

# Imaging Optics

This comprehensive and self-contained text presents the fundamentals of optical imaging from the viewpoint of both ray and wave optics, within a single volume. Comprising three distinct parts, it opens with an introduction to electromagnetic theory, including electromagnetic diffraction problems and how they can be solved with the aid of standard numerical methods such as RCWA or FDTD. The second part is devoted to the basic theory of geometrical optics and the study of optical aberrations inherent in imaging systems, including large-scale telescopes and high-resolution projection lenses. A detailed overview of state-of-the-art optical system design provides readers with the necessary tools to successfully use commercial optical design software. The final part explores diffraction theory and concludes with vectorial wave propagation, image formation and image detection in high-aperture imaging systems. The wide-ranging perspective of this important book provides researchers and professionals with a comprehensive and rigorous treatise on the theoretical and applied aspects of optical imaging.

**Joseph Braat** is Emeritus Professor of Optics at the Delft Technical University, The Netherlands, and a Fellow of the Royal Netherlands Academy of Arts and Sciences. Previously he was based at the Philips Research Laboratories in Eindhoven where he worked on optical disc systems for video and audio recording and on high-resolution optical lithography. His further research interests are diffraction theory, astronomical imaging and optical metrology.

**Peter Török** is Professor of Optical Physics at the Nanyang Technological University, Singapore and at Imperial College London, UK. His research is focused on the theory of diffraction, focusing and microscopy, with particular emphasis on confocal microscopy, spectroscopic imaging and polarisation. Throughout the years he has taught vector calculus, electromagnetism, optical imaging and optical design theory.

Cambridge University Press  
978-1-108-42808-8 — Imaging Optics  
Joseph Braat , Peter Török  
Frontmatter  
[More Information](#)

---

# Imaging Optics

JOSEPH BRAAT

Delft Technical University, The Netherlands

PETER TÖRÖK

Nanyang Technological University, Singapore

Imperial College of Science, Technology and Medicine, London



CAMBRIDGE  
UNIVERSITY PRESS

**CAMBRIDGE**  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781108428088](http://www.cambridge.org/9781108428088)

DOI: 10.1017/9781108552264

© Joseph Braat and Peter Török 2019

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2019

Printed in the United Kingdom by TJ International Ltd, Padstow Cornwall

*A catalogue record for this publication is available from the British Library.*

ISBN 978-1-108-42808-8 Hardback

Additional resources for this publication at [www.cambridge.org/imagingoptics](http://www.cambridge.org/imagingoptics).

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

# Contents

<i>Preface</i>	<i>page</i> ix
<i>Acknowledgements</i>	xi
<b>I Electromagnetic Theory in the Optical Domain</b>	
<b>1 Electromagnetic Wave Propagation in Isotropic Media</b>	<b>3</b>
1.1 Introduction	3
1.2 Maxwell's Equations as Experimental Laws	3
1.3 Maxwell's Equations in the Optical Domain	9
1.4 Electromagnetic Energy Density and Energy Transport	10
1.5 Potential Functions and the Electromagnetic Field Vectors	13
1.6 Harmonic Solutions and the Helmholtz Equation	18
1.7 Gaussian Beams	35
1.8 Wave Propagation at an Interface between Two Media	51
1.9 Transmission and Reflection in a Stratified Medium	68
1.10 Multilayer Reflection and Transmission Coefficients	72
1.11 The Scattering Matrix and the Impedance Matrix Formalism	81
1.12 Stratified Medium with Laterally Modulated Periodic Sublayers	87
<b>2 Wave Propagation in Anisotropic Media</b>	<b>107</b>
2.1 Introduction	107
2.2 Harmonic Electromagnetic Waves in an Anisotropic Medium	108
2.3 Plane Wave Solutions in Uniaxial and Biaxial Media	112
2.4 Solution of the Helmholtz Equation in an Anisotropic Medium	117
2.5 Energy Transport in a Medium with Linear Anisotropy	124
2.6 Eigenvalue Equations Involving the Ray Vector $\hat{s}$	127
2.7 The Functions $v_p(\hat{\mathbf{k}})$ , $n_p(\hat{\mathbf{k}})$ , $v_r(\hat{s})$ and $n_r(\hat{s})$	133
2.8 Conical Refraction	138
2.9 Optical Activity	143
2.10 Wave Propagation in an Anisotropic Medium Including Rotation	146
2.11 Energy Propagation in a General Anisotropic Medium	149
2.12 Reflection and Refraction at an Interface in Anisotropic Media	154
<b>3 Surface Waves, Metamaterials and Perfect Imaging</b>	<b>160</b>
3.1 Eigenmodes of a Metal–Dielectric Interface	161
3.2 Wave Propagation in Metamaterials	168
3.3 The Concept of the Perfect Lens	179

