

FORBIDDEN CONFIGURATIONS IN DISCRETE GEOMETRY

This book surveys the mathematical and computational properties of finite sets of points in the plane, covering recent breakthroughs on important problems in discrete geometry and listing many open problems. It unifies these mathematical and computational views using forbidden configurations, which are patterns that cannot appear in sets with a given property, and explores the implications of this unified view. Written with minimal prerequisites and featuring plenty of figures, this engaging book will be of interest to undergraduate students and researchers in mathematics and computer science.

Most topics are introduced with a related puzzle or brain-teaser. The topics range from abstract issues of collinearity, convexity, and general position to more applied areas including robust statistical estimation and network visualization, with connections to related areas of mathematics including number theory, graph theory, and the theory of permutation patterns. Pseudocode is included for many algorithms that compute properties of point sets.

David Eppstein is Chancellor's Professor of Computer Science at the University of California, Irvine. He has more than 350 publications on subjects including discrete and computational geometry, graph theory, graph algorithms, data structures, robust statistics, social network analysis and visualization, mesh generation, biosequence comparison, exponential algorithms, and recreational mathematics. He has been the moderator for data structures and algorithms on arXiv.org since 2006 and is a major contributor to Wikipedia's articles on mathematics and theoretical computer science. He was elected as an ACM fellow in 2012.

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Cambridge University Press
978-1-108-42391-5 – Forbidden Configurations in Discrete Geometry
David Eppstein
Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi - 110025, India
79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.
It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108423687
DOI: 10.1017/9781108529167

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First published 2018

Printed in the United States of America by Sheridan Books, Inc.

A catalogue record for this publication is available from the British Library

ISBN 978-1-108-42368-7 Hardback

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Acknowledgments

This book stemmed from an invitation to present the Erdős Memorial Lecture at the 29th Canadian Conference on Computational Geometry (held in Ottawa in July 2017) and would not have existed without that invitation. I would like to thank the many people who have given me helpful advice on it, especially Jean Cardinal, Sarel Har-Peled, Stefan Langerman, Joe O'Rourke, János Pach, Vijay Vazirani, and several anonymous reviewers. I am also grateful for the careful copyediting of Maureen Eppstein. The research in this work was supported in part by the US National Science Foundation under grants CCF-1618301 and CCF-1616248.