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Patrick Flandrin
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1 Introduction

One buzzword that has gained in popularity since the beginning of this century is "data science." What data science actually is, however, is a matter of debate (see, e.g., [8]). It can be argued that, since its objectives are to collect, analyze, and extract information from data, data science is ultimately a revamping of "statistics." A case could also be made for adding "signal processing" to the picture since, according to IEEE (the flagship society for the field), signal processing is "the science behind our digital life." As such, the very broad scientific coverage of signal processing makes it difficult to delineate for it a clear borderline with data science, which itself appears as one of the items listed by IEEE.

Whatever the terminology, the "data science/signal processing" main issue can be summarized as follows:

> Starting from some *data* (be they human-made or given by nature), the objective is to extract from them some information, assumed to exist and considered of interest.

Remark. The question of what is "of interest" or not makes the end user enter the process. A common distinction is made in signal processing between "signal" and "noise" (we will come back to this in Chapter 3), but it must be understood that this has only a relative meaning. For instance, in the case of the famous *cocktail-party problem*, which can be formulated in terms of source separation, it is clear that, when considering one specific conversation, this one becomes a "signal" while other conversations, although they are of the very same nature, are "noise." In a nutshell, signals to some are noise to others. Another example is given by the so-called *Cosmic Microwave Background* that can be seen as either a perturbation for communications (this was even the way it was discovered by Arno Penzias and Robert W. Wilson in 1965) or a scientific object per se, the analysis of which gives invaluable information about the early universe [9].

The process of extracting information from data encompasses many facets that may include acquisition, transformation, visualization, modeling, estimation, or classification. Signal processing (or data analysis) has therefore something to do with three main domains, namely *physics*, *mathematics*, and *informatics*. First, physics, which has to be understood at large, i.e., as in direct connection with the physical world where data live and/or are originated from (from this point of view, this also includes biological or even

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symbolic data). Then, mathematics (including statistics), which permits us to formalize transforms and manipulations of data, as well as assessing the performance of analysis and processing methods. And finally, informatics, which helps dealing with digitized data and turns processing methods into algorithms.

Remark. Signal processing is interdisciplinary by nature, making more difficult to appreciate its specificity when referring to classical categorizations of science (in the spirit of, say, Auguste Comte). Indeed, it took a while for it to be recognized as a discipline of its own. In its early days, i.e., during World War II and right after, signal processing was not necessarily named as such and was mostly considered through its applied and/or technological aspects, despite theoretical breakthroughs such as Norbert Wiener's "yellow peril" seminal book, which first took the form of a classified report in 1942 and was eventually published as a book in 1949 [10]. Though still facing practical problems (raised in particular by the U.S. Navy and the French Navy about background noise in underwater acoustics), several efforts were then pursued for giving signal processing solid grounds by bridging physics and mathematics. One landmark book in this respect is *Théorie des Fonctions Aléatoires* (i.e., *Theory of Random Functions*) by André Blanc-Lapierre and Robert Fortet [11]. This pioneering book established the field and launched a successful French school that later, in 1967, organized the first ever Groupe d'Etudes du Traitement du Signal (GRETSI) conference – the first one of the series of GRETSI symposia that are still run every two years – that was specifically dedicated to signal processing. This was followed about 10 years later by IEEE, which held its first International Conference on Acoustics, Speech and Signal Processing, or ICASSP, conference in 1976. Roughly speaking, considering countless conferences, books, and periodicals that are now flourishing, one can say that signal processing emerged as a recognized field during the 1970s.

In short, signal processing exists at the intersection of different fields, reducing to none of them but gaining from their confrontation an intrinsic value that goes beyond a simple addition of each. In "complex systems" terminology, we would say that "the whole is more than the sum of the parts" and, as illustrated in Figure 1.1:

> We claim that the success of a signal processing/data analysis method, as well as its acceptance by the scientific community, is based on a well-balanced importance of the three components of the "golden triangle" whose vertices are physics (data), mathematics (formalizations and proofs), and informatics (algorithms).

Remark. A companion interpretation of a similar "golden triangle" can be given by attaching to the bottom-right vertex the possibilities offered by informatics in terms of *simulation*. In such a case, the balance is among classical experiments rooted in physics, models expressed in mathematical terms, and numerical data generated by model-based equations governed by physics.

Let us support our claim about the "golden triangle" of Figure 1.1 by considering two examples that are closely related to the purpose of this book and to the methods that will be considered (we will later discuss some counterexamples).

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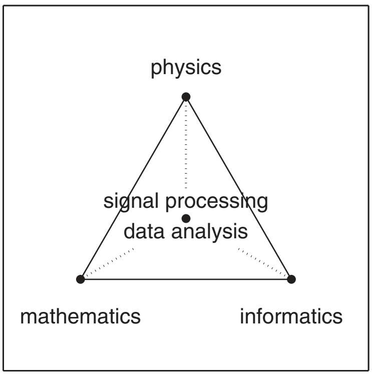

Figure 1.1 The "golden triangle." This symbolic representation states that signal processing (or data analysis) is based on interactions among three basic components rooted in the more classical fields of physics (data), mathematics (formalizations and proofs), and informatics (algorithms).

