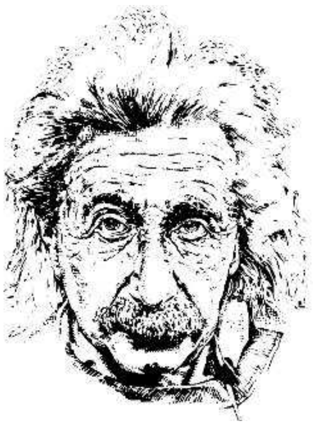

1 Nature of Light

"They could but make the best of it and went around with woebegone faces, sadly complaining that on Mondays, Wednesdays, and Fridays, they must look on light as a wave; on Tuesdays, Thursdays, and Saturdays, as a particle. On Sundays, they simply prayed."

The Strange Story of the Quantum
Banesh Hoffmann, 1947

INTRODUCTION

The words cited above—taken from a 1947 popular primer on the quantum world—delighted many readers who were just then coming into contact with ideas related to the nature of light and quanta. Hoffmann's amusing and informative account—involving in part the wave-particle twins "tweedledum" and "tweedledee"—captured nicely the level of frustration felt in those days about the true nature of light. And today, some 60 years later, the puzzle of tweedledum and tweedledee lingers. What is light? What is a photon? Indeed, in October of 2003, The Optical Society of America devoted a special issue of *Optics and Photonic News* to the topic "The Nature of Light: What is a Photon?" In this issue,[1] a number of renowned scientists, through five penetrating essays, accepted the challenge of describing the photon. Said Arthur Zajonc, in his lead article titled "Light Reconsidered":

> Light is an obvious feature of everyday life, and yet light's true nature has eluded us for centuries. Near the end of his life, Albert Einstein wrote, "All the 50 years of conscious brooding have brought me no closer to the answer to the question: What are light quanta?" We are today in the same state of "learned ignorance" with respect to light as was Einstein.

[1]"The Nature of Light: What is a Photon?" *OPN Trends*, Vol 3., No. 1, October 2003.

The evolution in our understanding of the physical nature of light forms one of the most fascinating accounts in the history of science. Since the dawn of modern science in the sixteenth and seventeenth centuries, light has been pictured either as particles or waves—seemingly incompatible models—each of which enjoyed a period of prominence among the scientific community. In the twentieth century it became clear that somehow light was both wave and particle, yet it was precisely neither. For some time this perplexing state of affairs, referred to as the *wave-particle duality*, motivated the greatest scientific minds of our age to find a resolution to these apparently contradictory models of light. In a formal sense, the solution was achieved through the creation of *quantum electrodynamics*, one of the most successful theoretical structures in the annals of physics. However, many scientists would agree, a comfortable *understanding* of the true nature of light is somewhat more elusive.

In our account of the developing understanding of light and photons, we will be content to sketch briefly a few of the high points. Certain areas of physics once considered to be disciplines apart from optics—electricity and magnetism, and atomic physics—are very much involved in this account. This alone suggests that the resolution achieved also constitutes one of the great unifications in our understanding of the physical world. The final result is that light and subatomic particles, like electrons, are both considered to be manifestations of energy and are governed by the same set of formal principles. In this introductory chapter, we begin with a brief history of light, addressing it alternately as particle and wave. Along the way we meet the great minds that championed one viewpoint or the other. We follow this account with several basic relationships—borrowed from quantum physics and the special theory of relativity—that describe the properties of subatomic particles, like electrons, and the *photon*. We close this chapter with an introductory glance at the electromagnetic spectrum and a survey of the *radiometric* units we use to describe the properties of electromagnetic radiation.

