1 Plasma Physics Fundamentals

Introduction

The accepted definition of plasma is that in an electrically conducting medium, all paired and closely positioned positive and negative charges shield each other from an externally applied electromagnetic field (Langmuir 1929). Therefore, plasma is the matter in a particular state that has the fundamental property of global electrical neutrality. The characteristic charge separation distance is the *Debye shielding length* and is the smallest length scale compared to all other macroscopic physical dimensions in plasma. Within this shielding sphere, the paired charged particles are not free from each other, but interact permanently, which leads to many intrinsic properties of the medium.

Charged particles in plasma interact with each other primarily by Coulomb force, magnetic induction, and collisions with neutral particles. The probability of collision can be expressed in terms of an effective momentum transfer cross-section area and the mean free path between collisions. The collision mechanism leads to the transport properties such as diffusion, mobility, and resistivity in the plasma. Plasma exists not only in gas and liquid but also in a solid conductor, except the electrons in a solid are closely bound but still can move within atomic or molecular structures between collisions. Although mass motion of charges does not generally take place, when an external electromagnetic field is applied, the dynamic effects can always be observed in electric conduction, the Hall effect, and polarization.

Plasma is electrically global neutral, and is dominated by interactions of charged particles of opposite polarity within the Debye shielding length. A strong electrostatic force exists between the paired charges; any small perturbation to the equilibrium separation distance will trigger a high-frequency oscillation by the restoring force. This oscillatory motion is referred to as the *plasma frequency*, which is distinguished from the lower-frequency oscillation involving mass motion. In addition, plasma does not naturally conform to its surroundings and will alter its domain according to local and distant conditions by the *Coulomb force* and the *Lorentz acceleration*. The other unique feature from the quasi-neutrality of plasma is that it stores inductive energy, and it contributes to resistance and inductance when the current circuit forms a closed loop. This characteristic is exemplified by the drastic change in *electric conductivity* σ through a strong response to electromagnetic fields.

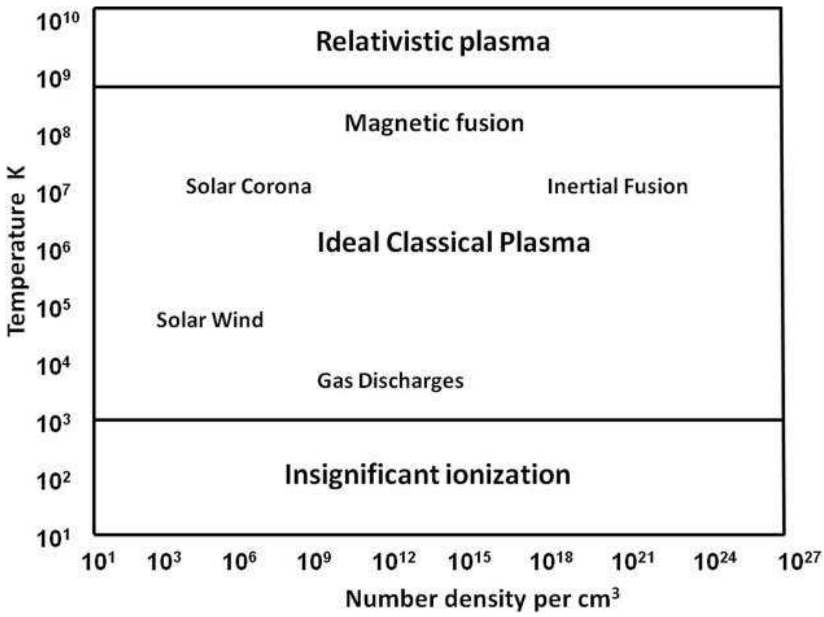

Figure 1.1 Classification for types of plasma.

Finally, the disturbance communication in plasma is not only by means of collision processes but also conveyed by transverse wave with phase velocities equal to the speed of light.

Plasma in the gas phase has a negligible shear stress and therefore does not have a definite shape or volume. The electric-conducting medium responds to electromagnetic fields, so plasma can be confined either by a solid container or by a magnetic field. Plasma appears in a wide range of formations such as electric arcs, filaments, micro-discharges, multiple layers in the presence of shock waves, and cellular structure in outer space. The spontaneous formations of unique spatial features on a wide range of length scales manifest the complexity of plasma. In fact, plasma is the most common state of matter by volume and is the fourth most common state of matter in rarefied and intergalactic environments. Description of plasma is therefore usually by its energy state and degree of ionization. The classification of plasma is commonly recognized by the electron number density and temperature in electron volts or the static temperature as displaced in Figure 1.1.

