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## Approach to Quantum Field Theory

In this introductory chapter, which actually is the core of the entire book, we make a serious effort to bring together a variety of ideas from philosophy and physics. We first lay the epistemological foundations for our approach to particle physics. These philosophical foundations, combined with tools borrowed from the amazingly sophisticated, but still not entirely satisfactory, mathematical apparatus of present-day quantum field theory and augmented by some new ideas, are then employed to develop a mathematical representation of fundamental particles and their interactions. In this process, also the relation between “fundamental particle physics” and “quantum field theory” is going to be clarified.

This first chapter consists of two voluminous sections: a number of *philosophical contemplations* followed by a discussion of *mathematical and physical elements*. The reader might wonder why these massive sections are not presented as two separate chapters. The reason for keeping them together is the desire to emphasize the intimate relation between these two sections: the selection and construction of the mathematical and physical elements is directly based on our philosophical contemplations. The beauty of the entire approach is a fruit of this intimate relationship.

The presentation of the material in this chapter is based on the assumption that the reader has a basic working knowledge of linear algebra and quantum mechanics. If that is not the case, the reader might want to consult the equally entertaining and serious introduction *Quantum Mechanics: The Theoretical Minimum* by Susskind and Friedman [4]. The book *Quantum Mechanics and Experience* by Albert [5] offers a fascinating introduction with a strong focus on the measurement problem. An almost equation-free discussion of the history and foundations of quantum mechanics can be found in the book *Einstein, Bohr and the Quantum Dilemma: From Quantum Theory to Quantum Information* by Whitaker [6]. Complex vector spaces, Hilbert space vectors and density matrices for describing states of quantum systems, bosons and fermions, canonical commutation relations, Heisenberg’s

uncertainty relation, the Schrödinger and Heisenberg pictures for the time evolution of quantum systems, as well as a basic idea of the measurement process are all referred or alluded to in Section 1.1; these basics of quantum mechanics are recapped only very briefly in Section 1.2. The crucial construct of Fock spaces is explained in a loose way in Section 1.1.3 and later elaborated in full detail in Section 1.2.1.

According to Henry Margenau [7], “[the epistemologist] is constantly tempted to reject all because of the difficulty of establishing any part of reality” (p. 287). But, again in the words of Margenau, “It is quite proper for us to assume that we know what a dog is even if we may not be able to define him” (p. 58). More classically, a similar idea has been expressed by David Hume: “NEXT to the ridicule of denying an evident truth, is that of taking much pains to defend it” (see p. 226 of [8]). In this spirit, I try to resist the temptation of raising more questions than one can possibly answer, no matter how fascinating these questions might be. Philosophy shall here serve as a practical tool for doing better physics. I try to use philosophy in a relevant and convincing way, but I am certainly not in a position to do frontier technical research in philosophy.

### 1.1 Philosophical Contemplations

We begin this chapter with some general remarks on the methodology of science, where we heavily rely on the epistemological ideas of Ludwig Boltzmann. The representation of space and time is then considered in the light of Immanuel Kant’s famous ideas. We further consider the more specific issues of infinity and irreversibility, and we conclude with some contemporary philosophic considerations about quantum field theory in its present form(s).

As a guideline for developing the mathematical and physical elements in Section 1.2 we condense our philosophical contemplations of the present section into four metaphysical postulates. Metaphysical principles may not be particularly popular among contemporary physicists, but consciously or unconsciously, they play an essential role in any science. We here prefer the conscious approach, which is eloquently recommended by Henry Margenau in his philosophy of modern physics on pp. 12–13 of [7]:

To deny the presence, indeed the necessary presence, of metaphysical elements in any successful science is to be blind to the obvious, although to foster such blindness has become a highly sophisticated endeavor in our time. Many reputable scientists have joined the ranks of the exterminator brigade, which goes noisily about chasing metaphysical bats out of scientific belfries. They are a useful crowd, for what they exterminate is rarely metaphysics—it is usually bad physics. Every scientist *must* invoke assumptions or rules of procedure which are not dictated by sensory evidence as such, rules whose application

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endows a collection of facts with internal organization and coherence, makes them simple, makes a theory elegant and acceptable.

