Lightwave Communications

This pioneering, course-tested text combines communications theory with the physics of optical communications. Comprehensive and rigorous, it brings together an in-depth treatment of the physical characteristics of the guided lightwave channel with the study of modern methods of algorithmic-based communication in time and space. The many different levels at which a lightwave communication signal can be described, (ray, wave, photon or quantum state), are integrated to provide a unified explanation of how a commonplace bit stream is transformed into a physical lightwave, how that lightwave travels through an optical fiber, and how it is then transformed back into the bit stream. Background fundamentals such as linear systems and electromagnetics are explained in relation to modern topics such as channel models, encoding, modulation, and interference, and end-of-chapter problems are provided throughout. This is an essential text for both graduates and senior undergraduates taking courses on optical communications, and professionals working in the area.

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To Theresa, Kate, and Alex

If you want to find the secrets of the universe, think in terms of energy, frequency, and vibration

Nikola Tesla

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

Contents

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

Preface

A lightwave communication signal can be described at many levels: a stream of photons, a bundle of rays, an electromagnetic wave, a quantum state, a modulated waveform, or a stream of information bits. Each description has its own vocabulary, traditions, and notation. The goal and challenge of this book is to seamlessly integrate these levels into a unified treatment of an information-bearing lightwave traveling in an optical fiber.

To the user of the communication system, however, such a discussion of the science of guided lightwave signal propagation is irrelevant. The user sees only the reliable transmission of a bit stream and is unaware that at the deepest level of the system, the bits have long lost their individual identity, buried deeply within the system and spread across time, wavelength, polarization, and space so as to create this reliable communication channel. Thus, the second goal and challenge of the book is to explain the many steps through which a commonplace bit stream is transformed into a physical lightwave and then back again to that bit stream.

This text combines a rigorous foundation of the physical characteristics of the guided lightwave channel with the study of modern methods of algorithm-based communication in time and space. Our view is that a fiber together with the lightwave it conducts is only one component of the larger lightwave communication system. The integration of this material into a single text that is accessible to readers from a wide range of backgrounds is necessary to facilitate the design of future lightwave communication systems.

The physics of a guided lightwave, as such, is a well-developed subject based on quantum theory and its two offspring, wave optics and photon optics. Within this richer environment, this book studies many traditional topics of digital communications extended to the theory of guided lightwaves. At these frequencies, the carrier has a dual wave/particle nature. Often the conventional theory of the communication topics must be enriched for these purposes. As a consequence, most chapters of the book contain some material that has been repurposed or reinterpreted. These individual chapters, in return, inform with new insights on how digital information can be conveyed. We believe that some readers who are not interested in guided lightwave communications will find value added to the understanding of conventional topics of digital communication. This book is addressed to them as well.

Early textbook treatments of lightwave communication systems have emphasized the physical characteristics of the lightwave channel and the components used to generate, amplify, and photodetect lightwave signals. This emphasis contrasts with the usual treatments of other communication systems at lower frequencies. In the more traditional lower-frequency communication systems, the exponential increase in the available processing power has enabled complex coding and detection algorithms that use many samples per modulation interval, and with many bits per modulation interval. This is due to the remarkable power and flexibility of modern methods of coded-modulation. Existing techniques operate close to the fundamental theoretical limit. These techniques include real-time estimation of the communication channel and may perform thousands of computational operations per symbol interval to yield data rates in excess of ten bits per second for every hertz of bandwidth or at an energy per bit smaller than the noise power density spectrum.

Historically, lightwave communication systems have used simpler methods of modulation and relied on wavelength-multiplexed systems to scale the capacity of a single fiber. However, the current rate of growth for data that can be transmitted on a single wavelength channel is now less than the rate of growth of the network traffic and is also less than the rate of growth of processing power. As a consequence, many of the communication techniques and algorithms developed for other multi-input multi-output communication systems that exploit both time and space are beginning to be applied to lightwave communication systems. These techniques will increasingly be used to expand the information rate carried in a single fiber. A concurrent trend is the rapid evolution from single-wavelength point-to-point links into multiple-wavelength networks where many network functions such as multiplexing and switching may be implemented in the optical domain.

Understanding and designing modern lightwave communication systems that reflect these trends requires a unifying system-level approach that is accessible to readers from a physics-oriented background, who have been the traditional developers of lightwave communication systems, as well as readers with a background in other types of communication systems and networks.

From a systems-level perspective, there are significant differences between current lightwave communication systems and other kinds of communication systems that operate at lower frequencies. In addition to the significantly higher data rates, the principal differences are (1) the physical sources that generate noise and interference, (2) mitigating signal impairments in multiple-wavelength and long-distance systems that are caused by nonlinearities in the fiber channel, and (3) the large energy of a lightwave photon compared with the mean thermal energy of the environment. The last difference brings elements of the quantum nature of a lightwave signal to the forefront.