1-1 A BRIEF HISTORY[2]

In the seventeenth century the most prominent advocate of a *particle theory* of light was Isaac Newton, the same creative giant who had erected a complete science of mechanics and gravity. In his treatise *Optics*, Newton clearly regarded rays of light as streams of very small particles emitted from a source of light and traveling in straight lines. Although Newton often argued forcefully for positing hypotheses that were derived only from observation and experiment, here he himself adopted a particle hypothesis, believing it to be adequately justified by his experience. Important in his considerations was the observation that light seemed to cast sharp *shadows* of objects, in contrast to water and sound waves, which bend around obstacles in their paths. At the same time, Newton was aware of the phenomenon now referred to as *Newton's rings*. Such light patterns are not easily explained by viewing light as a stream of particles traveling in straight lines. Newton maintained his basic particle hypothesis, however, and explained the phenomenon by endowing the particles themselves with what he called "fits of easy reflection and easy transmission," a kind of periodic motion due to the attractive and repulsive forces imposed by material obstacles. Newton's eminence as a scientist was such that his point of view dominated the century that followed his work.

Christian Huygens, a Dutch scientist contemporary with Newton, championed a view (in his *Treatise on Light*) that considered light as a wave, spreading out from a light source in all directions and propagating through an all-pervasive elastic medium called the *ether*. He was impressed, for example,

[2]A more in-depth historical account may be found, for example, in Vasco Ronchi, *The Nature of Light* (Cambridge: Harvard University Press, 1970).

by the experimental fact that when two beams of light intersected, they emerged unmodified, just as in the case of two water or sound waves. Adopting a wave theory, Huygens was able to derive the *laws of reflection and refraction* and to explain *double refraction* in calcite as well.

Within two years of the centenary of the publication of Newton's *Optics*, the Englishman Thomas Young performed a decisive experiment that seemed to demand a wave interpretation, turning the tide of support to the wave theory of light. It was the *double-slit experiment*, in which an opaque screen with two small, closely spaced openings was illuminated by monochromatic light from a small source. The "shadows" observed formed a complex interference pattern like those produced with water waves.

Victories for the wave theory continued up to the twentieth century. In the mood of scientific confidence that characterized the latter part of the nineteenth century, there was little doubt that light, like most other classical areas of physics, was well understood. In 1821, Augustin Fresnel published results of his experiments and analysis, which required that light be a transverse wave. On this basis, double refraction in calcite could be understood as a phenomenon involving *polarized light*. It had been assumed that light waves in an ether were necessarily longitudinal, like sound waves in a fluid, which cannot support transverse vibrations. For each of the two components of polarized light, Fresnel developed the *Fresnel equations*, which give the amplitude of light reflected and transmitted at a plane interface separating two optical media.

Working in the field of electricity and magnetism, James Clerk Maxwell synthesized known principles in his set of four *Maxwell equations*. The equations yielded a prediction for the speed of an *electromagnetic wave* in the ether that turned out to be the *measured speed of light*, suggesting its electromagnetic character. From then on, light was viewed as a particular region of the electromagnetic spectrum of radiation. The experiment (1887) of Albert Michelson and Edward Morley, which attempted to detect optically the earth's motion through the ether, and the special theory of relativity (1905) of Albert Einstein were of monumental importance. Together they led inevitably to the conclusion that the assumption of an ether was superfluous. The problems associated with transverse vibrations of a *wave in a fluid* thus vanished.

If the nineteenth century served to place the wave theory of light on a firm foundation, that foundation was to crumble as the century came to an end. The wave-particle controversy was resumed with vigor. Again, we mention only briefly some of the key events along the way. Difficulties in the wave theory seemed to show up in situations that involved the *interaction of light with matter*. In 1900, at the very dawn of the twentieth century, Max Planck announced at a meeting of the German Physical Society that he was able to derive the correct blackbody radiation spectrum only by making the curious assumption that atoms emitted light in discrete energy chunks rather than in a continuous manner. Thus *quanta* and *quantum mechanics* were born. According to Planck, the energy E of a quantum of electromagnetic radiation is proportional to the frequency ν of the radiation:

$$E = h\nu \tag{1-1}$$

where the constant of proportionality h, *Planck's constant*, has the very small value of 6.63×10^{-34} J-s. Five years later, in the same year that he published his theory of special relativity, Albert Einstein offered an explanation of the *photoelectric effect*, the emission of electrons from a metal surface when irradiated with light. Central to his explanation was the conception of light as a stream of light quanta whose energy is related to frequency by Planck's equation (1-1). Then in 1913, the Danish physicist Niels Bohr once more incorporated the *quantum of radiation* in his explanation of the *emission and absorption* processes of the hydrogen atom, providing a physical basis for understanding the hydrogen spectrum. Again in 1922, the model of light quanta came to the rescue for Arthur Compton, who explained the scattering of X-rays from electrons as particle-like collisions between light quanta and electrons in which both energy and momentum were conserved. In 1926, the chemist Gilbert Lewis suggested the name "photon" for the "quantum of light" and it has been so identified ever since.

All such victories for the photon or particle model of light indicated that light could be treated as a kind of particle, possessing both energy and momentum. It was Louis de Broglie who saw the other side of the picture. In 1924, he published his speculations that subatomic *particles* are endowed with *wave properties*. He suggested, in fact, that a particle with momentum p had an associated wavelength of

$$\lambda = \frac{h}{p} \tag{1-2}$$

where h was, again, Planck's constant. Experimental confirmation of de Broglie's hypothesis appeared during the years 1927–1928, when Clinton Davisson and Lester Germer in the United States and Sir George Thomson in England performed experiments that could only be interpreted as *the diffraction of a beam of electrons*.

Thus, the wave-particle duality came full circle. *Light behaves like waves in its propagation and in the phenomena of interference and diffraction; however, it exhibits particle-like behavior when exchanging energy with matter, as in the Compton and photoelectric effects.* Similarly, electrons often behaved like particles, as observed in the pointlike scintillations of a phosphor exposed to a beam of electrons; in other situations they were found to behave like waves, as in the diffraction produced by an electron microscope.

1-2 PARTICLES AND PHOTONS

Photons and electrons that behaved both as particles and as waves seemed at first an impossible contradiction, since particles and waves are very different entities indeed. Gradually it became clear, to a large extent through the reflections of Niels Bohr and especially in his *principle of complementarity*, that photons and electrons were neither waves nor particles, but something more complex than either.

In attempting to explain physical phenomena, it is natural that we appeal to well-known physical models like waves and particles. As it turns out, however, the complete nature of a photon or an electron is not exhausted by either model. In certain situations, wavelike attributes may predominate; in other situations, particle-like attributes stand out. We know of no simpler physical model that is adequate to handle all cases.

Quantum mechanics describes both light and matter and, together with special relativity, predicts that the momentum, p, wavelength, λ, and speed, v, for both material particles and photons are given by the same general equations:

$$p = \frac{\sqrt{E^2 - m^2c^4}}{c} \tag{1-3}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m^2c^4}} \tag{1-4}$$

$$v = \frac{pc^2}{E} = c\sqrt{1 - \frac{m^2c^4}{E^2}} \tag{1-5}$$

In these equations, m is the *rest mass* and E is the *total energy*, the sum of the rest-mass energy mc^2 and kinetic energy E_K, that is, the work done to accelerate the particle from rest to its measured speed. The proper expression for kinetic energy is no longer simply $E_K = \frac{1}{2}mv^2$, but rather is $E_K = mc^2(\gamma - 1)$, where $\gamma = 1/\sqrt{1 - (v^2/c^2)}$. This relativistic expression[3] for kinetic energy E_K approaches $\frac{1}{2}mv^2$ for $v \ll c$.