In his verbose philosophical exploration of science and nature, Simon Altmann [9] stresses the importance of metaphysical principles: "... science requires the use of certain *normative principles* that have a much greater generality than *physical laws* ..." (see p. 30 of [9]). He actually distinguishes between metaphysical and meta-physical normative principles, where the former are beyond experience and the latter are directly wedded to experience (without actually being *derivable* from it). We here use the conventional spelling but nevertheless claim that our metaphysical postulates are grounded in experience. Metaphysical postulates are used as a guideline for theory development, but they are themselves based on reflections on the evolving knowledge of physics. In the words of Cao (see p. 267 of [10]), "It [metaphysics] can help us to make physics intelligible by providing well-entrenched categories distilled from everyday life and previous scientific experiences. But with the advancement of physics, it has to move forward and revise itself for new situations: old categories have to be discarded or transformed, new categories have to be introduced to accommodate new facts and new situations."

It is important to realize that the metaphysical postulates to be developed in this section are not meant as rigorous fundamental principles, but rather as helpful intuitive guidelines. The reader should interpret them with benevolence and should consider them as an invitation to personal reflections, with the goal of increasing the awareness of how we are doing modern science. I will, however, try to elaborate how these metaphysical postulates affect the present approach to quantum field theory in a deep and decisive way. The style of the presentation is a compromise between the philosopher's cherished culture of multifaceted discourse and the physicist's impatient desire to get to the core of the story.

### 1.1.1 Images of Nature

Around 1900, the University of Vienna was a vivid center for agitated discussions about physics and philosophy, where the existence or nonexistence of atoms was one of the big topics. From 1895 to 1901, Ernst Mach held the newly created "chair for philosophy, especially for the history and theory of the inductive sciences." From 1893 to 1900 and from 1902 to 1906, Ludwig Boltzmann was the professor of theoretical physics at the University of Vienna. The fact that Boltzmann left Vienna and returned only after the retirement of Mach was not just a matter of coincidence but a consequence of enervating quarrels with Mach and other colleagues. In 1897, after Boltzmann, who was a leading proponent of atomic theory, had given a lecture at the Imperial Academy of Sciences in Vienna, Mach laconically declared:

“I don’t believe that atoms exist!” In 1903, while waiting for the faculty to propose candidates for Mach’s replacement, the ministry gave Boltzmann the gratifying assignment to lecture every semester for two hours per week on the “philosophy of nature and methodology of the natural sciences” to fill the gap that had existed since Mach’s retirement (actually Mach hadn’t been teaching after a stroke he suffered in 1898). Boltzmann’s philosophical lectures attracted huge audiences (some 600 students) and so much public attention that the Emperor Franz Joseph I (reigning in Austria from 1848 to 1916) invited him for a reception at the Palace to express his delight about Boltzmann’s return to Vienna. So, Boltzmann was not only a theoretical physicist of the first generation, but also an officially recognized part-time philosopher. For the last years of his life he focused on philosophical ideas to defend his pioneering work on the foundations of statistical mechanics and the kinetic theory of gases, which heavily relied on the existence of atoms.

In the very beginning of his very first philosophical lecture on October 26, 1903, Boltzmann stated that he had written only a single treatise with philosophical content in his entire life. He was referring to the article “On the Question of the Objective Existence of Processes in Inanimate Nature,” which had been published in 1897 (see essay 12 in [11]; an English translation is given on pp. 57–76 of [1]). However, Boltzmann had already made a number of oral contributions to the methodology of science that are clearly of epistemological content and would nowadays be classified as philosophical. For the subsequent discussion we actually rely on two such contributions dating from 1899. One of these contributions was an address to the meeting of natural scientists at Munich (“On the Development of the Methods of Theoretical Physics in Recent Times”), the other one a series of lectures given at Clark University in Worcester (“On the Fundamental Principles and Equations of Mechanics”); both contributions were published in his writings addressed to the public in 1905 (as items 14 and 16 in [11], translated in [1]; all the page numbers in the remainder of this section refer to the English translation [1] of his writings addressed to the public).

