

## Contents

	<i>Preface</i>	<i>page ix</i>
<b>1</b>	<b>Approximation of Univariate Functions</b>	<b>1</b>
1.1	Introduction	1
1.2	Trigonometric Polynomials	6
1.3	The Bernstein–Nicol’skii Inequalities. The Marcinkiewicz Theorem	17
1.4	Approximation of Functions in the Classes $W_{q,\alpha}^r$ and $H_q^r$	25
1.5	Historical Remarks	34
<b>2</b>	<b>Optimality and Other Properties of the Trigonometric System</b>	<b>36</b>
2.1	The Widths of the Classes $W_{q,\alpha}^r$ and $H_q^r$	36
2.2	Further Properties of the Trigonometric System	54
2.3	Approximation of Functions with Infinite Smoothness	62
2.4	Sampling and Numerical Integration	73
2.5	Historical Remarks	79
<b>3</b>	<b>Approximation of Functions from Anisotropic Sobolev and Nicol’skii Classes</b>	<b>81</b>
3.1	Introduction	81
3.2	Trigonometric Polynomials	82
3.3	The Bernstein–Nicol’skii Inequalities and Their Applications. A Generalization of the Marcinkiewicz Theorem	89
3.4	Approximation of Functions in the Classes $W_{\mathbf{q},\alpha}^r$ and $H_{\mathbf{q}}^r$	104
3.5	Estimates of the Widths of the Sobolev and Nicol’skii Classes	113
3.6	Sampling and Numerical Integration	121
3.7	Historical Remarks	126

vi	<i>Contents</i>	
<b>4</b>	<b>Hyperbolic Cross Approximation</b>	129
4.1	Introduction	129
4.2	Some Special Polynomials with Harmonics in Hyperbolic Crosses	138
4.3	The Bernstein–Nicol’skii Inequalities	151
4.4	Approximation of Functions in the Classes $\mathbf{W}_{q,\alpha}^r$ and $\mathbf{H}_q^r$	162
4.5	Some Further Remarks	186
4.6	Historical Comments	189
4.7	Open Problems	190
<b>5</b>	<b>The Widths of Classes of Functions with Mixed Smoothness</b>	191
5.1	Introduction	191
5.2	The Orthowidths of the Classes $\mathbf{W}_{q,\alpha}^r$ and $\mathbf{H}_q^r$	193
5.3	The Kolmogorov Widths of the Classes $\mathbf{W}_{q,\alpha}^r$ and $\mathbf{H}_q^r$	216
5.4	Universality of Approximation by Trigonometric Polynomials from the Hyperbolic Crosses	231
5.5	Historical Remarks	241
5.6	Open Problems	242
<b>6</b>	<b>Numerical Integration and Approximate Recovery</b>	244
6.1	Introduction	244
6.2	Cubature Formulas and Discrepancy	246
6.3	Optimal Cubature Formulas and Nonlinear Approximation	253
6.4	Lower Estimates	262
6.5	The Fibonacci Cubature Formulas	272
6.6	The Korobov Cubature Formulas	284
6.7	The Frolov Cubature Formulas	289
6.8	Universal Cubature Formulas	302
6.9	Recovery of Functions	305
6.10	Historical Notes, Comments, and Some Open Problems	315
6.11	Open Problems	320
<b>7</b>	<b>Entropy</b>	321
7.1	Introduction. Definitions and Some Simple Properties	321
7.2	Finite-Dimensional Spaces. Volume Estimates	323
7.3	Some Simple General Inequalities	325
7.4	An Inequality Between Entropy Numbers and Best $m$ -Term Approximations	328
7.5	Volume Estimates for Balls of Trigonometric Polynomials	333
7.6	Entropy Numbers of the Balls of Trigonometric Polynomials	345
7.7	Entropy Numbers for the $\mathbf{W}$ -Type Function Classes	363

<i>Contents</i>		vii
7.8	Entropy Numbers for the <b>H</b> -Type Function Classes	373
7.9	Discussion and Open Problems	380
7.10	Some Historical Comments	383
<b>8</b>	<b>Greedy Approximation</b>	387
8.1	Introduction	387
8.2	The Trigonometric System	394
8.3	Wavelet Bases	398
8.4	Some Inequalities for the Tensor Product of Greedy Bases	404
8.5	Weight-Greedy Bases	412
8.6	The Weak Chebyshev Greedy Algorithm	415
8.7	Sparse Approximation With Respect to General Dictionaries	422
8.8	Open Problems	447
<b>9</b>	<b>Sparse Approximation</b>	449
9.1	Introduction	449
9.2	Constructive Sparse Trigonometric Approximation	454
9.3	Constructive Sparse Trigonometric Approximation for Small Smoothness	472
9.4	Open Problems	495
9.5	Concluding Remarks	496
<i>Appendix</i>	<b>Classical Inequalities</b>	500
	<i>References</i>	520
	<i>Index</i>	532