EXTENSIONS OF $f(R)$ GRAVITY

Recent cosmological observations have posed a challenge for traditional theories of gravity: what is the force driving the accelerated expansion of the universe? What if dark energy or dark matter do not exist and what we observe is a modification of the gravitational interaction that dominates the universe at large scales?

Various extensions to Einstein's General Theory of Relativity have been proposed, and this book presents a detailed theoretical and phenomenological analysis of several leading, modified theories of gravity. Theories with generalized curvature-matter couplings are first explored, followed by hybrid metric-Palatini gravity. This timely book first discusses key motivations behind the development of these modified gravitational theories, before presenting a detailed overview of their subsequent development, mathematical structure, and cosmological and astrophysical implications.

Covering recent developments and with an emphasis on astrophysical and cosmological applications, this is the perfect text for graduate students and researchers.

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Extensions of $f(R)$ Gravity

Curvature-Matter Couplings and Hybrid Metric-Palatini Theory

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108428743
DOI: 10.1017/9781108645683

First published 2019

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Harko, Tiberiu, 1953– author. | Lobo, Francisco S. N., author.
Title: Extensions of f(R) gravity : curvature-matter couplings and hybrid metric-Palatini theory / Tiberiu Harko (University College London), Francisco S.N. Lobo (Universidade de Lisboa).
Description: Cambridge : Cambridge University Press, [2018] |
Series: Cambridge monographs on mathematical physics |
Includes bibliographical references and index.
Identifiers: LCCN 2018022347 | ISBN 9781108428743 (hardback : alk. paper)
Subjects: LCSH: Expanding universe. | Curvature cosmology. |
Large scale structure (Astronomy) | Gravitation. | Cosmology.
Classification: LCC QB991.E94 H37 2018 | DDC 523.1/8–dc23
LC record available at https://lccn.loc.gov/2018022347

ISBN 978-1-108-42874-3 Hardback

Cambridge University Press
978-1-108-42874-3 — Extensions of f(R) Gravity
Tiberiu Harko , Francisco S. N. Lobo
Frontmatter
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Preface

During the last few decades, cosmology has evolved from being mainly a theoretical area of physics to become a field supported by high-precision observational data. Recent experiments call upon state of the art technology in astronomy and astrophysics to provide detailed information on the contents and history of the universe, which has led to the measurement of parameters that describe our universe with increasing precision. The standard model of cosmology is remarkably successful in accounting for the observed features of the universe. However, a number of fundamental open questions remain at the foundation of the standard model. In particular, we lack a fundamental understanding of the recent acceleration of the universe [413, 436]. What is the so-called dark energy that is driving the cosmic acceleration? Is it vacuum energy or a dynamical field? Or is the acceleration due to infra-red modifications of Einstein's theory of General Relativity (GR)? How is structure formation affected in these alternative scenarios? What are the implications of this acceleration for the future of the universe?

The resolution of these fundamental questions is extremely important for theoretical cosmology. Dark energy models are usually assumed to be responsible for the acceleration of the cosmic expansion in most cosmological studies. However, it is clear that these questions involve not only gravity, but also particle physics. String theory provides a synthesis of these two branches of physics and is widely believed to be moving toward a viable quantum gravity theory. One of the key predictions of string theory is the existence of extra spatial dimensions. In the brane-world scenario, motivated by recent developments in string theory, the observed three-dimensional universe is embedded in a higher-dimensional spacetime [334]. The new degrees of freedom belong to the gravitational sector, and can be responsible for the late-time cosmic acceleration [162, 178]. On the other hand, generalizations of the Einstein–Hilbert Lagrangian, including quadratic Lagrangians, which involve second order curvature invariants, have also been extensively explored [103, 157, 324, 383, 471]. These modified theories of gravity not only provide an alternative explanation for the expansion history of the universe [99, 120, 377], but they also offer a paradigm fundamentally distinct from the simplest dark energy models of cosmic acceleration [140], even from those that perfectly mimic the same expansion history.

A large number of modified theories of gravity can be represented in a scalar-tensor formulation by means of appropriate metric rescalings and field redefinitions. It is, therefore, not surprising that we can think of scalar–tensor gravity theories as a first stepping stone to explore modifications of GR. They have the advantage of apparent simplicity and have been intensively analyzed in the literature. First proposed in its present form by Brans and Dicke for a single scalar field [88], they have been extensively generalized and have maintained the interest of researchers until the present day. For instance, with a conformal transformation, these theories can be recast as matter-interacting scalar fields in GR, where in this format, they play an important role in dark energy modeling. Recently, relative to scalar–tensor theory, much work has been invested in the Galileon models and their generalizations [161]. The latter models allow nonlinear derivative interactions of the scalar field in the Lagrangian and lead to second order field equations, thus removing any ghost-like instabilities. The Lagrangian was first written down by Horndeski in 1974 [254], which contains four arbitrary functions of the scalar field and its kinetic energy. The form of the Lagrangian is significantly simplified by requiring specific self-tuning properties, however, the screening is too effective, and will screen curvature from other matter sources as well as from the vacuum energy [130]. An alternative approach consists of searching for a de Sitter critical point for any kind of material content [342]. These models might alleviate the cosmological constant problem and can deliver a background dynamic that is compatible with the latest observational data.