The topics of degenerated quantum plasma and relativistic plasma are beyond the scope of plasma dynamics for aerospace engineering. Therefore, present discussions are focused on plasma that exists around atmospheric pressure, and on mixed thermal and nonthermal conditions. According to convention, the thermal condition of plasma is based on the relative temperatures between electrons, ions, and neutral components. The *nonthermal* or *cold plasma* means that the ions and neutral particles have a much lower temperature than the electrons. For most gas discharges generated by electron impact, the electron temperature is at most 3 eV

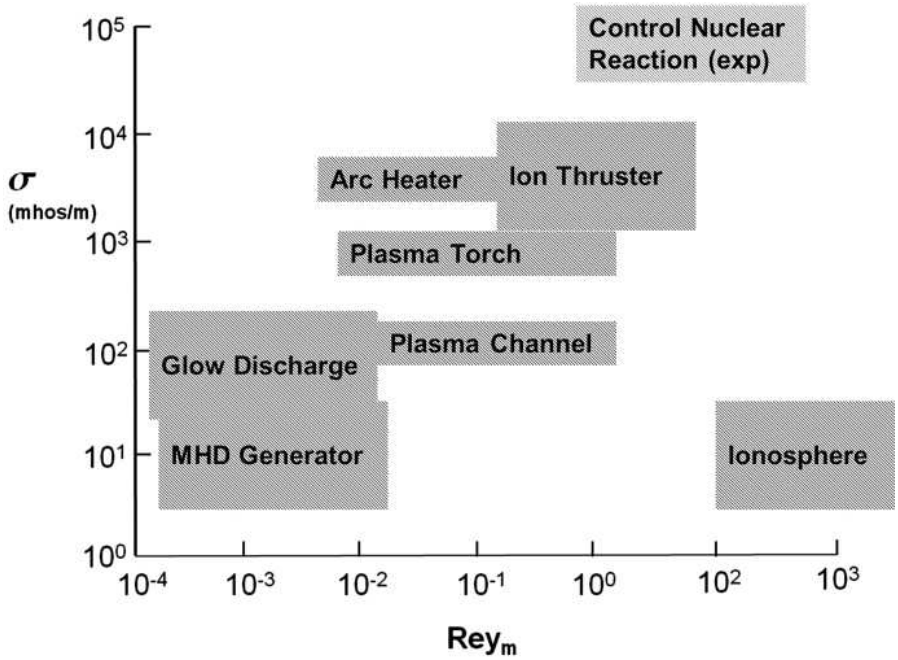

Figure 1.2 Range of electric conductivity and magnetic Reynolds numbers for common engineering applications.

(around 3×10^5 10^5 K), while the ions and neutrals are at near room temperature (Raizer 1991). When ionization is achieved by a strong shock compression in high pressure, the temperatures of the plasma composition are on the same order, around ten thousand degrees Kelvin or higher, but are not always in thermodynamic non-equilibrium. Therefore, the plasma of interest for aerospace applications is generally limited to an electron number density up to 10^{20}/cm^3 and an overall temperature lower than 10^6 K (Surzhikov 2013).

In engineering applications, the relative magnitude of the electromagnetic force and the inertia of gas motion have a strong bearing on the characteristics and structure of plasma. A classification of plasma by the *magnetic Reynolds number* becomes very important for this purpose, because the magnetic induction is generated by electric current, which is described by electric conductivity σ, charge particle velocity u, and the charge number density. The magnetic Reynolds number for some characteristic scale l is defined as $R_m = ul\mu\sigma$ in which $\sigma\mu$ is often considered as the magnetic diffusivity and μ is the *magnetic permeability* of the medium. The magnetic Reynolds number has often been interpreted as a measure of the ratio of the induced and total magnetic field of the plasma. For flows with a low magnetic Reynolds number, the convection by the magnetic field lines is negligible, and the induced magnetic field by electric current is also negligible. The relationship is presented in Figure 1.2 between the electrical conductive and magnetic Reynolds numbers for most engineering applications. In general, all the assembled engineering applications presented here have the same order of magnitude in characteristic lengths and velocity scales, and the ionized gas is generated mostly by an electrical field. Thus, the magnetic Reynolds number is often much less than unity. The condition of ionosphere is included as a comparative reference, because all other astrophysical applications always have a huge physical dimension that differs from the common aerospace engineering applications. As an example, the solar atmosphere has a similar value of electric conductivity in an arc heater, but its length scale is ten million times greater and leads to a proportional greater magnetic Reynolds number (Mitchner and Kruger 1973; Shang 2016).