### **“On the Development of the Methods of Theoretical Physics in Recent Times”**

After describing the evolution of the theory of electromagnetism, Boltzmann states, “Whereas it was perhaps less the creators of the old classical physics than its later representatives that pretended by means of it to have recognised the true nature of things, Maxwell wished his theory to be regarded as a mere picture of nature, a mechanical analogy as he puts it, which at the present moment allows one to give the most uniform and comprehensive account of the totality of phenomena” (p. 83). Regarding physical theories as pictures of nature is a very fundamental idea. I prefer to call them *images of nature* because imagination is exactly what theoretical physics should be about, with moral support from Einstein (quote from an interview

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Figure 1.1 Ludwig Boltzmann, 1844–1906.

given in 1929): “Imagination is more important than knowledge. Knowledge is limited. Imagination encircles the world.” Or, in my own simple words, imagination creates knowledge. Boltzmann elaborates the standing of images of nature in the following two paragraphs (pp. 90–91), emanating from the example of the theory of electromagnetism:

Maxwell had called Weber’s hypothesis a real physical theory, by which he meant that its author claimed objective truth for it, whereas his own account he called mere pictures of phenomena. Following on from there, Hertz makes physicists properly aware of something philosophers had no doubt long since stated, namely that no theory can be objective, actually coinciding with nature, but rather that each theory is only a mental picture of phenomena, related to them as sign is to designatum.

From this it follows that it cannot be our task to find an absolutely correct theory but rather a picture that is as simple as possible and that represents phenomena as accurately as possible. One might even conceive of two quite different theories both equally simple and equally congruent with phenomena, which therefore in spite of their difference are equally correct. The assertion that a given theory is the only correct one can only express our subjective conviction that there could not be another equally simple and fitting image. [Author: Note that here the German word ‘Bild’ is actually translated as ‘image’ rather than ‘picture.’]

Images of nature are never meant to be absolutely correct, and they should only be expected to cover a certain range of phenomena with a certain degree of

accuracy. More complete images can always arise so that we can ask with Margenau (see p. 171 of [7]): “But why, after all, should scientific truth be a static concept?” Or, in a beautiful formulation of William James (see p. x of [12]),<sup>1</sup> “The truth of an idea is not a stagnant property inherent in it. Truth *happens* to an idea. It *becomes* true, is *made* true by events.”

Different images can do equally well on a certain range of phenomena, but one of the images may lead to the discovery of new phenomena and hence turn out to be more successful than the other ones, without making them useless. Boltzmann illustrates this point with the theories of electromagnetic phenomena developed by Weber and by Maxwell (p. 83), “The phenomena known till then were equally well explained by both theories, but Maxwell’s went much beyond the old theory [of Weber].” The idea of electromagnetic waves emerged only from Maxwell’s theory replacing long-range interactions by close-range effects, thus leading to a deeper understanding of light and to new technological applications, such as “an ordinary optical telegraph.” Also, according to the philosopher Paul Feyerabend, “It must be asserted that the discussion of possibilities and of alternatives to a current theory plays a most important role in the development of our physical knowledge” (see p. 233 of [13]) and “*There is no way of singling out one and only one theory on the basis of observation*” (see p. 234 of [13]). Such a tolerant view about the fruitful coexistence of old and new theories is at variance with Thomas Kuhn’s more radical ideas about scientific revolutions (see p. 98 of [14]): “Einstein’s theory [of gravity] can be accepted only with the recognition that Newton’s was wrong.” Note that Altmann has criticized Kuhn’s restrictive ideas in profound ways (see chapter 20 of [9]).<sup>2</sup>

Boltzmann’s theoretical pluralism is the central topic in Videira’s analysis [15, 16] of Boltzmann’s philosophical works. Videira suggests that, by emphasizing the fundamental distinction between nature and its various representations, this theoretical pluralism is capable of counteracting dogmatic tendencies returning in modern science, for example, in twentieth-century cosmology. Actually, pluralism should be recognized as an enabling condition for progress in physics.<sup>3</sup> The various images of nature should compete in a Darwinistic sense. The idea of “evolutionary epistemology” has been expressed in a beautifully worded metaphor by van Fraassen (see p. 40 of [17]):

I claim that the success of current scientific theories is no miracle. It is not even surprising to the scientific (Darwinist) mind. For any scientific theory is born into a life of fierce

<sup>1</sup> There exist several online versions of this classical collection of writings first published in 1909.