Thus, a promising alternative to explain the late-time cosmic acceleration is to assume that at large scales Einstein's theory of GR breaks down, and a more general action describes the gravitational field. Thus, one may generalize the Einstein–Hilbert action by including second order curvature invariants such as R^2, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, $C^{\mu\nu\alpha\beta}C_{\mu\nu\alpha\beta}$, etc. Some of the physical motivations for these modifications of gravity were inspired on effective models raised in string theory, which indeed may lead to the possibility of a more realistic representation of the gravitational fields near curvature singularities [378]. Moreover, the quantization of fields in curved spacetimes tell us that the high-energy completion of the Einstein–Hilbert Lagrangian of GR involves higher-order terms on the curvature invariants above. This is in agreement with the results provided from the point of view of treating GR as an effective field theory [127]. Among these extensions of GR the so-called $f(R)$ gravity has drawn much attention over the last years, since it can reproduce the late-time cosmic acceleration, and, in spite of containing higher-order derivatives, it is free of the Ostrogradsky instability, as can be shown by its equivalence with scalar-tensor theories (for a review on $f(R)$ gravity see Refs. [103, 157, 324, 383, 471]).

Moreover, $f(R)$ gravity has also been proposed as a solution for the inflationary paradigm [37], where the so-called Starobinsky model is a successful proposal, since it satisfies the latest constraints released by Planck [10]. In addition, the equivalence of $f(R)$ gravity to some class of scalar–tensor theories has provided an extension of the so-called chameleon mechanism [279, 280] to $f(R)$ gravity, leading to some viable extensions of GR that pass the solar system constraints [260, 384]. Other alternative formulations for these extensions of GR have been considered in the literature, namely, the presence of nonminimal couplings between the scalar curvature and the matter Lagrangian density [44, 237]. Here, it was shown that an explicit coupling between an arbitrary function of the scalar curvature R and the Lagrangian density of matter generates a nonvanishing covariant derivative of the energy-momentum tensor, which implies non-geodesic motion and consequently leads to the appearance of an extra force [44]. These theories and generalizations will be extensively analyzed in Part II of the book. Another interesting approach to modified gravity involves the Palatini formalism, where the metric and affine connection are regarded as independent degrees of freedom, which yields an interesting phenomenology for cosmology [396]; and the metric-affine formalism, where the matter part of the action now depends and is varied with respect to the connection [472]. Recently, a novel approach to modified theories of gravity was proposed that consists of adding to the metric Einstein–Hilbert Lagrangian an $f(R)$ term constructed a la Palatini [224]. It was shown that the theory can pass the solar system observational constraints even if the scalar field is very light. This implies the existence of a long-range scalar field, which is able to modify the cosmological and galactic dynamics, but leaves the solar system unaffected. This hybrid metric-Palatini gravitational theory will be extensively analyzed in the Part III of the book.

Note that these modified theories of gravity are focused on extensions of the curvature-based Einstein–Hilbert action. Nevertheless, one could equally well modify gravity starting from its torsion-based formulation and, in particular, from the Teleparallel Equivalent of General Relativity (TEGR) [318]. The interesting point is that although GR is completely equivalent with TEGR at the level of the field equations, their modifications (for instance $f(R)$ and $f(T)$ gravities, where T is the torsion) are not equivalent, and they correspond to different classes of gravitational modifications. Hence, $f(T)$ gravity has novel and interesting cosmological implications, capable of describing inflation, the late-time acceleration, large scale structure, bouncing solutions, nonminimal couplings to matter, etc. [96, 245, 244].

An extremely important aspect of modern cosmology is the synergy between theory and observations. Dark energy models and modified gravity affect the geometry of the universe and cosmological structure formation, impacting the background expansion and leaving an imprint on the statistical properties of the large-scale structure. There are a number of well-established probes of cosmic evolution, such as type Ia supernovae, baryon acoustic oscillations (BAO), weak gravitational lensing, galaxy clustering, and galaxy clusters properties [504]. Different methods measure different observables, probing expansion and structure formation in different and often complementary ways and have different systematic effects. In particular, joint analyses with Cosmic Microwave Background (CMB) data are helpful in breaking degeneracies by constraining the standard cosmological parameters. Indeed, CMB has revolutionized the way we perceive the universe. The information encoded in its temperature and polarization maps provides one of the strongest evidences in favor of the hot Big Bang theory and has enabled ways to constrain cosmological models with unprecedented accuracy [11].