<b>II Geometrical Theory of Optical Imaging</b>	
<b>4 Foundations of Geometrical Optics</b>	191
4.1 Introduction	191
4.2 Geometrical Optics Derived from Maxwell's Equations	193
4.3 Characteristic Function of an Optical System	202
4.4 Angle Characteristic Function of a Single Surface	207
4.5 The First-order Angle Characteristic and the Paraxial Domain	210
4.6 Stigmatic Imaging and the Angle Characteristic Function	212
4.7 Construction of the Angle Characteristic Function of a System	214
4.8 Isoplanatism and Aplanatism of an Optical System	217
4.9 The Definition of Transverse and Wavefront Deviation	223
4.10 Paraxial Optics and the Matrix Analysis of Optical Systems	230
4.11 Radiometry and Photometry	257
<b>5 Aberration Analysis of Optical Systems</b>	266
5.1 Introduction	266
5.2 Classification of Aberrations	276
5.3 Calculation of the Seidel Aberration Coefficients	292
5.4 Aberration of a Thin Lens	309
5.5 Seidel Aberrations of a Plane-parallel Plate	320
5.6 Chromatic Aberration	324
5.7 Finite Ray-tracing	338
5.8 Total Aberration at a Single Surface; Formulas of Hopkins and Welford	357
5.9 Aperture- and Field-dependent Aberration Function of an Imaging System	359
5.10 Paraxial and Finite Ray-tracing in Inhomogeneous Media	372
5.11 Polarisation Ray-tracing in Anisotropic Media	378
<b>6 Analytic Design and Optimisation of Optical Systems</b>	383
6.1 Introduction	383
6.2 Analytic Aberration-free Design of an Optical System	384
6.3 Merit Function of an Optical System	395
6.4 Optimisation of Optical Systems	399
6.5 Optical Tolerancing	413
<b>7 Design Methods for Optical Imaging Systems</b>	425
7.1 Introduction	425
7.2 The Achromatic Doublet	426
7.3 The Photographic Landscape Lens	432
7.4 The Portrait Lens	443
7.5 Flat-field Imaging Systems	451
7.6 The Astronomical Telescope	474
7.7 Microscope Optics	496
7.8 Aspheric Objectives for Optical Disc Systems	504
7.9 Large-field Projection Systems with Diffraction-limited Quality	523
<b>III Diffraction Theory of Optical Imaging</b>	
<b>8 Vectorial and Scalar Theory of Diffraction and Focusing</b>	545
8.1 Foundation of Vector Diffraction	545