In contrast to lower-frequency communication systems, for which the assumption of additive thermal noise is usually a valid starting point, the physical sources of noise for lightwave sources, lightwave amplifiers, and some photodetectors are different, with the statistics associated with some of these processes being nongaussian and signal-dependent. The signal-dependent nature of both the gain and the noise makes the system-level analysis more involved.

At the network level, the use of lightwave signaling has a profound effect on the type of functionality that can be implemented. Currently, it is difficult to rapidly switch a large number of lightwave signals without first converting each lightwave signal into

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

an electrical signal, then reconverting the electrical signal, after switching, back into a lightwave signal. It is also practically impossible to store photons for meaningful periods of time because photons, as such, do not exist unless they are propagating in a medium or a vacuum. The design of future lightwave networks is facilitated by an understanding of how these basic aspects of lightwave communication systems differ from lower-frequency communication systems.

The material in this book is suitable for an advanced undergraduate course or a first-year graduate course. It is complementary to most introductory undergraduate communication courses. Extensive background and reference material on linear systems, random signals, and electromagnetics is provided in Chapter 2 for review as needed. Chapter 2 also serves to introduce the notation and terminology used throughout the text.

Our journey describing the elements of a lightwave communication system starts with a description of guided lightwaves in Chapter 3 and ends with a description of channel codes in Chapter 13, but the study does not end there. These eleven core chapters develop the theory in depth, but are directed towards the application rather than towards the science. Chapters 3 and 4 discuss the linear guided lightwave channel and dispersion, while Chapter 5 considers the nonlinear lightwave channel. Fundamental noise concepts, as well as the Poisson transform as a practical proxy for much of the relevant nature of quantum optics, are addressed in Chapter 6. Concepts from these early chapters are applied to discuss aspects of the components used for lightwave systems as described in Chapter 7. The electrical channel, which surrounds the lightwave channel, is developed in Chapter 8. The information channel, which surrounds the electrical channel, is developed in Chapter 9. These abstracted channel models include the encoding and modulation process at the transmitter and the demodulation, detection, and decoding process at the receiver. Modulation formats are discussed in Chapter 10. Interference is discussed in Chapter 11, with techniques that estimate the channel response parameters described in Chapter 12. Channel coding for lightwave channels is discussed in Chapter 13.

The last three chapters, Chapters 14, 15, and 16, discuss lightwave communications at a deeper and abstract level directed more towards the science of the topic. Chapter 14 provides a formal treatment of information theory, which is applied to determine the capacity of different types of lightwave channels. This novel chapter approaches the topic of information theory from the unique perspective of the lightwave, thereby unifying particle and wave descriptions of the same information channel. Chapter 15 is an interlude that presents background material for a quantum-optics signal model and discusses how this model is related to other signal models. The final chapter, Chapter 16, applies this quantum-optics signal model to study quantum-optics lightwave communication systems, a formal model whose presence is often seen in the background throughout the book. Chapters 15 and 16 present standard ideas of quantum information theory in an original way that is appropriate to our treatment of lightwave communications.

The draft manuscript of the book was developed and tested over many years in the classrooms at the University of Illinois, the University of California at San Diego, and the University of Pennsylvania. The core material in the book naturally divides into Chapters 3–7, which describe the physical aspects of a lightwave channel, and Chapters 8–13, which consider a lightwave communication system. Accordingly, several courses can be constructed, depending on the background of the students. Students with a strong electromagnetics background may prefer to bypass the early chapters, using them only for reference.

A one-semester course has been taught multiple times covering primarily Chapters 1–10. A one-quarter (ten-week) graduate-level course has been taught covering material from Chapters 8–11, augmenting several topics with material from earlier chapters. A one-semester (fifteen-week) graduate-level course might include additional material from Chapters 12–14 or a range of topics from Chapters 1–7, depending on the instructor. Moreover, individual chapters on various topics may provide supplementary material not found elsewhere for courses on those topics. An advanced graduate-level course on quantum-lightwave communications and quantum information theory can be constructed using Sections 6.1–6.3, part or all of Chapter 14, and Chapters 15 and 16. This material, with its unique perspective, provides a thorough introduction to quantum-lightwave communication systems that convey classical information using the earlier parts of the book as a reference.

A one-semester undergraduate-level course that uses linear system concepts and gaussian noise models can be constructed from the following core material: Sections 3.1–3.3, 4.1–4.4, 5.1, 5.2, 6.1.2–6.2, 7.1–7.4, 8.2–8.2.4, 9.5–9.5.2, 10.1–10.4, 11.1, and 11.2, with the depth and specific topics depending on the instructor. Finally, it should be mentioned that, although the chapters are integrated with a common underlying treatment, most chapters can be read independently without too much difficultly. This means that students of other subjects, such as physical optics, photonic devices, information theory, and communication systems, will find original material in the appropriate chapters.

