Explorations in Time-Frequency Analysis

An authoritative exposition of the methods at the heart of modern nonstationary signal processing from a recognized leader in the field. Offering a global view that favors interpretations and historical perspectives, it explores the basic concepts of time-frequency analysis and examines the most recent results and developments in the field in the context of existing, lesser-known approaches. Several example waveform families from bioacoustics, mathematics, and physics are examined in detail, with the methods for their analysis explained using a wealth of illustrative examples. Methods are discussed in terms of analysis, geometry, and statistics. This is an excellent resource for anyone wanting to understand the "why" and "how" of important methodological developments in time-frequency analysis, including academics and graduate students in signal processing and applied mathematics, as well as application-oriented scientists.

Patrick Flandrin is a research director at the CNRS (Centre national de la recherche scientifique), working in the Laboratoire de Physique of the École normale supérieure de Lyon. He is a Fellow of the IEEE and EURASIP, and a Member of the French Academy of Sciences.

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PATRICK FLANDRIN
École normale supérieure de Lyon

CAMBRIDGE
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108421027
DOI: 10.1017/9781108363181

First published 2018

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Flandrin, Patrick, author.
Title: Explorations in time-frequency analysis / Patrick Flandrin
(Ecole normale superieure de Lyon).
Description: Cambridge, United Kingdom ; New York, NY : Cambridge University Press, 2018. |
Includes bibliographical references and index.
Identifiers: LCCN 2018010021 | ISBN 9781108421027 (hardback) |
ISBN 1108421024 (hardback)
Subjects: LCSH: Signal processing–Mathematics. | Time-series analysis. | Frequency spectra.
Classification: LCC TK5102.9 .F545 2018 | DDC 621.382/20151955–dc23
LC record available at https://lccn.loc.gov/2018010021

ISBN 978-1-108-42102-7 Hardback

To Marie-Hélène, Lou, Margot, and Zoé,
for yesterday, for today, and for tomorrow.

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Cambridge University Press
978-1-108-42102-7 — Explorations in Time-Frequency Analysis
Patrick Flandrin
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Acknowledgments

The time-frequency explorations reported in this book have been shared for more than 30 years with a number of people that I am particularly pleased to thank for their collaboration, their criticisms (including comments on earlier drafts of this book, that I did my best to incorporate), and their friendship. Those include Paulo Gonçalves, Patrice Abry, Olivier Michel, Richard Baraniuk, and Franz Hlawatsch, with whom I had many time-frequency interactions in the early days of the SISYPH (Signals, Systems, and Physics) group in Lyon, and still have always fruitful discussions even if our scientific paths may have somehow moved apart; Éric Chassande-Mottin and François Auger, with whom we tirelessly pursued the quest for reassignment and its avatars, taking advantage of pointwise collaborations with Ingrid Daubechies and, more recently, Hau-tieng Wu; Pierre Borgnat and Nelly Pustelnik, who, among many other things, made compressed sensing and optimization enter the field; Cédric Richard, Paul Honeine, Pierre-Olivier Amblard, and Jun Xiao, with whom we revisited stationarity within the StaRAC project; Sylvain Meignen, Thomas Oberlin, Dominique Fourer, Jinane Harmouche, Jérémy Schmitt, Stephen McLaughlin, and Philippe Depalle, who helped opening new windows on signal disentanglement within the ASTRES project; Gabriel Rilling, Marcelo Colominas, Gaston Schlotthauer, and Maria-Eugenia Torres, who joined me (and Paulo Gonçalves) in the exploration of the foggy land of Empirical Mode Decomposition; Pierre Chainais and Rémi Bardenet, with whom I started discovering the fascinating territories of determinantal point processes and Gaussian Analytic Functions; and finally Géraldine Davis for her fresh and sharp eye.

A special mention is to be made of Yves Meyer: I had the privilege of meeting him during the earliest days of the wavelet adventure and benefiting from his scientific and human stature – not to mention his faultless support. I also thank Odile Macchi, whose work has (almost) nothing to do with the content of this book, but whose personality and scientific trajectory have always been an example for me, and whose day-to-day friendship is invaluable.

As a full-time researcher of CNRS (Centre national de la recherche scientifique, i.e., the French National Council for Scientific Research), I had the opportunity to pursue a research program in total freedom and the possibility of envisioning it in the longrun. I appreciate this privilege, as well as the excellent conditions I always found at École normale supérieure de Lyon and its Physics Department: it is my pleasure to warmly thank both institutions. Some recent aspects of the work reported here have been developed within the StaRAC and ASTRES projects, funded by Agence nationale de la recherche (under respective grants ANR-07-BLAN-0191 and ANR-13-BS03-0002-01), whose support is gratefully acknowledged. Finally, I would like to thank the Isaac Newton Institute for Mathematical Sciences, Cambridge, for support and hospitality during the “Statistical Network Analysis” program (supported by EPSRC grant no EP/K032208/1) where a large portion of the writing of this book has been undertaken and accomplished.

When not far from Internet and e-mail distractions in quiet Montgrenier, the rest of the writing was completed in Lyon, and in particular at a back table of Café Jutard, which is also gratefully acknowledged for its atmosphere and coffee quality.