Example 1 – Fourier. Let us start with the first success story, namely Fourier analysis, which is of considerable importance in signal processing. The starting point is clearly physics, since the purpose of Joseph Fourier's seminal work (completed in 1811, yet published only in 1822 [12]) was to answer the question, raised by the French Academy of Sciences, of establishing an analytic theory of heat. To this end, Fourier developed a mathematical technique of expansions based on sines and cosines that is commonly used today (Fourier series and integrals), and launched the whole field of harmonic analysis, which experienced tremendous developments during the nineteenth and twentieth centuries. Because of its potential in many applications, it soon became natural to look for "implementations" of Fourier analysis, leading to the invention of mechanical, optical, acoustical, and electrical devices [13]. Following the electronic revolution and the advent of computers, however, the true breakthrough came from algorithmics with the publication of the *Fast Fourier Transform* by James Cooley and John Tukey in 1965 [14]. Besides physics (with an unlimited range of applications to all kinds of data) and mathematics, this was the third key ingredient that really boosted Fourier analysis and made it enter the toolkit of every scientist. Incidentally, it is worth remarking that this point of view, which considers the three mentioned aspects altogether, was already at the heart of Fourier's program, as attested by two quotes [15] excerpted[1] from his *Théorie Analytique de la Chaleur* [12]. The first one is quite famous and states that "The deep study of nature is the most fruitful source of mathematical discoveries." The second one is less known, but nonetheless visionary, since it reads: "This difficult research required a special analysis, based on new theorems [. . .]. The proposed method ends up with nothing vague and undetermined in its solutions; it drives them to their ultimate numerical applications, a condition which is necessary for any research, and without which we would only obtain useless transformations." Nothing to add: physics, mathematics, numerics – all three are equally necessary in Fourier's words.

[1] The translation from French to English is mine.

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Example 2 – Wavelets. The second example is quite similar to Fourier analysis, and it is in some sense one of its avatars. As for Fourier and heat theory, *wavelet theory* also originated from a physics problem, namely the question of *vibroseismics* in geophysics. The technique, used for oil exploration, basically amounts to sending mechanical vibrations of increasing frequency into the ground from the surface and analyzing the returning echoes to produce information about the underlying geological structure. Because of the nonstationary nature of the excitation, Jean Morlet, a French engineer working for the Elf-Aquitaine group, was keen to use a time-frequency analysis, but he realized that the standard way, namely the Gabor expansion, had some shortcomings. Some of them were of a computational nature (typically, unstable reconstructions) and other ones came from physical considerations. Morlet's major objection to Gabor analysis was that it is based on a window of fixed duration, whatever the analyzed frequency. This means that, when analyzing "high" frequencies, the window may contain many oscillations, whereas when going down to "low" frequencies, the same window may contain only a fraction of one oscillation, questioning the concept of frequency itself. This echoes a remark of Norbert Wiener in his autobiography *Ex-Prodigy: My Childhood and Youth* [16]: "A fast jig on the lowest register of the organ is in fact not so much bad music as no music at all."

This physical observation prompted Morlet to look for a better mathematical decomposition, and his proposal was to forget about a duration-invariant window with a variable number of oscillations, preferring a *shape-invariant* waveform made of a few oscillations (hence the name "wavelet"), whose duration would be *locked* to the (inverse) analyzed frequency. This seemingly simple idea was first developed with Alex Grossmann from a mathematical physics point of view [17]. Soon after, it became a major topic in mathematics thanks to the pioneering works of Yves Meyer [18], Ingrid Daubechies [19], and Stéphane Mallat [20] (to name but a few), who put theory on firm and elegant grounds. As for Fourier and the Fast Fourier Transform (FFT), the impact of wavelets has been leveraged when wedding mathematics with electrical engineering, recognizing that wavelet bases can be given a filter bank interpretation, and that wavelet transforms can be equipped with fast and efficient algorithms [20]. Again, starting from physics and getting a proper mathematical formalization, closing the "golden triangle" with computational efficiency was instrumental in adopting and spreading wavelets in almost every domain of science and technology.

Coming back to data science in general, a current trend is to think "big." Indeed, we are now overwhelmed by a deluge of data that may take a myriad of forms and dimensionalities, be they multivariate, multimodal, hyperspectral, non-euclidian, or whatever. This has created a move of data analysis from classical signal processing or time series analysis toward new avenues that are paved with buzzwords such as data mining, large-scale optimization, or machine learning (preferably "deep"). Of course, the point is not to question those approaches that led to tremendous success stories. It is rather to consider that there still remains some room for a more "entomological" study of the fine structure of modest size waveforms, calling in turn for some "surgical" exploration of the methods dedicated to their analysis. This is what this book is about.

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> In this era of "big data," we propose to think about signals another way, to think "small"!