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Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

Notation

The choice of notation in a book that is designed to bridge several communities, each of which has its own well-established conventions, is challenging. Moreover, the book makes use of four separate signal models, often requiring different notation. A continuous signal model based on wave-optics is used whenever the noise sources can be accurately described using continuous quantities. A ray-optics signal model is used when the guiding structure is large compared with the wavelength. A discrete-energy model based on photon optics is used when some aspects of the quantum nature of the lightwave signal are evident. Finally, a quantum-optics signal model is developed and compared with the other signal models.

Fields that propagate in the $+z$ direction are expressed in complex notation using the form $\exp[\mathrm{i}(\omega t - \beta z)]$. Quantum-optics signals use the form $\exp[\mathrm{i}(\beta z - \omega t)]$. The orientation of polarization (i.e. left-handed or right-handed) is defined with respect to the field propagating towards the observer. Vectors and sequences are denoted using boldface. Matrices and linear transformations represented as matrices are defined using blackboard letters $\mathbb{M}$. Linear transformations for the quantum-optics signal are denoted with a caret $\widehat{\cdot}$. Random variables $\underline{x}$ and random sequences $\underline{\mathbf{n}}$ are denoted by an underbar. This underbar is omitted for a random process $n(t)$. Statistical expectations use $\langle\cdot\rangle$. Temporal averages use an overbar.

Sans serif symbols such as n, m and N are typically used for discrete quantities or expected values of discrete quantities. The amplitude of a cosinusoidal signal $s(t)$ is defined using a peak amplitude with the root-mean-squared (rms) amplitude being a factor of $\sqrt{2}$ smaller. The spectral characteristics are expressed using frequency f (Hz). The amplitude of a field or an envelope of a field that has both temporal and spatial dependence is defined using an rms amplitude with the spectral characteristics expressed using an angular frequency $\omega \doteq 2\pi f$ with units of radians/second.

Signals are defined in the optical domain or the electrical domain with the units depending on the signal model. The square-law characteristic of direct photodetection produces different scaling factors in the optical domain and the electrical domain for each signal model. In the optical domain, the scaling factor of the photon energy hf relates the signal energy E in the continuous wave-optics model to the expected number of signal photons $\mathsf{E} = \langle\underline{\mathsf{m}}\rangle$ in the discrete photon-optics signal model.

In the electrical domain after the lightwave signal has been photodetected, the units for each signal model and the scaling factors relating the two signal models are different. For wave optics, the directly photodetected lightwave energy W has units of charge and

Cambridge University Press
978-1-108-42756-2 — Lightwave Communications
George C. Papen , Richard E. Blahut
Frontmatter
More Information

is called the photocharge. Dividing the photocharge by the electron charge e produces the mean number of discrete photodetection events W caused by the detection of photons. These detected photon events are called photoelectrons.

The directly photodetected lightwave power has units of current and is called the photocurrent. Dividing the photocurrent by the electron charge e produces the mean photoelectron arrival rate μ. The relationships between these quantities are summarized in Table 6.2.

The units for the power density spectrum follow the same convention. For example, the lightwave power density spectrum of the spontaneous emission $N_{\text{sp}}(f)$ defined in (6.2.18) is expressed in units of energy or watts/Hz. The equivalent term $\mathsf{N}_{\mathsf{sp}}(f)$ in the photon-optics signal model is scaled by the photon energy hf and is expressed as the expected number of noise photons. The corresponding power density spectrum $N_{\text{opt}} = \mathcal{R}N_{\text{sp}}$ of the spontaneous emission generated by direct photodetection has units of charge when wave optics is used, where $\mathcal{R}$ is a scaling constant defined in (6.2.23), which is called the responsivity. For photon optics, the directly photodetected spontaneous emission $\eta\mathsf{N}_{\mathsf{sp}}$ has units of photoelectrons, where η is the probability that a photon will generate a photoelectron, which is called the quantum efficiency.

For wave optics, the electrical power density spectrum is typically expressed in units of amperes-squared per hertz, which is the power density spectrum per unit resistance R. For photon optics, the electrical power density spectrum has units of hertz.

