the crucial zero eccentricity and zero inclination conditions, allowing for the numerical integration of these equations without computer overflow and accuracy loss.

As such, the contents of this book will assist research students in aerospace engineering, as well as practitioners in the field of spaceflight dynamics, in developing simulation software to carry out trade studies useful in vehicle and mission design, and develop flight software for operational applications in autonomous mode, to actually transfer space vehicles from one orbit to another – the most important applications being for transfers between low Earth orbit (LEO) and geostationary Earth orbit (GEO), using electric ion or arcjet thrusters, being clear that the use of the popular classical elements will inevitably lead to errors and even computer run terminations because these orbits exhibit circular and equatorial conditions leading to singularities in the description of the system equations.

The main contributions to the solution of optimal low-thrust orbit transfers were historically provided by a team of Scientists at the Massachusets Institute of Technology's Draper Laboratory in Cambridge, Massachusets, led by Theodore Edelbaum, who made use of the nonsingular dynamic equations of orbital flight developed by Dr. Paul Cefola also of the Draper Laboratory, and Professor Roger Broucke of the University of Texas, at Austin.

The long duration of these powered flights necessitated the use of the averaging technique in the development of fast computer codes in which large integration steps spanning several orbital revolutions were made possible, in order to solve the two-point boundary value problem defining the transfer between two given orbits.

This rapid integration and consequent iterations using Newton's scheme, coupled to an indirect optimization method, needed to converge onto the initial values of the Lagrange multipliers or adjoints, and therefore onto the optimal solutions, were made possible by the use of the equinoctial elements, and because these elements were orbital elements, the technique of averaging could be applied. It was then sufficient to disregard the actual position of the space vehicle along its current orbit due to the averaging of the equations of motion which then described only the size, shape and orientation of the running orbit.

This five-element system of dynamic and adjoint differential equations is described in the first chapter with all its details and derivations along with many extensions, especially concerning the certain additional real-world spacecraft attitude constraints that affect the thrust vector orientation at each instant of time during an actual transfer.

The full six-state system of dynamic and adjoint equations for precision integration is then developed in the following chapters, while still using the same rotating orbital frame, also called the equinoctial frame, for the component resolution of the thrust acceleration vector, as well as the particular set of the equinoctial elements used by Edelbaum. It is thus possible to simulate "exact" transfer trajectories by taking advantage of the increased computational speed of modern computers, instead of generating only the approximate solutions of the averaged system of equations.

Chapters 2, 3 and 4 revisit earlier contributions by providing more accurate and error-free minimum-time and minimum-fuel simulation examples with additional numerical results and graphical representations, as well as performance comparisons with both the simplified analytic Edelbaum theory and also the averaged solutions. Both constant thrust and variable bounded thrust examples are thus depicted.

Chapters 5, 6 and 7 introduce the use of the epoch mean longitude and epoch eccentric longitude variables as the sixth orbital element, showing how the system and adjoint equations are related with the original current-time counterparts, for both mutual verification of the mathematics, the numerical simulations, and for possibly additional benefits in computation times for minimum-fuel transfers involving several coasting arcs. The savings in computation time would occur because the running orbit in the fundamental epoch-based systems as well as the epoch longitudes stay constant during coast in the absence of any thrust. The epoch-based systems are also convenient in orbit determination applications.

Chapters 8 and 9 introduce the use of the rotating Euler–Hill frame for component resolution of the thrust acceleration vector, as well as the adoption of the true longitude as the sixth orbital element for further simplification of the mathematics and corresponding software, while Chapters 10, 11 and 12 show how the second zonal harmonic J_2 of the Earth gravity model is accounted for, both within the frameworks of precision integration and averaging, using a variety of different sets of equinoctial elements for mutual verification and validation.

Chapter 13 derives the augmented system by also considering the perturbation effects of the higher zonal harmonics, namely J_3 and J_4, while the remaining chapters develop the mathematics of the further augmented system encompassing the gravity perturbations due to the Sun and the Moon, by adopting the analytic lunar theory of de Pontécoulant. The accelerations due to these third bodies are given in mathematical form by series expansions that converge rapidly at the third or at most fourth terms of the expansions, as is shown in the last chapter. The precession of the current orbit during the transfer of the space vehicle, due to the gravity perturbations imparted by the third bodies, can help reduce the effort spent to carry out the transfer, as shown by the examples in the last chapters.

A special effort was made to render each chapter as self-contained as possible for ease of reading, even though this necessitated the repetition of some definitions, crucial equations, and other relevant material, with the aim of preventing the reader from needing to turn pages back and forth too often between various chapters.