<sup>2</sup> We scientists seem to like Kuhn’s ideas because, whenever one of our papers gets rejected, we can feel as the misunderstood heroes of a scientific revolution hindered by conservative referees who are not yet ready for a paradigm shift.

<sup>3</sup> A. A. P. Videira, private communication (October 2015).

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competition, a jungle red in tooth and claw. Only the successful theories survive—the ones which *in fact* latched on to actual regularities in nature.

Boltzmann would presumably have questioned the concept of reality because ultimately we don't even know how to distinguish reality from our various mental representations. As an extreme example, he once made the following oral statement (cited from p. 213 of [18]):

You see, it doesn't make any difference to me if I say that the atomic model is only a picture. I don't mind this. I don't require that they have absolute real existence. I don't say this. 'An economic description' Mach said. Maybe the atoms are an economic description. This doesn't hurt me very much. From the viewpoint of the physicists this doesn't make a difference.

With a heavy heart, Boltzmann refrains from claiming reality even for atoms, which are the most important concept in his favorite image of nature. This statement shows how indispensable scientific pluralism is to Boltzmann. Instead of pluralism one can alternatively speak of an underdetermination of theories by empirical evidences (see p. 3 of [10]).

The following paragraphs clarify that Boltzmann's images of nature are meant to be of mathematical character (pp. 95–96):

Mathematical phenomenology at first fulfils a practical need. The hypotheses through which the equations had been obtained proved to be uncertain and prone to change, but the equations themselves, if tested in sufficiently many cases, were fixed at least within certain limits of accuracy; beyond these limits they did of course need further elaboration and refinement. . . .

Besides we must admit that the purpose of all science and thus of physics too, would be attained most perfectly if one had found formulae by means of which the phenomena to be expected could be unambiguously, reliably and completely calculated beforehand in every special instance; however this is just as much an unrealisable ideal as the knowledge of the law of action and the initial states of all atoms.

Phenomenology believed that it could represent nature without in any way going beyond experience, but I think this is an illusion. No equation represents any processes with absolute accuracy, but always idealizes them, emphasizing common features and neglecting what is different and thus going beyond experience.

How can one justify the fundamental mathematical equations adopted as an image of nature, how can one establish a theory as correct or true? Boltzmann answers these questions by essentially anticipating the ideas nowadays associated with the names of Duhem [2] and Quine [3] (see Preface), "He [Hertz] rightly points out that what convinces us of the correctness of all these equations is not, in mechanics, the few experiments from which its fundamental equations are usually derived, nor, in electrodynamics, the five or six basic experiments of Ampère, but rather their subsequent agreement with almost all hitherto known facts. He therefore

passes a judgment of Solomon that since we have these equations we had best write them down without derivation, compare them with phenomena and regard constant agreement between the two as the best proof that the equations are correct” (pp. 94–95). As truth is considered to be a property of a mathematical image, not of an existent object, we here adopt what can be classified as the pragmatist’s perspective on truth (see p. xv of [12]).

I would like to conclude the discussion of Boltzmann’s essay “On the Development of the Methods of Theoretical Physics in Recent Times” with a beautifully inspiring quote (p. 86): “Given this enormous variety of [electromagnetic] radiations we are almost tempted to argue with the creator for making our eyes sensitive for only so minute a range of them. This, as always, would be unjust, for in all areas only a small range of a great whole of natural phenomena is directly revealed to man, his intelligence being made acute enough to gain knowledge of the rest through his own efforts.”<sup>4</sup> Margenau’s entire philosophy of modern physics [7] is based exactly on the idea expressed in that quote: Starting from the plane of direct perception (sense data, immediate experience, nature), the field of valid rational constructs is obtained through so-called “rules of correspondence” or “epistemic correlations”; the field of constructs is subject to metaphysical requirements and empirical verification. Or in the words of Altmann, “Naked facts hardly exist at all: they are all processed by us through a network of theoretical constructs” (see p. 28 of [9]).