Primary Symbols

Symbol	*Description*	*Equation*
Lightwave Signal Terms		
$s(t)$	Lightwave signal	(8.1.1)
$P(t)$	Lightwave power	(1.2.5)
E	Lightwave energy	(1.2.1)
$\Phi(t)$	Photon arrival process	(6.2.19)
$\mathsf{R}(t)$	Photon arrival rate	(1.2.5)
$\mathsf{m}(t)$	Photon counting process	(6.2.20)
E	Expected number of signal photons	(6.2.22)
Lightwave Noise Terms		
N_{sp}	Spontaneous emission power density spectrum	(7.7.8)
P_n	Spontaneous emission noise power	(6.2.18)
N_0	Expected number of noise photons	(6.1.7)
N_{sp}	Expected number of additive-noise photons from spontaneous emission	(7.7.7)
Electrical Signal Terms		
$p(t)$	Electrical pulse	(8.2.24)
$\overline{r}(t)$	Noiseless electrical waveform	(9.2.5)
$q(t)$	Target electrical pulse used for sampling	(9.2.4)
Electrical Noise Terms		
N_0	Electrical power density spectrum for an additive-noise source for phase-synchronous demodulation	(8.2.3)
N_{shot}	Power density spectrum from shot noise	(6.7.8)
N_{RIN}	Power density spectrum from intensity noise	(7.8.10)

Symbols

a	Radius of fiber; constant
$\widehat{a}$	Coherent-state operator
$\underline{a}_k$	Expansion coefficient for spatial decorrelation
$\widehat{a}_I, \widehat{a}_Q$	In-phase and quadrature coherent-state operators
$\widehat{a}_N$	Image mode vacuum-state operator
$a(z, t)$	Root-mean-squared lightwave signal complex envelope
$\widetilde{a}(z, t)$	Modulated complex envelope
A, $A(t)$	Amplitude of a signal
$\mathcal{A}$	Area
$\mathcal{A}_{\text{fiber}}$	Area of a fiber core
$\mathcal{A}_{\text{coh}}$	Coherence region
$\mathcal{A}_{\text{overlap}}$	Overlap region
$\mathcal{A}_{\text{eff}}$	Effective area for a fiber
$\widehat{A}_{\text{homo}}$	Homodyne demodulation operator
$\widehat{A}_{\text{hetero}}$	Heterodyne demodulation operator
$A_{\text{rms}}(t)$	Root-mean-squared amplitude of a modulated signal
b	Normalized propagation contant
$\boldsymbol{\mathcal{B}}$	Magnetic flux density
B	Passband bandwidth
$\mathbb{B}$	Matrix with diagonal elements that are the singular values of the channel matrix $\mathbb{H}$
B_c	Number of coherence intervals per unit time
B_N	Noise-equivalent bandwidth
c_0; c	Phase velocity of light in vacuum (m/s); velocity of light in medium
cov	Covariance
C	Capacitance; channel spacing
$\mathcal{C}$	Power efficiency; bandlimited capacity of a channel (bits/second)
$\mathcal{C}_{\max}$	Maximum channel capacity (bits/second)
C	Single-letter capacity
$\mathsf{C}(f)$	Single-letter capacity as a function of frequency

C_p	Single-letter capacity associated with the discrete particle nature of lightwave
C_w	Single-letter capacity associated with the continuous wave nature of lightwave
C_band	Capacity of a bandlimited channel (bits)
C_mimo	Capacity of a multi-input multi-output channel (bits)
$C_x(f);\ C_x(\omega)$	Characteristic function of a random variable $\underline{x}$
$C(t_1, t_2)$	Covariance function
$\mathbb{C}$	Real covariance matrix
d	Distance; diameter
d	Hamming distance
$\boldsymbol{\mathsf{d}}$	Dataword
d_{ij}	Euclidean distance between two signals $s_i(t)$ and $s_j(t)$
d_{min}	Minimum euclidean distance
d_{min}	Minimum Hamming distance
D	Total group-velocity dispersion in wavelength units
$\mathbb{D}$	Polarization transformation matrix
D	Linear dispersion operator in the time domain
$\mathcal{D}$	Linear dispersion term in the frequency domain
$\widehat{\mathcal{D}}$	Displacement operator for a coherent state
$\widehat{D}$	Detection operator; dispersion operator
$\boldsymbol{\mathcal{D}}$	Electric flux density (C/m^2)
$\mathbf{D}$	Complex electric flux density: $\boldsymbol{\mathcal{D}}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{D}(\mathbf{r})e^{\mathrm{i}2\pi f t}]$
D_λ	Material dispersion coefficient in units of wavelength
$\mathbb{D}_\omega$	Derivative of the polarization transformation matrix
D_{guide}	Waveguide dispersion coefficient
e	Charge of an electron: 1.602×10^{-19} C; error
$\mathbf{e}$	Normalized electric field vector
$\mathbf{e}_t$	Normalized transverse electric field vector
$\boldsymbol{\mathsf{e}}$	Error pattern for a block code
$e(t)$	Error signal
e_x	Extinction ratio
$\widehat{\mathbf{e}}$	Unit vector such that $\widehat{\mathbf{e}} \times \widehat{\mathbf{h}} = \widehat{\mathbf{k}}$
$\widehat{\mathbf{e}}_\mathbf{1}, \widehat{\mathbf{e}}_\mathbf{2}$	Orthogonal unit vectors used to represent the polarization state
$\widehat{\mathbf{e}}_{\text{slow}}, \widehat{\mathbf{e}}_{\text{fast}}$	Unit vectors for slow and fast axes of a linear-birefringent material
E	Energy; electric field component of a monochromatic wave
E_b	Expected value of the energy in an uncoded bit
E_c	Expected value of the energy in a codebit
E_s	Expected value of the energy in a symbol s
E_g	Energy gap
E_{total}	Total signal energy
E	Expected number of signal photons
E_b	Expected number of signal photons in a bit