8.2	Boundary Value Problems in Diffraction	559
8.3	The Debye–Wolf and Related Diffraction Theories	562
8.4	Scalar Diffraction Theories	568
8.5	The Validity of the Debye–Wolf Theory	580
<b>9</b>	<b>The Aberrated Scalar and Vector Point-spread Function</b>	<b>582</b>
9.1	Introduction	582
9.2	Pupil Function Expansion Using Zernike Polynomials	584
9.3	The Point-spread Function and the Nijboer–Zernike Diffraction Theory	591
9.4	The Extended Nijboer–Zernike Diffraction Theory	609
9.5	Vector Point-spread Function and the ENZ Diffraction Theory	614
9.6	Energy and Momentum Density and Their Flow Components	640
<b>10</b>	<b>Frequency Analysis of Optical Imaging</b>	<b>657</b>
10.1	Introduction	657
10.2	Optical Transfer Function of a Classical Wide-field Imaging System	660
10.3	Frequency Transfer by a Scanning Imaging System	710
10.4	The Three-dimensional Transfer Function	725
10.5	Light Scattering and Frequency Transfer	767
<b>11</b>	<b>Theory of Vector Imaging</b>	<b>782</b>
11.1	Vector Ray Tracing – The Generalised Jones Matrix Formalism	783
11.2	Vectorial Point-spread Function	787
11.3	Focusing of Partially Coherent, Partially Polarised Light	794
11.4	Properties of High-numerical-aperture Imaging Systems	804
11.5	High-aperture Scanning Light Microscopes Imaging a Point Object	814
11.6	Theory of Multiphoton Fluorescence Microscopes	832
11.7	Extension of the Imaging Theory to More Complicated Optical Systems	847
11.8	Imaging of Arbitrary Objects	855
	<i>Appendix A Fourier Analysis, Complex Notation and Vector Formulas</i>	860
	<i>Appendix B Phase and Group Velocity of a Wave Packet</i>	879
	<i>Appendix C The Kramers–Kronig Dispersion Relations</i>	882
	<i>Appendix D Zernike Polynomials</i>	888
	<i>Appendix E Magnetically Induced Optical Rotation (Faraday Effect)</i>	907
	<i>Appendix F Vector Point-spread Function in a Multilayer Structure</i>	913
	<i>Appendix G V. S. Ignatowsky: Diffraction by a Lens of Arbitrary Aperture</i>	919
	<i>References*</i>	945
	<i>Author Index</i>	959
	<i>Subject Index</i>	963

Cambridge University Press  
978-1-108-42808-8 — Imaging Optics  
Joseph Braat , Peter Török  
Frontmatter  
[More Information](#)

---



## Preface

The idea of writing a specific book on ‘imaging optics’ came some ten years ago and was spurred by the experience which the present authors had acquired in teaching at a university and in guiding research of M.Sc. and Ph.D. students. We noticed that the more advanced optical subjects which have to be mastered by these students to successfully accomplish their studies are rather scattered over the literature. We felt that a comprehensive, well-organised book on the theory and practice of optical imaging, using the same notation and conventions for the various subjects, was lacking. The present book, which has been conceived in the past nine years, should make it easier for students to acquire specific knowledge in the field of optical imaging.

The book comprises three parts. The first introductory part provides the physical basis of optics by means of Maxwell’s equations and applies these equations to wave propagation in free space and to refraction and reflection at interfaces between media. A special topic is the propagation of fundamental and higher-order Gaussian beams. The principles needed for solving diffraction problems are explained with special attention to wave propagation and diffraction in stratified media. The rigorous coupled wave analysis and the finite-difference time-domain method are treated in some detail. The chapter on wave propagation in anisotropic media focuses on linear and circular birefringence as a preparation for polarisation aspects in imaging which are encountered in Part III of the book. Emphasis is put on the intriguing effect of conical refraction. Combined with the chapter on surface waves, the reader acquires a good overview of light diffraction and light scattering at an object surface or in an object volume of which an image has to be formed by the optical imaging system.

The second part of the book is devoted to geometrical optics, aberration theory and optical design. It provides the reader with a theoretical basis of ray optics and illustrates the limits on imaging quality based on this simplified light propagation model. Paraxial optics is treated by means of the matrix theory of refraction/reflection and ray propagation. An extensive chapter on aberration theory applied to a single surface, a single lens and to entire systems shows the practical limitations in imaging quality of an optical system. Throughout this chapter, the fundamental diffraction unsharpness in image space and the image blurring due to geometrical aberrations are jointly evaluated. In some cases the (partial) suppression of aberration in an optical system can be achieved by analytic methods. These methods are presented in some detail, together with the more widely used numerical optimisation methods. Imaging quality of an optical system can be further reduced by manufacturing errors. A statistical analysis is presented of the influence of opto-mechanical and mounting errors of lens elements and surfaces such that the expected quality of a real-world imaging system can be evaluated as well as the spread around this value. The second part ends with a longer chapter on optical design methods applied to a wide variety of low- and high-aperture optical imaging systems.