All physical phenomena of plasma are governed by the Maxwell equations, but once the plasma is treated as a continuum for engineering applications, it becomes an interdisciplinary subject at macroscopic scales. The global thermodynamic and kinematic properties of plasma must be described through a distribution function between microscopic and macroscopic dynamics. This approach is accurate when the microscopic structure can be linked approximately to macroscopic motion by the Maxwell-Boltzmann distribution. However, by this approach the detailed wave-particle interaction will be unresolved.

1.1 Intrinsic Electromagnetic Forces

Plasma always exists in electric and magnetic fields. The electric charged particles interact and respond to externally applied fields and always create induced field components; the total electromagnetic field is therefore a sum of the applied and the induced components. One of two elementary electromagnetic forces arises from the attraction and repulsion of charged particles of the opposite and same polarities known as the *electrostatic force*. A free charged particle motion will produce a magnetic field, and when it is moving within an applied magnetic field will be compelled by the *Lorentz force* to accelerate in the direction perpendicular to both the charged particle motion and the magnetic field.

The electrostatic force between two singly charged particles is described by *Coulomb's law*. The force between charged particles is collinear along the unit space vector between two charges separated by a distance r_{ij} as:

$$\boldsymbol{F}_{ij} = \frac{1}{2\pi\varepsilon_o}\frac{q_i q_j}{r_{ij}^2}\boldsymbol{r}_{ij} \tag{1.1}$$

In Equation (1.1), the symbol $\boldsymbol{r}_{ij}$ designates the unit vector between the two charged particles. The magnitude of the force is directly proportional to the product of the two electric charges and inversely proportional to the square of the distance between them. The symbol ε is designated as the *electric permittivity* and in the international system of units (SI) it has a value of 8.85×10^{-12} Farad/m (Krause 1953). The charges q_i and q_j are measured in Coulombs, which attract each other if they are of the opposite polarity, but repulse each other if they are the same. The resulting force per unit charge is usually defined as the electric field intensity $\boldsymbol{E}$, and the source of the electric intensity is derived from the electric charge. However, a conductor and an isolator will respond to the electric field differently. Through the definition of electric intensity; the electrostatic force can be given as:

$$\boldsymbol{F}_i = \boldsymbol{E} q_i \tag{1.2}$$

The SI unit of the electric field intensity is the Newton per Coulomb, however, in practical application, it is often given in Volt per meter. For multiple point charges, the principle of superposition applies so the force is the sum of all other particle charges. The entire electric field of space charges is the vector sum of the entire field from all the individual source charges, in which the symbol $\hat{r}_{ij}$ denotes the unit vector between the charges:

$$\boldsymbol{E}_i = \frac{1}{4\pi\varepsilon_o} \Sigma \frac{q_i}{r_{ij}^2} \boldsymbol{r}_{ij} \tag{1.3a}$$

In an electrically conducting medium, the free charge particle movement will produce electric current and exert additional force on each other. In an isolator, the molecules of a dielectric material will polarize to reduce the net local field intensity. The different effects on two kinds of media are described by a *dielectric constant* κ and Coulomb's law becomes:

$$\boldsymbol{F}_{ij} = \frac{1}{4\pi\varepsilon_o \kappa} \frac{q_i q_j}{r_{ij}^2} \boldsymbol{r}_{ij} \tag{1.3b}$$

The dielectric constant κ has a value of unity in vacuum and for most gas species of air. However, for isolators such as polystyrene, glass, and rubber, the constant has the values of 2.7, 4.7, and 6.7, respectively.