E_s	Expected number of signal photons or signal photoelectrons in a symbol s
$\mathcal{E}$	Energy spectral density; energy efficiency
$\boldsymbol{\mathcal{E}}$	Electric field vector
$\mathbf{E}$	Complex electric field: $\boldsymbol{\mathcal{E}}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})e^{\mathrm{i}2\pi f t}]$
$E_t(\mathbf{r})$	Transverse component of the electric field
f	Frequency; probability density function
f_c	Carrier frequency
f_d	Damping rate for laser diode; frequency difference from carrier
f_0	Relaxation oscillation frequency
$f_{\max}$	Frequency of a photon with energy E ($f_{\max} = E/h$); maximum frequency in a signal
$f_\lambda(\lambda)$	Normalized power density spectrum in wavelength units
f_{IF}	Intermediate frequency
f_{LO}	Local oscillator frequency
$f_{\underline{x}}(x)$	Probability density function of the random variable $\underline{x}$
F	SNR scaling constant: $F \doteq \langle \underline{A} \rangle^2 / 2\sigma^2$; excess noise factor for an avalanche photodetector
F_m	Proportion of the total power in a mode m
F_N; $F_N(f)$	Noise figure; spectral noise figure
F_{NP}	Noise figure defined using the photon number
FT	Fourier transform
FWHM	Full-width at the half-maximum
$F(x)$	Cumulative probability density function
g	Mode-group index $g = \nu + 2m$
$\mathsf{g}(t)$	Photoelectron arrival process
G; G	Gain
$\mathcal{G}$	Total number of mode-groups
$\mathcal{G}_T(f)$	Fourier transform of a sample function of photon arrival process $\Phi(t)$ truncated to a finite time T
h	Planck's constant: $h = 6.62607 \times 10^{-34}$ J-s
$\hbar$	Reduced Planck constant: $\hbar = h/2\pi = 1.05457 \times 10^{-34}$ J-s
$\widehat{\mathbf{h}}$	Unit vector for the magnetic field
$h(t)$	Impulse response
$\mathbb{h}(t)$	Time-domain channel matrix of a multi-input multi-output channel
$h_s(t)$	Photodetector impulse response
$h_{\mathrm{elec}}(t)$	Impulse response for the power using a noncoherent source
$h_m(t)$	Impulse response for mode m
$\mathbf{h}$	Normalized magnetic field vector
$\mathbf{h}_t$	Normalized transverse magnetic field vector
$\widehat{H}$	Hamiltonian
$\boldsymbol{\mathcal{H}}$	Magnetic field vector
$\mathbf{H}$	Complex magnetic field: $\boldsymbol{\mathcal{H}}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{H}(\mathbf{r})e^{\mathrm{i}2\pi f t}]$

$H(f)$, $H(\omega)$	Transfer function
$\mathbb{H}(\omega)$	Transfer function of multi-input multi-output channel
$H(\underline{x})$	Entropy of a random variable $\underline{x}$
$H_{\text{mode}}(x)$	Entropy of a mode
$H_{\text{total}}(x)$	Total entropy of a system
H_ℓ	Hypothesis that the ℓth symbol or symbol-state was transmitted
i	$\sqrt{-1}$
i; $i(t)$	Directly photodetected signal; current
i_{LO}	Directly photodetected lightwave local oscillator
i_{bias}	Bias current
i_{th}	Threshold current for a laser
$\widehat{I}$	Identity operator
$\mathbb{I}$	Identity matrix
$I(t)$	Optical intensity
$I(\mathsf{r};\mathsf{s})$	Mutual information
$I_m(\cdot)$	Modified Bessel function of the first kind of order m
Im	Imaginary part
$\mathbf{J}$	Jones vector
$J_m(\cdot)$	Bessel function of the first kind of order m
k	Wavenumber; Boltzmann's constant: 1.38×10^{-23} J/K
k_0	Free-space wavenumber
$\mathbf{k}$	Wavevector
k_r	Radial component of the wavevector
k_x, k_y, k_z	Components of the wavevector $\mathbf{k}$
K	Number of independent subsamples; spatial frequency of a ray in a graded-index fiber
$K_m(\cdot)$	Modified Bessel function of the second kind of order m
$\mathbb{K}$	Gram matrix
L	Length; number of symbol values
$\mathcal{L}_\Lambda(\cdot)$	Log-likelihood ratio
$\mathcal{L}^m(\cdot)$	Laguerre polynomial
$\mathcal{L}_n^m(\cdot)$	Generalized Laguerre polynomial
L_c	Coherence length
L_{pol}	Polarization decorrelation length
L_{comp}	Compensation length for a nonlinear equalizer
L_{eff}	Effective fiber length
L_{wo}	Walk-off length in a fiber
L_{NL}	Nonlinear length characterizing the fiber nonlinearity
L_D	Dispersion length
m	Number of photons or photoelectrons
$m(\beta)$	Total number of modes with propagation constants larger than β
$\mathbb{M}$	Signal constellation matrix
n	Index of refraction
n_c	Complex index of refraction