The diffraction of light is the subject of the third part of the book. Based on Maxwell’s equations, the first chapter starts with an in-depth treatment of vector diffraction models which are then, step by step, reduced to the older scalar diffraction theories. Various intermediate stages of approximation between the rigorous vector model and the simplest scalar diffraction model are presented such that the reader can decide which approximation is adequate for a specific diffraction problem at hand. The point-spread function, a basic building block for the construction of the image intensity of a composite object, is discussed for an ideal and an aberrated imaging system. The classical scalar diffraction theory of Zernike and Nijboer is used for the diffraction analysis of low-aperture, aberrated imaging systems. The region of validity of this classical diffraction theory is then extended to a much larger focal volume (Extended Nijboer–Zernike theory) and provides the reader with semi-analytic results which can replace the numerical methods used for the evaluation of a general diffraction integral. The extension of this theory to ideal and aberrated point-spread functions of imaging systems with high-aperture serves to describe image formation in non-ideal high-resolution imaging systems. The chapter on point-spread functions ends with a detailed vector-based analysis of the propagation of energy, linear momentum and

angular momentum in a high-aperture focused beam. Spatial-frequency analysis of the imaging by an optical system is the subject of a chapter in which the influence of the object illumination on the image is also studied. The van Cittert–Zernike coherence theorem is presented and applied to a certain number of experimental configurations. The classical two-dimensional frequency analysis is extended to the imaging of three-dimensional (volumetric) objects or object surfaces. The influence of light scattering in the optical system on the spatial frequency transfer from object to image space concludes this chapter. The final chapter of Part III discusses the systematic analysis of (vector) imaging systems. The general state of polarisation of the light radiated by the object is defined as well as the possible anisotropy of the imaging system itself, of specially inserted birefringent elements, polarisers, etc. The light propagation from object to image space is described by means of a modular, matrix-based model of wave propagation. The detection of light in image space is performed by means of a polarisation-dependent detector array in a high-aperture imaging geometry.

A number of appendices have been added and they explain, in more detail than was possible in the main body of the book, a certain number of basic definitions or analytic/numerical tools which are frequently used throughout the book. We mention the first appendix which emphasises the role of Fourier methods in modern optics. The fourth appendix provides the reader with an overview of the properties and the applications of Zernike polynomials in optics. A special appendix contains the English translation of an influential publication in the Russian language by V.S. Ignatowsky, dating back to the beginning of the twentieth century. It presents an analysis of the electromagnetic field in the focal region of a high-aperture beam focused by a lens. This publication has inspired many later researchers in this field.

The overview of subjects which is given above also shows which material associated with (classical) imaging optics is definitely missing from this book. Only incidentally and without much detail, we mention a few of the modern methods in low- and high-aperture imaging. Many acronyms circulate in the literature for special imaging methods adapted to a special type of object, illumination, state of polarisation, spectral composition of the light, interferometric detection mode, etc. Simply because of the size of this book, these methods could not be included. Another interesting topic that is missing is the (unique) retrieval of object properties from one or several recordings of an image of the object. This subject, both from the experimental and the numerical point of view, shows interesting progress, also with respect to the high-aperture imaging geometry.

The writing of a book requires continuous concentration on the subject and in this respect the first author (J.B.) was privileged because of his retired status. The absence of time-consuming managerial tasks, of proposal writing and of regular work on national or international committees permitted a permanent focus on book writing. The second author (P.T.) could not benefit from these favourable circumstances and his contribution has remained relatively small. Sections 1.2 and 1.5 and Subsection 1.8.3 of Chapter 1 and the entire Chapters 8 and 11 of the book bear his signature. The remaining part of the book and the Appendices A to F have been written by the first author.

We are confident that this textbook will be a welcome companion on the desk of a masters or graduate student in general optics and in optical imaging in particular. Part of the book can also serve as teaching material for an advanced optics course to such students. Finally, the professional in optical research and development will have at his disposal a reference book covering a wide variety of subjects in advanced optical imaging.