A crucial difference between particles like electrons and neutrons and particles like photons is that the latter have *zero rest mass*. Equations (1-3) to (1-5) then take the simpler forms for photons:

$$p = \frac{E}{c} \tag{1-6}$$

$$\lambda = \frac{h}{p} = \frac{hc}{E} \tag{1-7}$$

$$v = \frac{pc^2}{E} = c \tag{1-8}$$

Thus, while nonzero rest-mass particles like electrons have a *limiting* speed of c, Eq. (1-8) shows that zero rest-mass particles like photons must travel with the constant speed c. The energy of a photon is not a function of its speed but rather of its frequency, as expressed in Eq. (1-1) or in Eqs. (1-6) and (1-7), taken together. Notice that for a photon, because of its zero rest mass, there is no distinction between its total energy and its kinetic energy. The following example helps clarify the differences in the momentum, wavelength, and speed of electrons and photons of the same total energy.

Example 1-1

An electron is accelerated to a kinetic energy E_K of 2.5 MeV. (a) Determine its relativistic momentum, de Broglie wavelength, and speed. (b) Determine the same properties for a photon having the same total energy as the electron.

Solution

The electron's total energy E must be the sum of its rest mass energy mc^2 and its kinetic energy E_K. The rest mass energy is
$mc^2 = (9.11 \times 10^{-31}\text{ kg})(3 \times 10^8\text{ m/s})^2 = 8.19 \times 10^{-14}\text{ J}$.
Since $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$, we have $mc^2 = 5.11 \times 10^5\text{ eV} = 0.511\text{ MeV}$. Thus,

$$E = mc^2 + E_K = 0.511\text{ MeV} + 2.5\text{ MeV} = 3.011\text{ MeV}$$

or

$$E = 3.011 \times 10^6\text{ eV} \times (1.602 \times 10^{-19}\text{ J/eV}) = 4.82 \times 10^{-13}\text{ J}$$

The other quantities are then calculated in order. Working with SI units we obtain, from Eq. (1-3):

$$p = \frac{\sqrt{E^2 - (mc^2)}}{c} = \frac{\sqrt{(4.82 \times 10^{-13}\text{ J})^2 - (8.19 \times 10^{-14}\text{ J})^2}}{3 \times 10^8\text{ m/s}}$$
$$= 1.58 \times 10^{-21}\text{ kg·m/s}$$

from Eq. (1-4):

[3]This discussion is not meant to be a condensed tutorial on relativistic mechanics, but, with the help of Eqs. (1-3) to (1-8), a summary of some basic relations that unify particles of matter and light.

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{1.58 \times 10^{-21}\ \text{kg}\cdot\text{m/s}} = 4.19 \times 10^{-13}\ \text{m} = 0.419\ \text{pm}$$

and from Eq. (1-5):

$$v = \frac{pc^2}{E} = \frac{(1.58 \times 10^{-21}\ \text{kg}\cdot\text{m/s})(3 \times 10^8\ \text{m/s})^2}{4.82 \times 10^{-13}\ \text{J}}$$

$$= 2.95 \times 10^8\ \text{m/s}$$

For the photon, with $m = 0$, we get instead,
from Eq. (1-6):

$$p = \frac{E}{c} = \frac{4.82 \times 10^{-13}\ \text{J}}{3 \times 10^8\ \text{m/s}} = 1.61 \times 10^{-21}\ \text{kg}\cdot\text{m/s}$$

from Eq. (1-7):

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{1.61 \times 10^{-21}\ \text{kg}\cdot\text{m/s}} = 0.412\ \text{pm}$$

and from Eq. (1-8):

$$v = c = 3.00 \times 10^8\ \text{m/s}$$

There is another important distinction between electrons and photons. Electrons obey *Fermi-Dirac statistics*, whereas photons obey *Bose-Einstein statistics*. A consequence of Fermi-Dirac statistics is that no two electrons in the same interacting system can be in the same *state*, that is, have precisely the same physical properties. Bose-Einstein statistics impose no such prohibition, so that identical photons with the same energy, momentum, and polarization can occur together in large numbers, as they do, for example, in a laser cavity.