$\mathtt{n}_2$	Nonlinear index of refraction coefficient
n_a	Index of refraction at core/cladding interface for a graded-index fiber
n_b	Background index of refraction
n_o, n_e	Ordinary and extraordinary indices of refraction
n_0	Maximum index of refraction of a graded-index fiber
n_{t+1}	Number of error patterns with $t+1$ errors
$n(t)$	Complex-baseband noise process
$n_e(t)$	Electrical noise process
$n_o(t)$	Spontaneous emission noise process
n_{sp}	Spontaneous emission noise factor for a lightwave amplifier
$\overline{n}$	Average number of nearest neighbors in a signal constellation
N	Carrier density in a semiconductor; group index
N	Nonlinear term in the time domain;
$\widehat{N}$	Photon-number-state operator; nonlinear operator
N_1	Number density of the lower-energy state
N_2	Number density of the higher-energy state
N_a	Power density spectrum at the amplifier input
N_c	Group index at the center of a graded-index fiber
N_{in}	Power density spectrum of the electrical noise at an input
N_{sp}	Power density spectrum of the lightwave power from spontaneous emission
N_{shot}	Power density spectrum of the photodetected shot noise
$N_{\mathrm{sp}_\mathrm{e}}$	Power density spectrum from spontaneous emission using phase-synchronous demodulation
N_0	Total electrical power density spectrum
N_0	Expected number of noise photons
N_{opt}	Photodetected lightwave noise power density spectrum or the expected photocharge from lightwave noise
N_{RIN}	Power density spectrum from relative intensity noise
NA	Numerical aperture
OSNR	Optical signal-to-noise ratio
OSIR	Optical signal-to-interference ratio
OSNIR	Optical signal-to-noise-plus-interference ratio
p	Probability; photon momentum; decay rate for cladding solutions
$\mathbf{p}$	Prior; input probability vector; input Jones vector
$\widehat{p}$	Momentum operator
p_c	Probability of a correct detection event
p_e	Probability of a detection error
$p(t)$	Demodulated electrical pulse
$\mathbb{P}_m$	Projection matrix
$\widehat{P}_m$	Projection operator
P	Lightwave power
P	Syndrome
$P_s;\ P_s(t)$	Lightwave signal power

P_n; $P_n(t)$	Lightwave noise power
$P_{\text{in}}(t)$; $P_{\text{out}}(t)$	Input and output lightwave power in a pulse
P_{coh}	Lightwave signal power in a spatial coherence region $\mathcal{A}_{\text{coh}}$
$P_e(t)$	Electrical power
P_{in}; P_{out}	Input and output lightwave signal power
P_{max}	Peak lightwave signal power
P_L	Lightwave power from linear interference
P_{NL}	Lightwave power from nonlinear interference
$\mathcal{P}(\boldsymbol{\alpha})$	P-representation of a quantum-optics signal
$\mathbb{P}(\omega)$	Polarization-dependent part of the multi-input multi-output channel matrix
$\boldsymbol{\mathcal{P}}$	Material polarization vector
$\mathbf{P}$	Complex material polarization vector
q	Charge; spatial frequency for waveguide solution in the cladding
$q(t)$	Target pulse after the detection filter
$\mathcal{Q}$	Figure-of-merit for evaluating error probabilities (see (9.5.28))
$\mathbb{Q}$	Channel transition matrix
$Q(\alpha_I, \alpha_Q)$	Husimi quasi-probability distribution
r	Position vector in space; radial coordinate in cylindrical coordinates; radius
$r(t)$	Received noisy complex waveform; filtered complex waveform
$\overline{r}(t)$	Received noise-free complex waveform
$\widetilde{r}(t)$	Received noisy passband waveform
$\overline{r}_k$	Noise-free sampled output after the detection filter
$\underline{r}_k$	Noisy sampled output after the detection filter
$\widehat{\boldsymbol{r}}, \widehat{\boldsymbol{\psi}}, \widehat{\boldsymbol{z}}$	Unit vectors in cylindrical coordinates
$\mathfrak{r}$	Ratio of the prior probabilities
R	Data rate; resistance; normalized radius of a fiber
R	Photon-optics signal arrival rate
R_c	Code rate
R_s	Sample rate
$\mathbb{R}$	Covariance matrix at the channel output
$R_P(\tau)$	Autocorrelation function of the lightwave power
$R_s(\tau)$	Autocorrelation function of the complex lightwave signal
$R_e(\tau)$	Autocorrelation function of the electrical signal
$R_n(\tau)$	Autocorrelation function of a noise process
$\mathcal{R}$	Responsivity of photodetector; decision region; decision subspace region
Re	Real part
$\mathbf{s}$	Stokes vector; signal point in signal space
$\boldsymbol{\mathsf{s}}$	Codeword; scaled signal point in signal space ($|\boldsymbol{\mathsf{s}}|^2 = |\mathbf{s}|^2/\hbar\omega$)
$s_0,\ s_1,\ s_2,\ s_3$	Stokes parameters
s_λ	The slope of the wavelength-dependent group delay at the carrier wavelength λ_c