The free electric charge in motion produces a conductive electric current. In metallic conductors the charge is carried by electrons with an elementary negative charge of 1.61×10^{-19} Coulomb. In liquid conductors, such as electrolytes, the charge is carried by both positive and negative ions. The electric field compels the free charges into continuous motion and results in an electric current that can be defined in terms of the electric flux vector per unit area. The conductive current density is different from the convective and displacement current densities. The convective current does not involve an electrically conducting medium and consequently does not satisfy Ohm's law. The convective current flows through an isolating medium such as a liquid, rarified gas, or vacuum, and the best example is an electron beam within a vacuum tube (Jahn 1968). The displacement current arises from the time-varying electric field and is introduced by Maxwell to account for the generation of a magnetic field when the conductive current is zero. Without the concept of the displacement current, the electromagnetic wave propagation would be impossible (Stratton 1953).

The conductive electric current density $\boldsymbol{J}$ has a physical dimension of coulomb per second or ampere. It is a vector for which its orientation is dictated by the vector sum of all electric particle velocities:

$$\boldsymbol{J} = \Sigma n_i q_i \boldsymbol{u}_i = \Sigma \rho_i \boldsymbol{u}_i \tag{1.4}$$

where n_i denotes the charge particle number density and $\boldsymbol{u}_i$ is the averaged velocity of the charged particles. The current charge density ρ_i is defined as the sum of the total electric charges per unit volume of species *i*.

A magnetic field generated by an electric current and the orientation of the induced magnetic field is defined by the right-hand rule. The current may be due to an externally applied electromagnetic field, or an electron beam, or a conductive current in current-carrying wire. Analogous to the electrostatic force, the *magnetic field intensity* $\boldsymbol{H}$ is defined by the field intensity at a distance r from the source. In fact, the differential magnetic field intensity is governed by the *Biot-Savart law* for magnetostatics, which is the counterpart of Coulomb's law for electrostatics. The field intensity is proportional to the product of a differential current element $\boldsymbol{J}d\boldsymbol{l}$, direction sine of the angle between the points of interest, and inversely proportional to the square of the mutual distance. The induced magnetic field line is continuous. In vector form the Biot-Savart law gives (Jackson 1999):

$$d\boldsymbol{H} = \frac{\boldsymbol{J} \times d\boldsymbol{l}}{4\pi r^2} \tag{1.5a}$$

A current-carrying wire produces a magnetic field perpendicular to the electric current, and the magnetic field strength $\boldsymbol{H}$ has the physical unit of Amp/m^2. The magnetic field orientation is defined by the right-hand rule with respect to the generating electric current. Similar to electrostatics, the intermediary magnetic field variable is the *magnetic flux density* $\boldsymbol{B}$ or is also called the *magnetic induction*, which has the physic unit of Weber/m^2 or Amp/m^2. The connection between the magnetic field strength $\boldsymbol{H}$ and the magnetic flux density $\boldsymbol{B}$ in an isotropic medium is the constitute relation; $\boldsymbol{B} = \mu \boldsymbol{H}$ and μ is referred to as the *magnetic permeability*. In free space, it has the dimensional value of $\mu = 4\pi \times 10^{-1}$ Hennery/meter.

From Equation (1.5a), the magnetic flux density $\boldsymbol{B}$ therefore can be given as:

$$\boldsymbol{B} = \frac{\mu}{4\pi} \iiint \frac{\boldsymbol{J} \times d\boldsymbol{l}}{r^2} dv \tag{1.5b}$$

Like that of electrostatics, the force on a current element becomes:

$$d\boldsymbol{F}_{ij} = J_i d\boldsymbol{l}_i \times d\boldsymbol{B}_{ij} \tag{1.5c}$$

Note that the induced forces components $\boldsymbol{F}_{ij}$ and $\boldsymbol{F}_{ji}$ are symmetric, thus opposite each in directions. However, the electric current–induced forces still can contradict *Newton's third law* by the free charge movement and by the transient electric current in an incomplete circuit (Jahn 1968).