In the theory called *quantum electrodynamics*, which combines the principles of quantum mechanics with those of special relativity, photons interact only with charges. An electron, for example, is capable of both absorbing and emitting a photon. There is no conservation law for photons as there is for the charge associated with particles. As indicated in the preceding example, in this theory the wave-particle duality becomes reconciled in the sense that both classical waves (i.e., light) and classical particles (i.e., electrons) are seen to have the same basic nature, which is neither wholly wave nor wholly particle. Essential distinctions between photons and electrons are removed and both are subject to the same general principles. Nevertheless, the complementary aspects of particle and wave descriptions of light remain, justifying our use of one or the other when appropriate. The wave description of light will be found adequate to describe most of the optical phenomena treated in this text.

In this brief comparison we have remarked on some of the differences and similarities between classical particles and light and have provided several fundamental relations that apply to both. The notion that light interacts with matter by exchanging photons of definite energy, momentum, and polarization will serve us well several chapters hence when we consider laser operation.

1-3 THE ELECTROMAGNETIC SPECTRUM

In this text we are concerned with the properties and applications of light. Following the pivotal work of James Clerk Maxwell, for whom the equations that govern electricity and magnetism are named, "light" is identified as an *electromagnetic wave* having a frequency in the range that human eyes can detect and interpret. All electromagnetic waves are made up of time-varying electric and magnetic fields. Electromagnetic (EM) waves are produced by

Cambridge University Press
978-1-108-42826-2 — Introduction to Optics
Frank L. Pedrotti , Leno M. Pedrotti , Leno S. Pedrotti
Excerpt
More Information

accelerating charge distributions, carry energy, and exert forces on charged particles upon which they impinge. The properties of electromagnetic waves are discussed in more detail in Chapter 4 and elsewhere throughout this text. In the last two sections of this introductory chapter, we wish to introduce only the most basic characteristics of electromagnetic waves.

In free (that is, empty) space all electromagnetic waves travel with the same speed, commonly given the symbol c. This speed emerges naturally from Maxwell's equations and is given, approximately, as $c = 3 \times 10^8$ m/s. As with all waves, the frequency of an electromagnetic wave is determined by the frequency of the source of the wave. An electromagnetic disturbance that propagates through space as a wave may be *monochromatic*, that is, characterized for practical purposes by a single frequency, or *polychromatic*, in which case it is represented by many frequencies, either discrete or in a continuum. The distribution of energy among the various constituent waves is called the *spectrum* of the radiation, and the adjective *spectral* implies a dependence on wavelength. Various regions of the *electromagnetic spectrum* are referred to by particular names, such as *radio waves*, *cosmic rays*, *light*, and *ultraviolet radiation*, because of differences in the way they are produced or detected. Most of the common descriptions of the various frequency ranges are given in Figure 1-1, in which the electromagnetic spectrum is