In this work, we perform a detailed theoretical and phenomenological analysis of specific modified theories of gravity and investigate their astrophysical and cosmological applications. The theories that we present are essentially two largely explored extensions of $f(R)$ gravity, namely: (i) modified gravity with curvature-matter coupling; and (ii) the hybrid metric-Palatini gravitational theory. The extensions can be read independently of one another. This book is outlined in the following manner:

1. As this work is also aimed at the advanced undergraduate level, in Part I, we present a review of the General Theory of Relativity and of $f(R)$ gravity. In Chapter 1 we present an Introduction to GR, and in Chapter 2, we review the mathematical formalism in order to describe the gravitational field in the framework of GR. The Einstein gravitational field equations, which are defined in a Riemannian geometry, establish a deep connection between the geometric properties of the spacetime, and its matter content, provide a full description of both the geometric characteristics of the spacetime and of the dynamics of particles. In Chapter 3, we derive the Einstein field equations, through the variational principle, and discuss their mathematical properties, as well as some of their physical implications, such as the weak field limit and the gravitational field equations in spherical symmetry. In Chapter 4, we review the fundamental observational tests of GR at the solar system level. Furthermore, we will discuss some of the basic properties of compact, stellar-type astrophysical objects, and we will provide a brief introduction to the description of the accretion processes. In Chapter 5, we review some of the basic results and models in modern cosmology. We start our investigation in the framework of Newtonian cosmology, whose results, interestingly enough, coincide, in some particular situations, with those of GR. We will then proceed to a brief presentation of the general relativistic cosmology, and of its more significant results, focusing on the problems of dark matter and of dark energy. Finally, in Chapter 6, we go beyond Einstein's General Relativity and briefly review the basic mathematical formalism of $f(R)$ modified gravity theory, and we will discuss some of its cosmological and

astrophysical applications. This chapter will serve as an introduction and as a bridge to the following parts of the book.

2. In Part II, we review a plethora of modified theories of gravity with generalized curvature-matter couplings. The explicit nonminimal couplings – for instance, between an arbitrary function of the scalar curvature R and the Lagrangian density of matter – induce a nonvanishing covariant derivative of the energy-momentum tensor, implying non-geodesic motion and consequently leading to the appearance of an extra force. Applied to the cosmological context, these curvature-matter coupling lead to interesting phenomenology, where one can obtain a unified description of the cosmological epochs. We also consider the possibility that the behavior of the galactic flat rotation curves can be explained in the framework of the curvature-matter coupling models, where the extra-terms in the gravitational field equations modify the equations of motion of test particles, and induce a supplementary gravitational interaction.

   Thus, we review these generalized curvature-matter coupling modified theories of gravity in Part II, which is outlined in the following manner: In Chapter 7, we extensively motivate the approach, and in Chapter 8, we introduce the linear curvature-matter coupling and present some of its interesting features. In Chapter 9, we generalize the latter linear curvature-matter coupling by considering the maximal extension of the Einstein–Hilbert action, which results in the $f(R, L_m)$ gravitational theory, and extend the theory with the inclusion of general scalar field and kinetic term dependencies. In Chapter 10, we consider another extension of GR, namely, $f(R,T)$ modified theories of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of the energy-momentum tensor T. Furthermore, we include an explicit invariant Ricci-energy-momentum tensor coupling, and explore some of its astrophysical and cosmological phenomenology. This latter theory is motivated as considering a traceless energy-momentum tensor, $T = 0$, whereby the gravitational field equations for the $f(R,T)$ theory reduce to that of $f(R)$ gravity and all nonminimal couplings of gravity to the matter field vanish, while the inclusion of the $R_{\mu\nu}T^{\mu\nu}$ term still allows a nonminimal coupling. The possibility of explaining dark matter as a consequence of the curvature-matter coupling is considered in Chapter 11. In Chapter 12, we consider the application of the thermodynamics of open systems for the physical interpretation of modified gravity with a curvature-matter coupling, namely, for $f(R, L_m)$ and $f(R,T)$ theories of gravity. We begin our analysis by presenting the basic ideas of the irreversible thermodynamics and of the description of matter creation in a full covariant formulation. In Chapter 13, we present a brief introduction to the study of the quantum cosmology of $f(R,T)$ gravity, and some of its physical and theoretical implications. In Chapter 14, we present the theoretical and cosmological consequences of the modification of gravity that can be obtained by considering the quantum fluctuations of the gravitational metric, where the basic idea of this approach is the assumption that the quantum metric can be generally represented as the sum of a classical and of a fluctuating part, the latter being of a quantum (or stochastic) nature. Finally, in Chapter 15, we conclude.