If two current-carrying wires are brought into the vicinity of each other, each wire is surrounded by two individual magnetic fields, leading to a force that acts on the wires. When the wires are carrying current in the same direction, the wires are attracted to each other; when the wires are carrying current in opposite directions, the wires are repelled. The interaction of two electric elements is depicted in

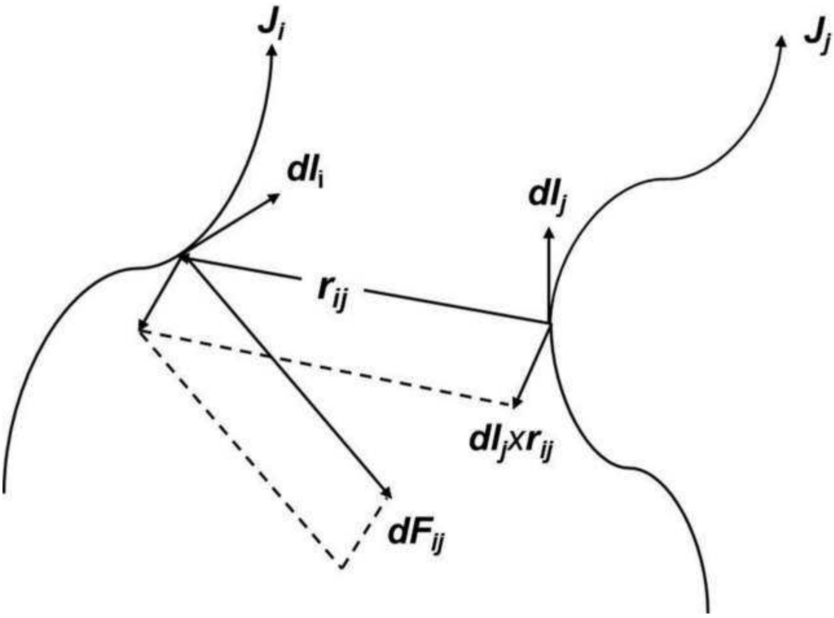

Figure 1.3 Induced magnetic force by two electric current elements.

Figure 1.3. The incremental force existing between the elements of the current path J_i and J_j is:

$$d\boldsymbol{F}_{ij} = \frac{\mu}{4\pi}\frac{J_i J_j}{r_{ij}^2}[d\boldsymbol{l}_i \times (d\boldsymbol{l}_j \times \hat{r}_{ij})] \tag{1.5d}$$

Basically, there are two modes of the electromagnetic body force in an electrically conducting medium that can exert on the charged particles. The interaction of the electric field with the free charge density of the medium $\boldsymbol{F}_{\mathbf{e}} = \rho_e \boldsymbol{E}$ is known as the *electrostatic force*. The interaction with an externally applied magnetic field by an electric current that is driven by a force within the medium is $\boldsymbol{F}_m = \boldsymbol{J} \times \boldsymbol{B}$, known as the Lorentz force, both $\boldsymbol{F}_e$ and $\boldsymbol{F}_m$ have the physical unit of Newton per meter square. From Equation (1.4), the current density is directly related to charge particle motion then $\boldsymbol{F}_m = \boldsymbol{J} \times \boldsymbol{B}$ and it is the well-known *Lorentz force*. It is also often considered that the electromagnetic or Lorentz force is $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})$.

1.2 Charged Particle Motion

The moving charged particles in an electrostatic field will induce a magnetic force perpendicular to the orientations of the applied electric field; therefore these are always associated with an *electromagnetic field*. According to Coulomb's law and the Biot-Savart law, the motion of an electrically charged particle always consists of a rectilinear and rotational component. The rectilinear acceleration is aligned with the externally applied electric field and a gyration intrinsically revolves around an induced and applied magnetic field vector. In the absence of an electric field, the angular acceleration is restricted in the plane perpendicular to the magnetic field. A moving charge q with a velocity $\boldsymbol{u}$ in a steady and uniform electromagnetic field with an applied magnetic flux density $\boldsymbol{B}$ will be pushed by a force normal to $\boldsymbol{B}$, and also directly accelerated by the electric field intensity $\boldsymbol{E}$. The force that is perpendicular to the directions of the charged particle velocity and the magnetic field is the Lorentz force or acceleration, $\boldsymbol{u} \times \boldsymbol{B}$. As a consequence the equation of motion for a charged particle with a velocity of $\boldsymbol{u}$ in an electromagnetic field is:

$$\boldsymbol{F} = m(d\boldsymbol{u}/dt) = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \tag{1.6a}$$