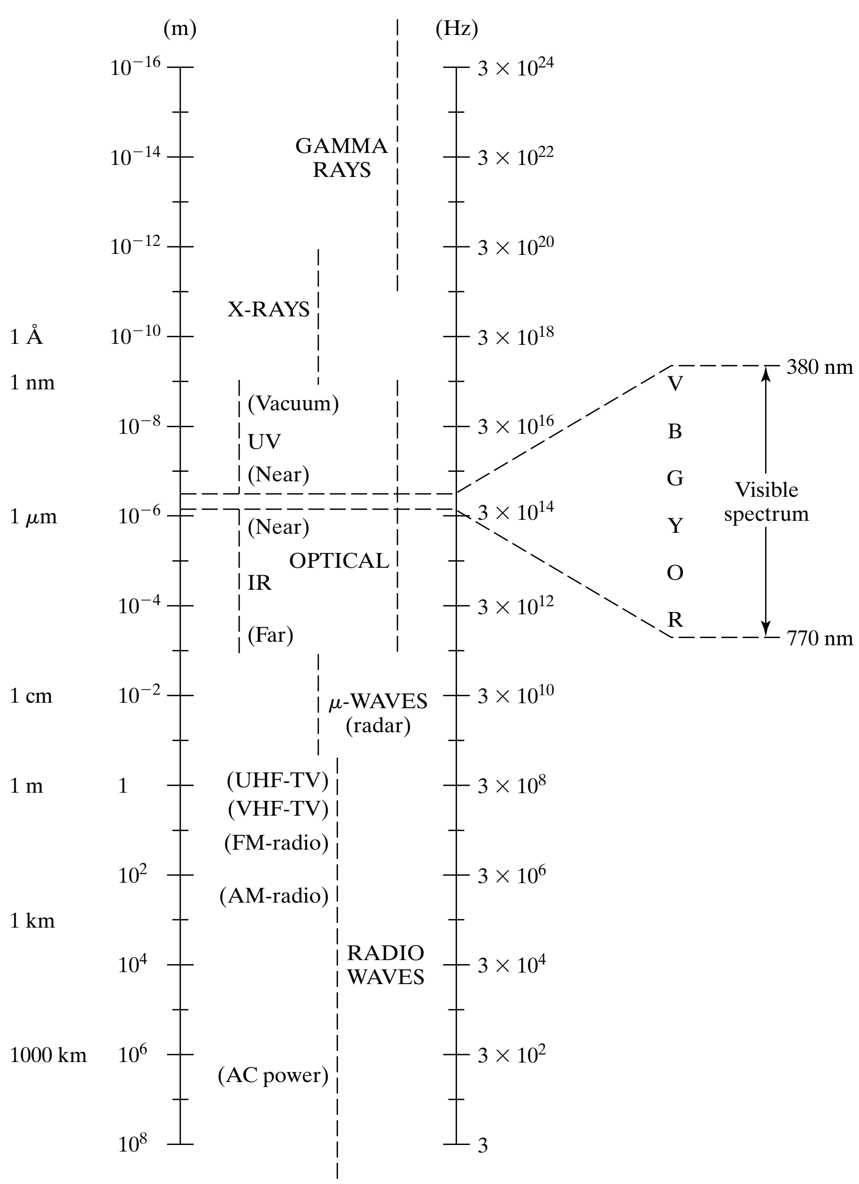

Figure 1-1 Electromagnetic spectrum, arranged by wavelength in meters and frequency in hertz. The narrow portion occupied by the visible spectrum is highlighted.

displayed in terms of both frequency ν and wavelength, λ. Recall that these two quantities are related, as with all types of wave motion, through the velocity c:

$$c = \lambda\nu \tag{1-9}$$

As indicated in Figure 1-1, common units for wavelength are the *angstrom* ($1\ \text{Å} = 10^{-10}$ m), the *nanometer* ($1\ \text{nm} = 10^{-9}$ m), and the *micrometer* ($1\ \mu\text{m} = 10^{-6}$ m). The regions ascribed to various types of waves, as shown, are not precisely bounded. Regions may overlap, as in the case of the continuum from X-rays to gamma rays. The choice of label will depend on the manner in which the radiation is either produced or used. The narrow range of electromagnetic waves from approximately 380 to 770 nm is capable of producing a visual sensation in the human eye and is properly referred to as "light." It is not surprising that this *visible region* of the spectrum corresponds to the frequencies of electromagnetic radiation that predominate in the output of the sun. Humans "see" different wavelengths of light as different colors. The visible spectrum of colors ranges from red (long-wavelength end) to violet (short-wavelength end) and is bounded by the invisible *ultraviolet* and *infrared* regions, as shown. The three regions taken together comprise the *optical spectrum*, that region of the electromagnetic spectrum of special interest in a textbook on optics. In addition, atoms and molecules have resonant frequencies in this optical spectrum and so EM waves in this frequency range interact most strongly with atoms and molecules. We now provide brief sketches of the various invisible regions of the electromagnetic spectrum.

