Modern Quantum Mechanics

Second Edition

Modern Quantum Mechanics is a classical graduate level textbook, covering the main quantum mechanics concepts in a clear, organized and engaging manner. The author, J. J. Sakurai, was a renowned theorist in particle theory. The Second Edition, revised by Jim Napolitano, introduces topics that extend the text's usefulness into the 21st century such as advanced mathematical techniques associated with quantum mechanical calculations, while at the same time retaining classic developments such as neutron interferometer experiments, Feynman path integrals, correlation measurements, and Bell's inequality. A solution manual for instructors using this textbook can be downloaded from www.cambirdge.org/napolitano under the resources tab.

The late **J.J. Sakurai**, noted theorist in particle physics, was born in Tokyo, Japan in 1933. He received his B.A. from Harvard University in 1955 and his PhD from Cornell University in 1958. He was appointed as an assistant professor at the University of Chicago, where he worked until he became a professor at the University of California, Los Angeles in 1970. Sakurai died in 1982 while he was visiting a professor at CERN in Geneva, Switzerland.

Jim Napolitano earned an undergraduate Physics degree at Rensselaer Polytechnic Institute in 1977, and a PhD in Physics from Stanford University in 1982. Since that time, he has conducted research in experimental nuclear and particle physics, with an emphasis on studying fundamental interactions and symmetries. He joined the faculty at Rensselaer in 1992 after working as a member of the scientific staff at two different national laboratories. Since 2014 he has been Professor of Physics at Temple University. He is author and co-author of over 150 scientific papers in refereed journals. Professor Napolitano maintains a keen interest in science education in general, and in particular physics education at both the undergraduate and graduate levels. He has taught both graduate and upper-level undergraduate courses in Quantum Mechanics, as well as an advanced graduate course in Quantum Field Theory.

Modern Quantum Mechanics

Second Edition

J. J. Sakurai

Deceased

Jim Napolitano

Temple University, Philadelphia

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Foreword to the First Edition

J. J. Sakurai was always a very welcome guest here at CERN, for he was one of those rare theorists to whom the experimental facts are even more interesting than the theoretical game itself. Nevertheless, he delighted in theoretical physics and in its teaching, a subject on which he held strong opinions. He thought that much theoretical physics teaching was both too narrow and too remote from application: "…we see a number of sophisticated, yet uneducated, theoreticians who are conversant in the LSZ formalism of the Heisenberg field operators, but do not know why an excited atom radiates, or are ignorant of the quantum theoretic derivation of Rayleigh's law that accounts for the blueness of the sky." And he insisted that the student must be able to use what has been taught: "The reader who has read the book but cannot do the exercises has learned nothing."

He put these principles to work in his fine book *Advanced Quantum Mechanics* (1967) and in *Invariance Principles and Elementary Particles* (1964), both of which have been very much used in the CERN library. This new book, *Modern Quantum Mechanics*, should be used even more, by a larger and less specialized group. The book combines breadth of interest with a thorough practicality. Its readers will find here what they need to know, with a sustained and successful effort to make it intelligible.

J. J. Sakurai's sudden death on November 1, 1982 left this book unfinished. Reinhold Bertlmann and I helped Mrs. Sakurai sort out her husband's papers at CERN. Among them we found a rough, handwritten version of most of the book and a large collection of exercises. Though only three chapters had been completely finished, it was clear that the bulk of the creative work had been done. It was also clear that much work remained to fill in gaps, polish the writing, and put the manuscript in order.

That the book is now finished is due to the determination of Noriko Sakurai and the dedication of San Fu Tuan. Upon her husband's death, Mrs. Sakurai resolved immediately that his last effort should not go to waste. With great courage and dignity she became the driving force behind the project, overcoming all obstacles and setting the high standards to be maintained. San Fu Tuan willingly gave his time and energy to the editing and completion of Sakurai's work. Perhaps only others close to the hectic field of high-energy theoretical physics can fully appreciate the sacrifice involved.

For me personally, J. J. had long been far more than just a particularly distinguished colleague. It saddens me that we will never again laugh together at physics and physicists and life in general, and that he will not see the success of his last work. But I am happy that it has been brought to fruition.

John S. Bell
CERN, Geneva

Preface to the Revised Edition

Since 1989 the editor has enthusiastically pursued a revised edition of *Modern Quantum Mechanics* by his late great friend J. J. Sakurai, in order to extend this text's usefulness into the twenty-first century. Much consultation took place with the panel of Sakurai friends who helped with the original edition, but in particular with Professor Yasuo Hara of Tsukuba University and Professor Akio Sakurai of Kyoto Sangyo University in Japan.