If the electric field is negligible, the magnetic flux density is applied along the z-coordinate. Under this circumstance, the motion of a charged particle along the z-coordinate is unaltered, thus the normal force component of the charge particles is always restricted in the plane perpendicular to a constant and steady magnetic $\boldsymbol{B}$. The velocity components in three-dimensional space can be combined into the component that is parallel $\boldsymbol{u}_{\parallel}$ and perpendicular $\boldsymbol{u}_{\perp}$ to the magnetic field. The velocity component $\boldsymbol{u}_{\parallel}$ that is parallel to $\boldsymbol{B}$ will not be affected by the magnetic field, but the velocity component $\boldsymbol{u}_{\perp}$ normal to $\boldsymbol{B}$ will gyrate around it. The scalar components for the equation of a charge motion in the x-y plane perpendicular to an applied magnetic field and aligned with the z-coordinate becomes:

$$\begin{aligned} du_x/dt &= (qB/m)u_y, \\ du_y/dt &= -(qB/m)u_x, \\ du_z/dt &= 0 \end{aligned} \tag{1.6b}$$

Here and further: $B = |\mathbf{B}|,\ u_x = \mathbf{i}\cdot\mathbf{u}_{\perp},\ u_y = \mathbf{j}\cdot\mathbf{u}_{\perp}, u_z = \mathbf{k}\cdot\boldsymbol{u}_{\parallel},...;\ \mathbf{i},\mathbf{j},\mathbf{k}$

are the unit vectors of the Cartezian reference system. The general solution to the first-order partial differential equation in time; Equation (1.6b) is:

$$\begin{aligned} u_x &= u_{\perp} \exp(i\omega_b t) \\ u_y &= u_{\perp} \exp(i\omega_b t) \\ u_z &= c \end{aligned} \tag{1.6c}$$

In Equation (1.6c), $u_{\perp} = |\boldsymbol{u}_{\perp}| = \sqrt{u_x^2 + u_y^2}$ the symbol ω_b is denoted as the *electron cyclotron* frequency or the *Larmor frequency* as (for electron e=|q|):

$$\omega_b = \mathrm{e}B/m = 1.76 \times 10^{11} B \text{ (rad/s)} \tag{1.6d}$$

Using Equation (1.6d) for calculating the Larmor frequency of a single charge, the magnetic flux density $\boldsymbol{B}$ needs to be in the SI unit of Weber/m^2.

Equation (1.6c) actually describes the single charged particle motion by a simple harmonic oscillator with the *cyclotron* or *Lamor frequency*. The trajectory of the charged particle in a pure magnetic field can be obtained by integrating the equation of motion, Equation (1.6c), and taking the real component of the results to get the location of the particle in the x- and y-coordinates:

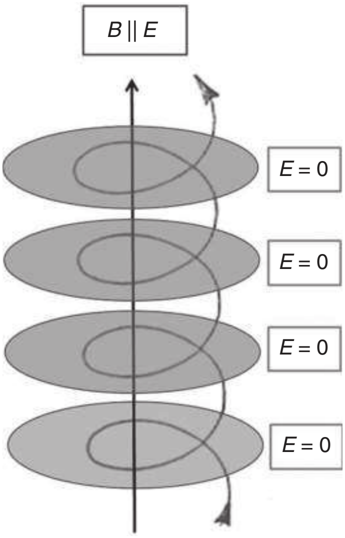

Figure 1.4 Trajectory of electron motion in electromagnetic field.

$$x - x_o = (u_\perp / \omega_b)\cos(\omega_b t)$$
$$y - y_o = (u_\perp / \omega_b)\sin(\omega_b t) \quad (1.6e)$$

As before by defining a gyro radius as $r_b = u_\perp / \omega_b$, the charge particle motion in the normal plane to the magnetic field must follow a circular path and the charged particles will execute a spiral motion. The gyro radius can be determined easily by the balance of the centrifugal acceleration and the electromagnetic force exerted on a singly charged particle, $eu_\perp B = mu_\perp^2 / r_b$. The radius of the circular motion of a group of charged particles in the plane perpendicular the magnetic field is $r_b = mu_\perp / eB$, with an angular gyro velocity $\omega_b = eB/m$. In short, the gyro radius is often referred to as the Larmor or cyclotron radius and can be evaluated based on a single electron, $r_b = mu_\perp / eB$. Similarly, the rate of the angular gyro velocity has also been widely referred to as the Larmor or cyclotron frequency. The charge particle trajectory in a three-dimensional electromagnetic field is then a helix with its axis parallel to **B** and a pitch of $2\pi u_\parallel / \omega B$. When the electric field vector is applied in addition to the magnetic field, the charged particles will execute a persisted spiral trajectory as shown in Figure 1.4. The orientation of the resultant helical trajectory is dictated by the direction of the applied electric field intensity. In fact, the direction of gyration is opposite to the applied magnetic field and tends to reduce the applied magnetic field strength, a process known as *diamagnetism*.