That is also the reason why references are kept at the end of each chapter instead of displaying them at the end of the book. Thus the relevant references for each chapter are shown at the end of the particular chapter.

In order to validate each formulation mathematically, it is necessary to compare results for mutual verification and validation of the mathematics. For example, when the J_2 perturbation is first discussed in the book, the mean longitude formulation is used in Chapters 10 and 11, and the examples are compared later with formulations using the eccentric longitude in Chapter 12, and also with both the mean longitude and the true longitude formulations by also including the higher zonal harmonics J_3 and J_4 in Chapter 13, again for mutual verification of the mathematics. The true longitude formulation is what most current researchers are using in their studies. For Chapters 15 and 16, being rather large, it was decided to leave them separate. Readers would readily know that they are indeed related.

The analyses and simulations presented in this book were carried out in the Flight Mechanics and Astrodynamics Departments at the Aerospace Corporation in El Segundo, California between 1989 and 2011, under contract with the United States Air Force Space and Missile Systems Center.

Finally, it is appropriate to recognize the extraordinary teachers I was fortunate to be associated with, starting with the late Professors Baudouin Fraeijs de Veubeke, Jean Smolderen and Charles Massonnet at the Faculté des Sciences Appliquées de L'Université de Liège in Belgium, and later Professor John V. Breakwell at the Department of Aeronautics and Astronautics at Stanford University, who directed my thesis. The two classic theoretical books on Astronautics, by Dr. Richard Battin of the MIT Draper Laboratory and Dr. Jean-Pierre Marec of the French ONERA, namely *Astronautical Guidance* and *Optimal Space Trajectories*, respectively, with the latter initially published in 1973 as Lecture Notes at the Ecole Nationale Supérieure de L'Aéronautique et de L'Espace under the title *Trajectoires Spatiales Optimales*, were extremely influential and inspirational.

The challenging and difficult work of formatting the contents of this book were carried out by Mary Villanueva of the Aerospace Corporation in El Segundo, California, and the transcription of many of the figures into electronic Adobe Illustrator files was expertly carried out by Yvonne Crane of the Rand Corporation in Arlington, Virginia. Their contribution is gratefully acknowledged.

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Analytic Astronautics
La Canada, California

Cambridge University Press
978-1-108-47236-4 — Applied Nonsingular Astrodynamics
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More Information

Nomenclature

θ^*	true anomaly
a	semimajor axis
β	$1/(1+G)$
e	eccentricity
E	eccentric anomaly
$\mathbf{f}$	acceleration vector due to J_2 (context dependent)
$\mathbf{f}$	thrust vector, N
F	eccentric longitude $= E + \omega + \Omega$, rad
f	thrust magnitude, N
F'	$1 + G = 1/\beta$
$\hat{\mathbf{f}}, \hat{\mathbf{g}}, \hat{\mathbf{w}}$	unit vectors along axes of direct equinoctial frame
f_t	thrust acceleration, km/s^2
G	$(1 - h^2 - k^2)^{1/2}$
$\mathbf{g}$	gravity acceleration vector, km/s^2
h	$e \sin(\omega + \Omega)$
i	orbit inclination
J_2	Earth second zonal harmonic $= 1.08263 \times 10^{-3}$
K	$1 + p^2 + q^2$
k	$e \cos(\omega + \Omega)$
λ	mean longitude = $M + \omega + \Omega$
λ_0	mean longitude at epoch, $M_0 + \omega + \Omega$
L	true longitude, $\theta^* + \omega + \Omega$
M	mean anomaly
m	spacecraft mass, kg
μ	Earth gravity constant $= 398,601.3$ (km^3/s^2)
M_0	mean anomaly at epoch
n	orbit mean motion $= \mu^{1/2} a^{-3/2}$ (rad/s)
p	$\tan(i/2) \sin \Omega$
p'	semi-latus rectum $= a(1 - e^2)$
q	$\tan(i/2) \cos \Omega$
R	equatorial radius of the Earth $= 6378.14$ km
r	radial distance, km
$\mathbf{r}, \dot{\mathbf{r}}$	position and velocity vectors, km and km/s

$\ddot{\mathbf{r}}$	acceleration vector, km/s^2
$\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\mathbf{h}}$	unit vectors along axes of the Euler–Hill frame
s_F, c_F	$\sin F$, $\cos F$, etc.
$\hat{\mathbf{u}}$	unit vector in the direction of thrust
v	$\lvert\dot{\mathbf{r}}\rvert$, velocity vector magnitude, km/s
ω	argument of perigee
Ω	right ascension of ascending node