This book is intended for the first-year graduate student who has studied quantum mechanics at the junior or senior level. It does not provide an introduction to quantum mechanics for the beginner. The reader should have had some experience in solving time-dependent and time-independent wave equations. A familiarity with the time evolution of the Gaussian wave packet in a force-free region is assumed, as is the ability to solve one-dimensional transmission-reflection problems. Some of the general properties of the energy eigenfunctions and the energy eigenvalues should also be known to the student who uses this text.

The major motivation for this project is to revise the main text. There are three important additions and/or changes to the revised edition, which otherwise preserves the original version unchanged. These include a reworking of certain portions of Section 5.2 on time-independent perturbation theory for the degenerate case, by Professor Kenneth Johnson of M.I.T., taking into account a subtle point that has not been properly treated by a number of texts on quantum mechanics in this country. Professor Roger Newton of Indiana University contributed refinements on lifetime broadening in Stark effect and additional explanations of phase shifts at resonances, the optical theorem, and the non-normalizable state. These appear as "remarks by the editor" or "editor's note" in the revised edition. Professor Thomas Fulton of the Johns Hopkins University reworked his Coulomb scattering contribution (Section 7.13); it now appears as a shorter text portion emphasizing the physics, with the mathematical details relegated to Appendix C.

Though not a major part of the text, some additions were deemed necessary to take into account developments in quantum mechanics that have become prominent since November 1, 1982. To this end, two supplements are included at the end of the text. Supplement I is on adiabatic change and geometrical phase (popularized by M. V. Berry since 1983) and is actually an English translation of the supplement on this subject written by Professor Akio Sakurai for the Japanese version of *Modern Quantum Mechanics* (copyright © Yoshioka-Shoten Publishing of Kyoto). Supplement II on nonexponential decays was written by my colleague here, Professor Xerxes Tata, and read over by Professor E. C. G. Sudarshan of the University of Texas at Austin. Although nonexponential decays have a long

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history theoretically, experimental work on transition rates that tests such decays indirectly was done only in 1990. Introduction of additional material is of course a subjective decision on the part of the editor; readers can judge its appropriateness for themselves. Thanks to Professor Akio Sakurai, the revised edition has been diligently searched to correct misprint errors of the first ten printings of the original edition. My colleague Professor Sandip Pakvasa provided me overall guidance and encouragement throughout this process of revision.

In addition to the acknowledgments above, my former students Li Ping, Shi Xiaohong, and Yasunaga Suzuki provided the sounding board for ideas on the revised edition when taking may graduate quantum mechanics course at the University of Hawaii during the spring of 1992. Suzuki provided the initial translation from Japanese of Supplement I as a course term paper. Dr. Andy Acker provided me with computer graphics assistance. The Department of Physics and Astronomy, and particularly the High Energy Physics Group of the University of Hawaii at Manoa, again provided both the facilities and a conducive atmosphere for me to carry out my editorial task. Finally I wish to express my gratitude to physics (and sponsoring) senior editor Stuart Johnson and his editorial assistant Jennifer Duggan as well as senior production coordinator Amy Willcutt, of Addison-Wesley for their encouragement and optimism that the revised edition would indeed materialize.

San Fu Tuan
Honolulu, Hawaii

Preface to the Second Edition

Quantum mechanics fascinates me. It describes a wide variety of phenomena based on very few assumptions. It starts with a framework so unlike the differential equations of classical physics, yet it contains classical physics within it. It provides quantitative predictions for many physical situations, and these predictions agree with experiments. In short, quantum mechanics is the ultimate basis, today, by which we understand the physical world.

Thus, I was very pleased to be asked to write the next revised edition of *Modern Quantum Mechanics*, by J. J. Sakurai. I had taught this material out of this book for a few years and found myself very in tune with its presentation. Like many other instructors, however, I found some aspects of the book lacking and therefore introduced material from other books and from my own background and research. My hybrid class notes form the basis for the changes in this new edition.

Of course, my original proposal was more ambitious than could be realized, and it still took much longer than I would have liked. So many excellent suggestions found their way to me through a number of reviewers, and I wish I had been able to incorporate all of them. I am pleased with the result, however, and I have tried hard to maintain the spirit of Sakurai's original manuscript.

Chapter 1 is essentially unchanged. Some of the figures were updated, and reference is made to Chapter 8, where the relativistic origin of the Dirac magnetic moment is laid out.

Material was added to **Chapter 2**. This includes a new section on elementary solutions including the free particle in three dimensions; the simple harmonic oscillator in the Schrödinger equation using generating functions; and the linear potential as a way of introducing Airy functions. The linear potential solution is used to feed into the discussion of the WKB approximation, and the eigenvalues are compared to an experiment measuring "bouncing neutrons." Also included is a brief discussion of neutrino oscillations as a demonstration of quantum-mechanical interference.