In a steady state the electromagnetic force components are balanced; thus from Equation (1.6a) we shall have $\boldsymbol{E} = -\boldsymbol{u} \times \boldsymbol{B}$. By taking the cross-product with respect to B of the equation, using a vector identity, $(\boldsymbol{u} \times \boldsymbol{B}) \times \boldsymbol{B} \equiv (\boldsymbol{B} \cdot \boldsymbol{u})\boldsymbol{B} - (\boldsymbol{B} \cdot \boldsymbol{B})\boldsymbol{u}$ and recognizing that $\boldsymbol{B} \cdot \boldsymbol{u} \equiv 0$, it yields:

$$\boldsymbol{E} \times \boldsymbol{B} = \boldsymbol{u} B^2 \quad (1.6f)$$

The resultant velocity $\boldsymbol{u} = (\boldsymbol{E} \times \boldsymbol{B})/B^2$ is the well-known *drift velocity* of $\boldsymbol{E} \times \boldsymbol{B}$, which moves in a direction perpendicular to both the electric and magnetic fields. The drift velocity leads to the well-known *Hall effect* (Hall 1897).

Meanwhile, in a magnetic field that is increasing in strength, the kinetic energy parallel to the field converts into rotational energy of the particle and increases its Larmor radius. However, the energy of the system is invariant because the magnetic field does no work to change the total kinetic energy of the charged particle. When the magnetic field increases to a point, the velocity parallel to the magnetic field vanishes; it leads to a unique phenomenon of the *magnetic mirror* for plasma by the Lorentz force (Goldston and Rutherford 1995; Goebel and Katz 2008).

In a large group of charged particles, the charged particles will encounter numerous collisions with each other and neutrals in partially ionized plasma. The charged particles' dynamics are untraceable, but the global behavior for the system of particle has been thoroughly studied by the field of statistic thermodynamics. The global effect of collisions can be determined by the velocity distribution function for each species in the microscopic motions to the macroscopic behavior of plasma. In the absence of the other forces, these particles can be characterized by a speed solely as a function of the group temperature and mass of the species. The most probable distribution of velocity in thermal equilibrium is the Maxwell distribution:

$$f(u) = (m/2\pi kT)^{3/2} \exp[-mu^2/2kT] \tag{1.7a}$$

Where k is the Boltzmann constant, in SI units $k = 1.3806 \times 10^{-23}\, m^2 kg / s^2 K$, and in plasma dynamic applications the value of $k = 8.61733 \times 10^{-8}\, eV / K$ is frequently used. In Boltzmann distribution Equation (1.7a), the notation T is the system temperature in a thermal equilibrium state. From this distribution function, the average velocity of the system can be found as:

$$u_{avg} = \int_0^\infty 4\pi u (m/2\pi kT)^{3/2} \exp[-mu^2/2kT]\, du \tag{1.7b}$$

The classic result of the average velocity of a particle in a thermodynamic equilibrium state is obtained by integrating Equation (1.7b), which is proportional to the square root value of the temperature and inversely proportional to the unit mass of the particle. The result reveals the huge difference between the average velocities of electrons and ions due to the difference in masses of electrons and ions.

$$u_{avg} = (8\pi T/\pi m)^{1/2} \tag{1.7c}$$

It becomes clear that the stationary charges generate an electrostatic field; in fact, the remotely acting Coulomb force is the genesis of the intrinsic characteristics of plasma for the Debye shielding length and the plasma frequency. While the direct current produces a magnetostatic field, the dynamics of the electromagnetic field must more often be accompanied by a time-varying electric current density. In short, within a static electromagnetic field, the electric and magnetic