3. In Part III, we explore another extension of $f(R)$ gravity, which has been motivated by the possibility of accounting for the self-accelerated cosmic expansion without invoking dark energy sources. Indeed, it has been established that both metric and Palatini versions of $f(R)$ gravity have interesting features but also manifest severe and different downsides. A hybrid combination of theories, containing elements from both of these formalisms, turns out to be very successful in accounting for the observed phenomenology, and it is able to avoid some drawbacks of the original approaches. Thus, the second part of this work explores the formulation of this hybrid metric-Palatini approach and its main achievements in passing the local tests and in applications to astrophysical and cosmological scenarios, where it provides a unified approach to the problems of dark energy and dark matter. Thus, we review the formulation and the main applications of hybrid gravity models in late-time cosmological and astrophysical scenarios.

   In Chapter 16, we start the discussion, and in Chapter 17 we consider the action and the field equations of the hybrid metric-Palatini formalism. In particular, we discuss the scalar–tensor representation, the Cauchy problem, and more general hybrid theories. Chapter 18 is devoted to hybrid-gravity cosmology. We derive the Friedmann equations, construct the related dynamical system, and briefly consider some solutions. Furthermore, we analyze the cosmological perturbations in order to understand structure formation in these theories. We focus on the evolution of perturbations in the matter dominated era and vacuum fluctuations relevant to inflation. Chapter 19 is dedicated to the astrophysical applications of the hybrid metric-Palatini gravitational theory. More specifically, the weak field behavior that is crucial for the solar system precision tests of gravity is considered. We also discuss the galactic phenomenology and the astrophysical applications of hybrid gravity as an alternative to dark matter. In particular, we study the stellar dynamics and the theory of orbits, the generalization of the virial theorem, the flat rotation curves of spiral galaxies, and the galactic clusters starting from the relativistic Boltzmann equation for collisionless systems of particles. In Chapter 20, we investigate the properties of the relativistic high density compact astrophysical stellar objects, and in Chapter 21, we analyze wormhole geometries in the scalar–tensor formulation of hybrid metric-Palatini gravity, and conclude in Chapter 22.

The definitions used throughout this book are the following:

We consider the $(-,+,+,+)$ metric signature.

The *Riemann curvature tensor*, $R^{\sigma}_{\lambda\mu\nu}$, a fourth order tensor, is defined in terms of the Christoffel symbols as

$$R^{\sigma}{}_{\lambda\mu\nu} = \frac{\partial \Gamma^{\sigma}_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\sigma}_{\lambda\mu}}{\partial x^{\nu}} + \Gamma^{\tau}_{\nu\lambda}\Gamma^{\sigma}_{\mu\tau} - \Gamma^{\tau}_{\mu\lambda}\Gamma^{\sigma}_{\nu\tau}.$$

From the Riemann tensor, by contraction, we obtain a second order tensor $R_{\lambda\nu}$, called the *Ricci tensor*, given by

$$R_{\lambda\nu} = R^{\sigma}_{\lambda\sigma\nu} = \frac{\partial \Gamma^{\sigma}_{\lambda\nu}}{\partial x^{\sigma}} - \frac{\partial \Gamma^{\sigma}_{\lambda\sigma}}{\partial x^{\nu}} + \Gamma^{\tau}_{\nu\lambda}\Gamma^{\sigma}_{\sigma\tau} - \Gamma^{\tau}_{\sigma\lambda}\Gamma^{\sigma}_{\nu\tau}.$$

Contracting the Ricci tensor gives the *Ricci scalar*, defined as

$$R = R^{\lambda}_{\lambda} = g^{\lambda\mu}R_{\lambda\mu} = g^{\lambda\nu}g^{\mu\sigma}R_{\lambda\mu\nu\sigma}.$$

Acknowledgments

We would like to thank our collaborators who have accompanied us in the elaboration of the theories presented in this book, in particular, Orfeu Bertolami, Christian Boehmer, Salvatore Capozziello, Bogdan Danilă, Zahra Haghani, Tomi Koivisto, Zoltán Kovács, José Lemos, Shidong Liang, Xing Liu, Man Kwong Mak, José Pedro Mimoso, Shin'ichi Nojiri, Sergey D. Odintsov, Gonzalo J. Olmo, Jorge Páramos, Diego Pavón, João Luís Rosa, Shahab Shahidi, Sergey Sushkov, Stefano Vignolo, and Min-Xing Xu.

We are very grateful to Bogdan Danilă for his help in the preparation of the figures. TH would like to thank the Yat-Sen School, of the Sun Yat-Sen University in Guangzhou, P. R. China, for the kind hospitality offered during the last stages of the writing of the book. FSNL also acknowledges financial support of the Fundação para a Ciência e Tecnologia through an Investigador FCT Research contract, with reference IF/00859/2012, funded by FCT/MCTES (Portugal).