Chapter 3 now includes solutions to Schrödinger's equation for central potentials. The general radial equation is presented and is applied to the free particle in three dimensions with application to the infinite spherical well. We solve the isotropic harmonic oscillator and discuss its application to the "nuclear potential well." We also carry through the solution using the Coulomb potential with a discussion on degeneracy. Advanced mathematical techniques are emphasized.

A subsection that has been added to **Chapter 4** discusses the symmetry, known classically in terms of the Lenz vector, inherent in the Coulomb problem. This

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provides an introduction to SO(4) as an extension of an earlier discussion in Chapter 3 on continuous symmetries.

There are two additions to **Chapter 5**. First, there is a new introduction to Section 5.3 that applies perturbation theory to the hydrogen atom in the context of relativistic corrections to the kinetic energy. This, along with some modifications to the material on spin-orbit interactions, is helpful for comparisons when the Dirac equation is applied to the hydrogen atom at the end of the book.

Second, a new section on Hamiltonians with "extreme" time dependences has been added. This includes a brief discussion of the sudden approximation and a longer discussion of the adiabatic approximation. The adiabatic approximation is then developed into a discussion of Berry's Phase, including a specific example (with experimental verification) in the spin $\frac{1}{2}$ system. Some material from the first supplement for the previous addition has found its way into this section.

The end of the book contains the most significant revisions, including reversed ordering of the chapters on *Scattering* and *Identical Particles*. This is partly because of a strong feeling on my part (and on the part of several reviewers) that the material on scattering needed particular attention. Also, at the suggestion of reviewers, the reader is brought closer to the subject of quantum field theory, both as an extension of the material on identical particles to include second quantization, and with a new chapter on relativistic quantum mechanics.

Thus, **Chapter 6**, which now covers scattering in quantum mechanics, has a nearly completely rewritten introduction. A time-dependent treatment is used to develop the subject. Furthermore, the sections on the scattering amplitude and Born approximation are rewritten to follow this new flow. This includes incorporating what had been a short section on the optical theorem into the treatment of the scattering amplitude, before moving on to the Born approximation. The remaining sections have been edited, combined, and reworked, with some material removed, in an effort to keep what I, and the reviewers, felt were the most important pieces of physics from the last edition.

Chapter 7 has two new sections that contain a significant expansion of the existing material on identical particles. (The section on Young tableaux has been removed.) Multiparticle states are developed using second quantization, and two applications are given in some detail. One is the problem of an electron gas in the presence of a positively charged uniform background. The other is the canonical quantization of the electromagnetic field.

The treatment of multiparticle quantum states is just one path toward the development of quantum field theory. The other path involves incorporating special relativity into quantum mechanics, and this is the subject of **Chapter 8**. The subject is introduced, and the Klein-Gordon equation is taken about as far as I believe is reasonable. The Dirac equation is treated in some detail, in more or less standard fashion. Finally, the Coulomb problem is solved for the Dirac equation, and some comments are offered on the transition to a relativistic quantum field theory.

The **Appendices** are reorganzied. A new appendix on electromagnetic units is aimed at the typical student who uses *SI* units as an undergraduate but is faced with *Gaussian* units in graduate school.

I am an experimental physicist, and I try to incorporate relevant experimental results in my teaching. Some of these have found their way into this edition, most often in terms of figures taken mainly from modern publications.

- Figure 1.6 demonstrates the use of a Stern-Gerlach apparatus to analyze the polarization states of a beam of cesium atoms.

- Spin rotation in terms of the high-precision measurement of $g - 2$ for the muon is shown in Figure 2.1.

- Neutrino oscillations as observed by the KamLAND collaboration are shown in Figure 2.2.

- A lovely experiment demonstrating the quantum energy levels of "bouncing neutrons," Figure 2.4, is included to emphasize agreement between the exact and WKB eigenvalues for the linear potential.

- Figure 2.10 showing gravitational phase shift appeared in the previous edition.

- I included Figure 3.6, an old standard, to emphasize that the central-potential problems are very much applicable to the real world.

- Although many measurements of parity violation have been carried out in the five decades since its discovery, Wu's original measurement, Figure 4.6, remains one of the clearest demonstrations.

- Berry's Phase for spin $\frac{1}{2}$ measured with ultra-cold neutrons, is demonstrated in Figure 5.6.

- Figure 6.6 is a clear example of how one uses scattering data to interpret properties of the target.

- Sometimes, carefully executed experiments point to some problem in the predictions, and Figure 7.2 shows what happens when exchange symmetry is not included.