Preface

Paterson: [voice over]
"I go through
trillions of molecules
that move aside
to make way for me
while on both sides
trillions more
stay where they are."
—Ron Padgett

Time-frequency can be considered the natural language of signal processing. We live in an ever-changing world, with time as a marker of events and evolutions. And from light to gravitation, from biological clocks to human activities, our everyday experience is faced with waves, oscillations, and rhythms, i.e., with frequencies.

Thinking of audio, the first attempts to record and transcribe the human voice go back to 1857, with the invention of the "phonautograph" by Édouard-Léon Scott de Martinville [1, 2]. The recording of speech as a signal was a success, but the transcription from the only waveform was a failure. The modern way out only came almost one century later with the invention of the "sound spectrograph" by W. Koenig, H. K. Dunn, and D. Y. Lacy [3], who in 1946 opened a new window on speech analysis by unfolding waveforms into time-frequency images. Starting therefore with "visible speech" [4] questions, the applications of time-frequency analysis later happened to be not only unlimited, but also instrumental in almost all areas of science and technology, culminating somehow in the pivotal role recently played by wavelet-like methods in the first detection of gravitational waves [5, 6].

The development of time-frequency analysis that has been observed since the 1980s has led to the writing of many articles as well as of a number of books, including by the present author [7], and one may question the need to add one more piece to the existing literature. In fact, some of the books written at the turn of the century were originally research monographs, and over the years, their content has either become standard material or has been sidestepped or superseded by new developments. New achievements have appeared and, for newcomers to the field, have been adopted mainly in comparison to the current state of the art, which itself resulted from a cumulative construction that most often lost track of the earliest attempts. One of the motivations for the writing of the present book is therefore to be found in the desire to offer the reader an overview of key concepts and results in the field by bridging new advances and older ideas that, even if they have not been fully followed per se, have been instrumental in deriving the techniques that were eventually adopted. In doing so, this book presents a series of *explorations* that mix elementary, well-established facts that are at the core of time-frequency analysis with more recent variations, whose novelty can be rooted in much older ideas.

Notation

STFT	Short-Time Fourier Transform
EMD	Empirical Mode Decomposition
AM	Amplitude Modulation
FM	Frequency Modulation
1D, 2D, 3D	one-dimensional, two-dimensional, three-dimensiional
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
i	$\sqrt{-1}$
$\mathrm{Re}\{z\}$	real part of $z \in \mathbb{C}$
$\mathrm{Im}\{z\}$	imaginary part of $z \in \mathbb{C}$
$\mathbb{P}(.)$	probability function
$p(.)$	probability density function
$\mathbb{E}\{X\}$	expectation value of X
$\mathrm{var}\{X\}$	variance of X
$\mathrm{cov}\{X, Y\}$	covariance of X and Y
$\mathcal{H}(p)$	Shannon entropy of probability density function p
$\mathcal{H}_\alpha(p)$	α-order Rényi entropy of probability density function p
t	time
ω	(angular) frequency
$x(t)$, $X(\omega)$	Fourier transform pair
$x^*(t)$	complex conjugate of $x(t)$
E_x	energy of $x(t)$
τ	time delay
ξ	Doppler shift
$\|x\|_p$	L_p-norm of $x(t)$
$L^p(\mathbb{R})$	space of L_p-integrable functions
$\langle x, y\rangle$	inner product of $x(t)$ and $y(t)$
$\overline{F}^\rho$	average of F with respect to the density ρ
$\delta(.)$	Dirac's "δ-function"
δ_{nm}	Kronecker's symbol (= 1 if $n = m$ and 0 otherwise)
$e_\omega(t)$	monochromatic wave of frequency ω
$\mathbf{1}_T(t)$	indicator function of interval T (= 1 if $-T/2 \leq t \leq +T/2$ and 0 otherwise)
$\gamma_x(\tau)$	stochastic correlation function of $x(t)$
$\tilde{\gamma}_x(\tau)$	deterministic correlation function of $x(t)$

$\Gamma_x(\omega)$	spectrum density of $x(t)$
$r_x(\tau)$	relation function of $x(t)$
$(\mathbf{F}x)(t)$	Fourier transform of $x(t)$
$(\mathbf{H}x)(t)$	Hilbert transform of $x(t)$
$(\mathbf{M}x)(t)$	Mellin transform of $x(t)$
$(\mathbf{T}_{\tau,\xi}x)(t)$	time-frequency shift operator acting on $x(t)$
$h_k(t)$	Hermite function
$g(t)$	circular Gaussian window
$\mathcal{F}_x(z)$	Bargmann transform of $x(t)$
$F_x^{(h)}(t,\omega)$	STFT of $x(t)$ with window $h(t)$
$S_x^{(h)}(t,\omega)$	spectrogram of $x(t)$ with window $h(t)$
$\hat{S}_x^{(h)}(t,\omega)$	reassigned spectrogram of $x(t)$ with window $h(t)$
$\hat{t}(t,\omega)$, $\hat{\omega}(t,\omega)$	reassignment time, reassignment frequency
$\hat{\mathbf{r}}_x(t,\omega)$	reassignment vector field of $x(t)$
$\tilde{F}_x^{(h)}(t,\omega)$	synchrosqueezed STFT of $x(t)$ with window $h(t)$
$W_x(t,\omega)$	Wigner distribution of $x(t)$
$A_x(\xi,\tau)$	ambiguity function of $x(t)$
$C_x(t,\omega;\varphi)$	Cohen's class distribution of $x(t)$ with kernel $\varphi(\xi,\tau)$