Ultraviolet

On the short-wavelength side of visible light, this electromagnetic region spans wavelengths ranging from 380 nm down to 10 nm. Ultraviolet light is sometimes subdivided into three categories: UV-A refers to the wavelength range 380–315 nm, UV-B to the range 315–280 nm and UV-C to the range 280–10 nm. The sun emits significant amounts of electromagnetic radiation in all three UV bands but, due to absorption in the *ozone layer* of the earth's atmosphere, roughly 99% of the UV radiation that reaches the earth's surface is in the UV-A band. UV radiation from the sun is linked to a variety of health risks. UV-A radiation, generally regarded as the least harmful of the three UV bands, can contribute to skin aging and is possibly linked to some forms of skin cancer. UV-A radiation does not contribute to sunburns. UV-B radiation has been linked to a variety of skin cancers and contributes to the sunburning process. The link between UV-B radiation and skin cancer is a primary reason for the concern related to ozone depletion, which is believed to be in part caused by human use of so-called chlorofluorocarbon (CFC) compounds. Ozone (O_3) is formed when UV-C radiation reacts with oxygen in the stratosphere and, as mentioned, plays an important role in the filtering of UV-B and UV-C from the electromagnetic radiation that reaches the earth's surface. CFC compounds can participate in chemical processes that lead to the conversion of ozone into "ordinary" oxygen (O_2). The concern that CFCs and other similar chemicals may contribute to the depletion of the ozone layer and thus increase the risk of skin cancer led to protocols calling for the reduction of the use of refrigerants, aerosol sprays, and other products that release these chemicals into the atmosphere. Sunblock and sunscreen lotions are intended in part to block harmful UV-B radiation. On the other hand, UV radiation has the beneficial effect of inducing vitamin D production in the skin.

X-rays

X-rays are EM waves with wavelengths in the 10 nm to 10^{-4} nm range. These can be produced when high-energy electrons strike a metal target and are used as diagnostic tools in medicine to see bone structure and as treatments for certain cancers. X-ray diffraction serves as a probe of the lattice structure of crystalline solids and X-ray telescopes provide important information from astronomical objects.

Gamma Rays

This type of EM radiation has its origin in nuclear radioactive decay and certain other nuclear reactions. Gamma rays have very short wavelengths in the range from 0.1 nm to 10^{-14} nm. Like X-rays, penetrating gamma rays find use in the medical area, often in the treatment of localized cancers.

Infrared Radiation

On the long-wavelength side of the visible spectrum, infrared (IR) radiation has wavelengths spanning the region from 770 nm to 1 mm. Objects in thermal equilibrium at terrestrial temperatures emit radiation that has its energy output peak in the IR range. Consequently, infrared radiation is sometimes termed "heat radiation" and finds application in nightvision scopes that detect the IR emitted from objects in absolute "darkness" and in infrared photography wherein objects at different temperatures (and so with different peak wavelengths of emitted radiation) are imaged as areas of contrasting brightness. Images such as these can be used to map the temperature variation across the surface of the earth, for example. Infrared radiation is used as a treatment for sore muscles and joints, and, more recently, lasers that emit IR radiation have been used to treat the eye for vision abnormalities. Infrared radiation is also used in optical fiber communication systems and in a variety of remote control devices.

Microwaves

Beyond infrared radiation we find microwaves, with wavelengths from 1 mm to 30 cm or so. Microwave ovens, which have become a common kitchen appliance, use microwaves to heat food. In addition, microwaves play an important role in radar systems both on the ground and in the air, in telecommunications, and in spectroscopy.