- Quantization of the electromagnetic field is demonstrated by data on the Casimir effect (Figure 7.9) and in the observation of squeezed light (Figure 7.10).

- Finally, some classic demonstrations of the need for relativistic quantum mechanics are shown. Carl Anderson's original discovery of the positron is shown in Figure 8.1. Modern information on details of the energy levels of the hydrogen atom is included in Figure 8.2.

In addition, I have included a number of references to experimental work relevant to the discussion topic at hand.

My thanks go out to so many people who have helped me with this project. Colleagues in physics include John Cummings, Stuart Freedman, Joel Giedt, David Hertzog, Barry Holstein, Bob Jaffe, Joe Levinger, Alan Litke, Kam-Biu Luk, Bob

McKeown, Harry Nelson, Joe Paki, Murray Peshkin, Olivier Pfister, Mike Snow, John Townsend, San Fu Tuan, David Van Baak, Dirk Walecka, Tony Zee, and also the reviewers who saw the various drafts of the manuscript. At Addison-Wesley, I have been guided through this process by Adam Black, Katie Conley, Ashley Eklund, Deb Greco, Dyan Menezes, and Jim Smith. I am also indebted to John Rogosich and Carol Sawyer from Techsetters, Inc., for their technical expertise and advice. My apologies to those whose names have slipped my mind as I write this acknowledgment.

In the end, it is my sincere hope that this new edition is true to Sakurai's original vision and has not been weakened significantly by my interloping.

Jim Napolitano
Troy, New York

In Memoriam

Jun John Sakurai was born in 1933 in Tokyo and came to the United States as a high school student in 1949. He studied at Harvard and at Cornell, where he received his Ph.D. in 1958. He was then appointed assistant professor of physics at the University of Chicago and became a full professor in 1964. He stayed at Chicago until 1970 when he moved to the University of California at Los Angeles, where he remained until his death. During his lifetime he wrote 119 articles on theoretical physics of elementary particles as well as several books and monographs on both quantum and particle theory.

The discipline of theoretical physics has as its principal aim the formulation of theoretical descriptions of the physical world that are at once concise and comprehensive. Because nature is subtle and complex, the pursuit of theoretical physics requires bold and enthusiastic ventures to the frontiers of newly discovered phenomena. This is an area in which Sakurai reigned supreme, with his uncanny physical insight and intuition and also his ability to explain these phenomena to the unsophisticated in illuminating physical terms. One has but to read his very lucid textbooks on *Invariance Principles and Elementary Particles* and *Advanced Quantum Mechanics*, or his reviews and summer school lectures, to appreciate this. Without exaggeration I could say that much of what I did understand in particle physics came from these and from his articles and private tutoring.

When Sakurai was still a graduate student, he proposed what is now known as the V-A theory of weak interactions, independently of (and simultaneously with) Richard Feynman, Murray Gell-Mann, Robert Marshak, and George Sudarshan. In 1960 he published in *Annals of Physics* a prophetic paper, probably his single most important one. It was concerned with the first serious attempt to construct a theory of strong interactions based on Abelian and non-Abelian (Yang-Mills) gauge invariance. This seminal work induced theorists to attempt an understanding of the mechanisms of mass generation for gauge (vector) fields, now recognized as the Higgs mechanism. Above all it stimulated the search for a realistic unification of forces under the gauge principle, since crowned with success in the celebrated Glashow-Weinberg-Salam unification of weak and electromagnetic forces. On the phenomenological side, Sakurai pursued and vigorously advocated the vector mesons dominance model of hadron dynamics. He was the first to discuss the mixing of ω and ϕ meson states. Indeed, he made numerous important contributions to particle physics phenomenology in a much more general sense, as his heart was always close to experimental activities.

I knew Jun John for more than 25 years, and I had the greatest admiration not only for his immense powers as a theoretical physicist but also for the warmth

and generosity of his spirit. Though a graduate student himself at Cornell during 1957–1958, he took time from his own pioneering research in K-nucleon dispersion relations to help me (via extensive correspondence) with my Ph.D. thesis on the same subject at Berkeley. Both Sandip Pakvasa and I were privileged to be associated with one of his last papers on weak couplings of heavy quarks, which displayed once more his infectious and intuitive style of doing physics. It is of course gratifying to us in retrospect that Jun John counted this paper among the score of his published works that he particularly enjoyed.

The physics community suffered a great loss at Jun John Sakurai's death. The personal sense of loss is a severe one for me. Hence I am profoundly thankful for the opportunity to edit and complete his manuscript on *Modern Quantum Mechanics* for publication. In my faith no greater gift can be given me than an opportunity to show my respect and love for Jun John through meaningful service.

San Fu Tuan