$\widetilde{s}(t), s(t)$	Modulated lightwave signal and the complex-baseband equivalent
S	Expected number of photons; syndrome
$\mathcal{S}(f), \mathcal{S}_\lambda(f)$	Power density spectrum in frequency and wavelength
$\mathcal{S}_{\mathrm{d}\phi}(f)$	Power density spectrum for the derivative of the phase noise
$\mathcal{S}_n(f)$	Power density spectrum of the noise
$\mathcal{S}(\widehat{\rho})$	von Neumann entropy
$\boldsymbol{\mathcal{S}}_{\mathbf{ave}}$	Time-averaged Poynting vector
$\mathbf{S}_{\mathbf{ave}}$	Average Poynting vector for a monochromatic wave
SNR	Signal-to-noise ratio
$\bar{t}$	Temporal mean
$\mathcal{T}$	Transmittance
$\mathbb{T}$	Coupling matrix
$\widehat{T}$	Quantum-lightwave channel transformation
T_0	Temperature
T_{rms}	Temporal root-mean-squared width of a pulse
T_s	Sample time
TB	Timewidth–bandwidth product
$u(t)$	Unit-step function
$u(r)$	Ratio of posterior probability distributions
$u(\alpha_I)$	Quantum wave function for the in-phase signal component
$U(\alpha_Q)$	Quantum wave function for the quadrature signal component
$\widehat{U}$	Unitary transformation
$U_T(f)$	Fourier transform of a sample function $u_t(t)$
$\widetilde{U}(\mathbf{r}, t)$	Passband field amplitude
$\mathbf{U}(\mathbf{r}, t)$	Complex field envelope
v	Voltage; velocity
v_g	Group velocity
V	Normalized frequency or the V-parameter
$\mathbb{V}$	Complex covariance matrix
$\mathbb{V}_N$	Complex covariance matrix of the noise
$\mathbb{V}_s$	Complex covariance matrix of the signal
$\mathcal{V}$	Volume
w_n	Photocharge from the noise at the input to a gain process
w_s	Photocharge from the signal at the input to a gain process
W, W	Photocharge; mean number of photoelectrons $W = e\mathsf{W} = \mathcal{R}E$
$W_s, \mathsf{W}_\mathsf{s}$	Signal photocharge; mean number of signal photoelectrons
$W_n, \mathsf{W}_\mathsf{n}$	Noise photocharge; mean number of noise photoelectrons
$W_b, \mathsf{W}_\mathsf{b}$	Photocharge in a bit; mean number of photoelectrons in a bit
$W_p, \mathsf{W}_\mathsf{p}$	Photocharge in a pulse; mean number of photoelectrons in a pulse
$\mathsf{W}_{\mathrm{dark}}$	Mean number of photoelectrons from dark current
$\mathcal{W}$	Baseband bandwidth
$\mathcal{W}_{\mathrm{rms}}$	Root-mean-squared baseband bandwidth
$\mathcal{W}_h$	Half power or -3 dB bandwidth

