

A First Course in Quantitative Finance

This new and exciting book offers a fresh approach to quantitative finance and utilizes novel new features, including stereoscopic images which permit 3D visualization of complex subjects without the need for additional tools.

Offering an integrated approach to the subject, *A First Course in Quantitative Finance* introduces students to the architecture of complete financial markets before exploring the concepts and models of modern portfolio theory, derivative pricing, and fixed-income products in both complete and incomplete market settings. Subjects are organized throughout in a way that encourages a gradual and parallel learning process of both the economic concepts and their mathematical descriptions, framed by additional perspectives from classical utility theory, financial economics, and behavioral finance.

Suitable for postgraduate students studying courses in quantitative finance, financial engineering, and financial econometrics as part of an economics, finance, econometric, or mathematics program, this book contains all necessary theoretical and mathematical concepts and numerical methods, as well as the necessary programming code for porting algorithms onto a computer.

Professor Dr. Thomas Mazzone has lectured at the University of Hagen and the Dortmund Business School and is now based at the University of Greifswald, Germany, where he received the 2014 award for excellence in teaching and outstanding dedication.

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