

Cambridge Elements

Elements in the Philosophy of Mathematics

edited by

Penelope Rush

University of Tasmania

Stewart Shapiro

The Ohio State University

A CONCISE HISTORY OF MATHEMATICS FOR PHILOSOPHERS

John Stillwell

University of San Francisco



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108456234
DOI: 10.1017/9781108610124

© John Stillwell 2019

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2019

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-45623-4 Paperback
ISSN 2399-2883 (online)
ISSN 2514-3808 (print)

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

A Concise History of Mathematics for Philosophers

Elements in the Philosophy of Mathematics

DOI: 10.1017/9781108610124
First published online: June 2019

John Stillwell
University of San Francisco

Author for correspondence: John Stillwell, stillwell@usfca.edu

Abstract: This Element aims to present an outline of mathematics and its history, with particular emphasis on events that shook up its philosophy. It ranges from the discovery of irrational numbers in ancient Greece to the nineteenth- and twentieth-century discoveries on the nature of infinity and proof. Recurring themes are intuition and logic, meaning and existence, and the discrete and the continuous. These themes have evolved under the influence of new mathematical discoveries, and the story of their evolution is, to a large extent, the story of philosophy of mathematics.

Keywords: mathematics, philosophy

© John Stillwell 2019

ISBNs: 9781108456234 (PB), 9781108610124 (OC)
ISSNs: 2399-2883 (online), 2514-3808 (print)

Contents

Preface	1
1 Irrational Numbers and Geometry	2
2 Infinity in Greek Mathematics	8
3 Imaginary Numbers	15
4 Calculus and Infinitesimals	21
5 Continuous Functions and Real Numbers	30
6 From Non-Euclidean Geometry to Arithmetic	35
7 Set Theory and Its Paradoxes	44
8 Formal Systems	51
9 Unsolvability and Incompleteness	57
Bibliography	66