$W(q, p)$, $W(\alpha_I, \alpha_Q)$	Wigner quasi-probability distribution
$\underline{x}$	Random variable
$x(t)$	Transmit pulse
$\widehat{x}$	Position operator
$\widehat{x}(t)$	Hilbert transform of $x(t)$
$\widehat{\mathbf{x}}, \widehat{\mathbf{y}}, \widehat{\mathbf{z}}$	Unit vectors in cartesian coordinates
$X(\mathbf{r}, t)$	Susceptibility of the material
$y(t)$	Detection filter
Υ	Expected number of received photons
$\widehat{Y}$	Generalized measurement operator
α	Index-of-refraction power-law profile for a graded-index fiber; linear interference constant
α_e, α_h	Ionization coefficients for an avalanche photodiode
α_s	Scattering loss
α_m	Absorption coefficient
α_{NL}	Nonlinear interference power scaling coefficient
$\boldsymbol{\alpha}$	Glauber number
$\lvert\boldsymbol{\alpha}\rangle$	Coherent state specified by the Glauber number $\boldsymbol{\alpha}$
α_I, α_Q	Measured values of the in-phase and quadrature components of a coherent state
β, $\boldsymbol{\beta}$, $\widehat{\boldsymbol{\beta}}$	Propagation constant, propagation vector, and unit propagation vector
β_2	Group velocity dispersion coefficient
$\delta_{\mathrm{slow}}, \delta_{\mathrm{fast}}$	Phase shift along the fast axis and the slow axis
$\delta(t)$	Dirac impulse
δ_{ij}	Kronecker impulse
Δ	Normalized index difference
$\Delta\tau$	Differential group delay
$\Delta\vec{\tau}$	Polarization-mode dispersion vector
ε	Permittivity
ε_0	Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C}^2/(\mathrm{N} \cdot \mathrm{m}^2)$
ε_r	Relative permittivity
ϕ	Phase; angular coordinate in cylindrical coordinates
ϕ_e	Phase error
$\varphi(\tau)$; $\varphi_I(\tau)$	Coherence function; intensity coherence function
$\phi(t)$, $\phi(\omega)$	Phase functions in time and frequency; phase-noise random process
ϕ_{NL}	Nonlinear phase
ϕ_{SPM}	Nonlinear phase shift from self-phase modulation
ϕ_{XPM}	Nonlinear phase shift from cross-phase modulation
$\Phi(t)$	Photon arrival process
γ	Small-signal gain of an lightwave amplifier; separation constant; sample signal-to-noise ratio; fiber nonlinear coefficient; Euler's constant = 0.5772

Γ	Net small-signal gain; phase-noise parameter
κ	Attenuation coefficient of a fiber; coupling coefficient; inner product of two signal states
κ_p	Polarization-dependent loss coefficient of a fiber
κ_{blk}	Inner product between two block-symbol states
κ_{sym}	Inner product between two component-symbol states
η_0	Impedance of free space: $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\,\Omega$
η	Quantum efficiency; coupling efficiency; impedance
$\lvert\eta_k\rangle$	Sampling eigenstate
λ	Wavelength; likelihood ratio
λ_0	Free-space wavelength
Λ	Wavelength of an acoustic wave; spatial period; likelihood function
$\mu(t)$	Photoelectron arrival rate
μ_{dark}	Generation rate for dark current
μ_0	Permeability of free space: $\mu_0 \doteq 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
ρ	Crossover probability for a binary symmetric channel
ρ_{10}	Correlation coefficient between two signals
$\widehat{\rho}$	Density matrix
$\widehat{\rho}_{\mathrm{s}}$	Density matrix of a signal state
$\widehat{\rho}_{\mathrm{in}}, \widehat{\rho}_{\mathrm{out}}$	Density matrix at the channel input; channel output
$\widehat{\rho}_{\mathrm{blk}}$	Density matrix of a block-symbol state
$\widehat{\rho}_{\mathrm{sym}}$	Density matrix of a component-symbol state
$\widehat{\rho}_{\mathrm{vac}}$	Density matrix of a vacuum state
σ_a	Absorption cross section
σ_e	Emission cross section
σ_s	Scattering cross section; root-mean-square value of the variance for the signal
σ^2	Variance
$\boldsymbol{\sigma}_n$	Channel state vector
σ_P^2	Variance of the lightwave signal power
$\sigma_{\mathrm{inter}}^2$	Mean-squared width of the impulse response caused by mode-dependent intermodal dispersion
$\sigma_{\mathrm{intra}}^2$	Mean-squared width of the impulse response caused by wavelength-dependent intramodal dispersion
σ_{shot}^2	Variance from shot noise
σ_{ISI}^2	Variance from intersymbol interference
σ_{sp}^2	Spontaneous emission noise power per quadrature component per polarization component
σ_λ^2	Variance of the power density spectrum $\mathcal{S}_\lambda(\lambda)$ for a modulated lightwave signal in wavelength units
τ	Delay; group delay
$\tau_+; \tau_-$	Group delay for the principal polarization states

τ_c	Coherence time
τ_m	Group delay for mode m
$\tau_{\max}$	Maximum delay spread
τ_{total}	Total group delay in the presence of polarization mode dispersion
χ	Holevo information; susceptibility; angle describing polarization state
χ_{NL}	Nonlinear susceptibility
θ_c	Critical angle
$\theta_{\max}$	Maximum launch angle for a guided ray
Θ	Threshold used for detection
ω	Angular frequency in radians/second
ω_c	Carrier frequency
Ω	Resistance; solid angle; Fourier transform variable
$\|\psi_\ell\rangle$	Signal state
ξ	Angle describing polarization state; loss factor for a channel number of photoelectrons; eigenvalue of $\mathbb{H}(\omega)\mathbb{H}(\omega)^\dagger$
ζ	Damping parameter for a phase-locked loop