To achieve this program, a specific perspective will be adopted, namely that of describing signals *jointly in time and frequency*; this is similar to the method of analysis used in the wavelet technique mentioned previously, but is not restricted to only this type of analysis. It is well known that a signal most often takes the form of a "time series," i.e., a succession of values that reflect the temporal evolution of some quantity. This may concern speech, music, heartbeats, earthquakes, or whatever we can imagine as the output of some sensor. Most data often involve rhythms, cycles, and oscillations; it is also well known that there exists a powerful mathematical tool, the *Fourier transform*, that allows for a complementary description of the very same data in a dual, frequency-based domain. This yields a different yet equivalent representation, and we can go back and forth from one to the other without losing any information. However, what each of those representations tells us about some data is not only of a different nature but also *orthogonal* in the sense that they are exclusive of each other: a frequency spectrum just ignores time, mirroring the "natural" representation in time that makes no direct reference to a frequency content.

As powerful as it has proven to be from a mathematical point of view, this alternative contrasts with our everyday experience, which seems to indicate that time and frequency should be able to interact and exchange information; after all, whistling does seem to make frequency vary with time. While this is an idea that nobody really has a problem with, it is something that a strict Fourier representation cannot easily handle. The classic analogy that is used when speaking of overcoming this Fourier limitation is often that of a *musical score*, i.e., a symbolic representation that makes use of two dimensions for writing down a musical piece: time on the one hand for the occurrence and duration of different notes, and frequency on the other hand for their pitch. It should be noted, however, that a musical score is a *prescription* for what a signal (the musical piece when actually played) should be.

> *Time-frequency* analysis goes somehow the other way, its very purpose being the *writing* of the musical score, given a recording.

There is unfortunately no unique and completely satisfactory way of achieving this program, but there are a number of methods whose detailed study permits a meaningful characterization of signals that reconciles mathematical description and physical intuition, hopefully closing the "golden triangle" with efficient algorithms. This is also what this book is about.

One more word. This book is not intended to be a comprehensive treatise of time-frequency analysis that would cover all aspects of the field (this can be found elsewhere; see, e.g., [21], [22], [7], or [23], to name but a few). It must rather be seen as an *exploration*, a *journey* in which stops are made when needed for addressing specific issues. The construction is in no way axiomatic or linear. What is privileged is *interpretation*, at the expense of full proofs and, sometimes, full rigor.

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Roadmap. The book is organized as follows. Following this Introduction, Part I is devoted to "Basics and Constraints," i.e., to a general presentation of the fundamental problems and tools that motivate the use of a time-frequency analysis. This begins in Chapter 2 with a discussion of specific signals (encountered in physics, bioacoustics, or mathematics) that are in some sense emblematic of a need for time-frequency analysis and that will be considered again at the end of the book. Chapter 3 then discusses notions of noise, in contrast with what is expected from signals as more structured objects. The Fourier description of signals in time *or* frequency is addressed in Chapter 4, with special emphasis on Gaussian waveforms for the pivotal role they play in many questions of time-frequency analysis. This is in particular the case for the uncertainty relations that put a limit on joint localization, and that are exposed under different forms in Chapter 5. Based on the previous considerations, Chapter 6 enters the core of the subject, introducing in a mostly interpretative way basic time *and* frequency representations and distributions. This offers the possibility of revisiting uncertainty in Chapter 7, from a completely time-frequency-oriented perspective. Finally, Chapter 8 discusses the key concept of (non-)stationarity, with a revisiting of time-frequency that allows for an operational definition.

Part II of this book is concerned with a more detailed exploration of the structure of time-frequency distributions in terms of "Geometry and Statistics." Chapter 9 focuses on the geometry of the spectrogram and its interpretation. This leads on to a discussion in Chapter 10 of a number of variations aimed at sharpening a spectrogram, based on ideas of reassignment, synchrosqueezing, or sparsity. Such approaches are indeed reminiscent of alternative techniques related to the so-called Hilbert–Huang Transform, which this book digresses to examine in Chapter 11. Chapter 12 comes back to the mainstream of the book, with a deeper interpretation of the spectrogram geometry in the Gaussian case, deriving spatial organizations in the plane from the structure of the reassignment vector field. Whereas the underlying construction rules apply equally to any waveform, the noise case is more specifically addressed in Chapter 13, with uncertainty revisited in terms of statistical correlation. This is detailed further in Chapter 14, which proposes a simple (randomized lattice) model for the distribution of local maxima considered as a 2D point process. Similar considerations are followed in Chapter 15 for zeros, in connection with the theory of Gaussian Analytic Functions. The importance of spectrogram zeros is stressed by the proposal of a zeros-based algorithm for time-frequency filtering, as well as by "universal" properties attached to such characteristic points. With all the techniques discussed so far at our disposal, Chapter 16 comes back to the examples of Chapter 2, elaborating on their possible time-frequency analyses and on the information that can be gained from them.

Finally, a short Conclusion is followed by a series of commented-upon links to free software tools permitting actual implementation of most of the techniques discussed in the book.

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