# Acknowledgements

The first author (J.B.) would like to thank various institutions and persons who facilitated his work during the long period from the first written page up to the final wrap-up of the manuscript. During these nine years, I was able to benefit from the hospitality offered by the Department of Applied Physics of Delft University. In particular, the optics research group of the department gave me the opportunity to work in a concentrated way on this book manuscript. The ‘guest’ room was a very efficient environment, facing a metallic grey-coloured dead wall and giving a very restricted view of the sky. There were no perturbing sources of distraction during working-time, apart from the social contacts with the group members and students during coffee breaks. I would like to thank Professor Lucas van Vliet and Professor Paul Urbach for their efficient support by offering me this monastic room (nonetheless having worldwide internet access to scientific publications and libraries).

More directly related to the book-writing I would like to thank Jurgen Rusch of Philips Research Laboratories who offered to transfer to another platform a graphics program that I had written for my own optical software. In a surprisingly short amount of time he forwarded me a pdf-based graphics program. All figures in this book related to optical systems have been produced with his new plotting routines. I would also like to thank Dr. A.J.E.M. Janssen who has been a frequent counsellor on and solver of mathematical problems related to the contents of this book. In particular, he has laid the foundation for the Extended Nijboer–Zernike diffraction theory. Especially in this field I have greatly profited from his unique analytical skills. He also kindly agreed to critically read the appendix on Zernike polynomials. Among the Ph.D. students that have graduated in the optics group of Delft University, I would like to mention three persons in particular, Arthur van de Nes, Aura Nugrowati and Sven van Haver. The long-standing collaboration with them was a great pleasure for me and, in several instances, the book content has benefited from their research results.

Two persons have contributed to the final quality of the book in an essential way. In an ideal world, the choice of expert critical readers should have been made by the author of a book before even writing the very first sentence of that book. I had the good luck to find at a later stage of the writing process two critical readers who were willing to spend much time on reviewing the manuscript. Matthew Foreman took care of most of the electromagnetic and physical optics part of the book. His sharp mind and rigorous scientific attitude, combined with his broad knowledge of the presented material, have substantially improved the level of the book and the way of presenting the material. Peter Nuyens has been an extremely strict and precise critical reader of Part II of the book. During the almost two years that we have collaborated on further improving the manuscript I have highly appreciated Peter’s analytic dissection of formulas, sentences and paragraphs, his suggestions for shortening my sometimes too verbose presentation and his thorough inspection of the presented optical design examples. Peter, you used a powerful magnifying glass! I would like to apologise to both critical readers for presenting to them in several instances book material that should have been polished to a higher degree beforehand by me. Last but certainly not least, I thank my wife Anna who supported me as a sometimes absent-minded companion in life during these book-writing years.

The second author (P.T.) is grateful to a number of colleagues who contributed to developing his understanding of optics and imaging. Given that I should only have room for a contribution-proportional acknowledgement, these few lines will not be sufficient to mention everybody. Colin Sheppard played an important part in developing the consistent derivation of optical diffraction in 1998–2000 when I visited his Department in Sydney on several occasions. Peter Varga contributed to developing various focusing theories. Emmanouil Kriezis and I used to sit in the Engineering Common room in Oxford discussing how the theory of imaging extended objects could be developed. A string of amazing students inspired me and contributed to a significant degree to developing the mathematical tools and physical understanding discussed in chapters of this book. These included Peter Munro, Carlos Macías Romero and Matthew Foreman. I am truly honoured that they chose to come to work with me. As Joseph so eloquently said above, Matt has done the majority

of proof reading of the text picking up typos and mistakes for which I am really grateful. I am also profoundly thankful to Joseph that he has put up with my hectic schedule and lack of progress in writing during these years. I am not sure if I could have found any other co-author willing to do what he did. Finally, I would like to give thanks to my family. Gina and Zoli are the very inspiration for my every-day existence. My wife, Janey has been the most loving, encouraging and understanding companion. I am truly blessed to have you all!

Both authors would like to acknowledge the smooth collaboration with the staff of Cambridge University Press during the publishing process, from initial manuscript transfer to copy-editing and proofreading. We especially thank Richard Smith for copy-editing the book manuscript and Roisin Munnely for her professional guidance and assistance during the entire last year before publication.