Radio Waves

Radio waves are long-wavelength EM radiations produced, for example, by electrons oscillating in conductors that form antennas of various shapes. Radio waves have wavelengths ranging from meters to thousands of meters. Used commonly in radio and television broadcasts, they include the AM radio band (540–1600 kHz) with wavelengths ranging from 188 to 556 m as well as the FM radio band (88–108 MHz) with wavelengths from 2.78 to 3.41 m.

We have indicated that EM waves may lose and gain energy only in discrete amounts that are multiples of the energy associated with the energy quanta that have come to be called photons. Equation (1-1) gives the energy of a photon as $h\nu$. When EM wave energy is detected, the detector can record only energies that are multiples of a photon's energy. As the following example indicates, for macroscopic light sources, the energy of a photon is typically far less than the total detected energy, and so, in such a case, the restriction that the detected energy must be only a multiple of a photon's energy goes unnoticed. Since the energy per photon decreases with increased wavelength, for a given total energy, the energy "graininess" is less for long-wavelength radiation than for short-wavelength radiation. To understand the interaction of light with individual atoms and molecules, it is important to keep in mind that EM waves gain and lose energy in discrete amounts proportional to the frequency of the radiation. Consider the following example.

Example 1-2

A certain sensitive radar receiver detects an electromagnetic signal of frequency 100 MHz and power (energy/time) 6.63×10^{-16} J/s.

a. What is the wavelength of a photon with this frequency?
b. What is the energy of a photon in this signal? Express this energy in J and in eV.
c. How many photons/s would arrive at the receiver in this signal?
d. What is the energy (in J and in eV) of a visible photon of wavelength 555 nm?
e. How many visible ($\lambda = 555$ nm) photons/s would correspond to a detected power of 6.63×10^{-16} J/s?
f. What is the energy (in J and in eV) of an X-ray of wavelength 0.1 nm?
g. How many X-ray ($\lambda = 0.1$ nm) photons/s would correspond to a detected power of 6.63×10^{-16} J/s?

Solution

a. $$\lambda = c/\nu = \frac{3 \times 10^8 \text{ m/s}}{100 \times 10^6 \text{ Hz}} = 3 \text{ m}$$

b. $$E = h\nu = (6.63 \times 10^{-34} \text{ J}\cdot\text{s})(100 \times 10^6 \text{ Hz}) = 6.63 \times 10^{-26} \text{ J}$$

$$E = 6.63 \times 10^{-26} \text{ J}\left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}\right) = 4.14 \times 10^{-7} \text{ eV}$$

c. The number of detected photons per second N would be

$$N = \frac{\text{Power}}{\text{Energy/photon}} = \frac{6.63 \times 10^{-16} \text{ J/s}}{6.3 \times 10^{-26} \text{ J}} = 10^{10}/\text{s}$$

So each photon contributes but one part in 10 billion of the total power in the radar wave even for this very weak signal. In such a case, the "graininess" of the power in the signal is likely to go undetected.

d. $$E_{555} = h\nu = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{555 \times 10^{-9} \text{ m}}$$

$$= 3.58 \times 10^{-19} \text{ J} = 2.2 \text{ eV}$$

e. $N_{555} = \dfrac{\text{Power}}{\text{Energy/photon}} = \dfrac{6.63 \times 10^{-16} \text{ J/s}}{3.58 \times 10^{-19} \text{ J}} = 1850/\text{s}$. The effect of addition or removal of a single photon would perhaps be noticeable.

f. $$E_{\text{X-ray}} = h\nu = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{0.1 \times 10^{-9} \text{ m}}$$

$$= 1.99 \times 10^{-15} \text{ J} = 12{,}400 \text{ eV}$$

g. $$N_{\text{X-ray}} = \frac{\text{Power}}{\text{Energy/photon}} = \frac{6.63 \times 10^{-16} \text{ J/s}}{1.99 \times 10^{-15} \text{ J}} = 0.33/\text{s}.$$

One X-ray would be detected every 3 seconds or so. The discreteness of the energy of light quanta would be very evident in this case.