**“On the Fundamental Principles and Equations of Mechanics”** Whereas the idea of regarding physical theories as images of nature should be sufficiently elaborated by now, Boltzmann’s 1899 lectures at Clark University further clarify the process of creating images and the idea that only the fully developed image with all its possible consequences, rather than the basic hypotheses from which it was derived, should be tested against the facts of experience (pp. 107–108):

Some pictures were built up only gradually over centuries through the joint efforts of many enquirers, for example the mechanical theory of heat. Some were found by a single scientific genius, though often by very intricate detours, only then could other scientists illuminate them from various angles. Maxwell’s theory of electricity and magnetism discussed above is one such. Now there is no doubt a particular mode of representation that has quite peculiar advantages, though it has its defects too. This mode consists in starting to operate only with mental abstractions, in tune with our task of constructing only internal mental pictures. In this we do not yet take account of facts of experience. We merely endeavour to develop our mental pictures as clearly as possible and to draw from them all

<sup>4</sup> This remark nicely points to the biological origin of our cognitive faculties, adapted in response to our environment.

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possible consequences. Only later, after complete exposition of the picture, do we test its agreement with the facts of experience; it is, then, only after the event that we give reasons why the picture had to be chosen thus and not otherwise, a matter on which we give not the slightest prior hint. Let us call this deductive representation. Its advantages are obvious. For a start, it forestalls any doubt that it aims at furnishing not things in themselves but only an internal mental picture, its endeavours being confined to fashioning this picture into an apt designation of phenomena. Since the deductive method does not constantly mix external experience forced on us with internal pictures arbitrarily chosen by us, this is much the easiest way of developing these pictures clearly and consistently. For it is one of the most important requirements that the pictures be perfectly clear, that we should never be at a loss how to fashion them in any given case and that the results should always be derivable in an unambiguous and indubitable manner. It is precisely this clarity that suffers if we bring in experience too early, and it is best preserved if we use the deductive mode of representation. On the other hand, this method highlights the arbitrary nature of the pictures, since we start with quite arbitrary mental constructions whose necessity is not given in advance but justified only afterwards. There is not the slightest proof that one might not excogitate other pictures equally congruent with experience. This seems to be a mistake but is perhaps an advantage at least for those who hold the above-mentioned view as to the essence of any theory. However, it is a genuine mistake of the deductive method that it leaves invisible the path on which the picture in question was reached. Still, in the theory of science especially it is the rule that the structure of the arguments becomes most obvious if as far as possible they are given in their natural order irrespective of the often tortuous path by which they were found.

In the preceding quote, the word *clear* occurs four times, and in addition, the words *clarity*, *consistent*, *unambiguous*, and *indubitable* appear. Obviously the clarity and consistency of a mathematical image of nature is of greatest importance to Boltzmann. The role of experience in theorizing has been described by Feyerabend in a way that nicely reflects Boltzmann's deductive mode of representation (see pp. 226–227 of [13]): “Indeed the whole tradition of science from Galileo (or even from Thales) up to Einstein and Bohm is incompatible with the principle that ‘facts’ should be regarded as the unalterable basis of any theorizing. In this tradition the results of experiment are not regarded as the unalterable and unanalyzable building stones of knowledge. They are regarded as capable of analysis, of improvement (after all, no observer, and no theoretician collecting observations is ever perfect), and it is assumed that such analysis and improvement is absolutely necessary.”

In the context of quantum field theory, mathematical consistency is a particularly serious concern raised even by its most famous proponents. In his Nobel lecture (1965), Feynman, in a catchy metaphorical statement, expressed the possible concern that renormalization “is simply a way to sweep the difficulties of the divergences of [quantum] electrodynamics under the rug.” Modern renormalization-group theory [19] has certainly provided a better understanding. But, in the words of the insistent critic Dirac [20], “the quantum mechanics that most physicists are using nowadays [in quantum field theory] is just a set of

working rules, and not a complete dynamical theory at all.” Dirac felt that “some really drastic changes” in the equations were needed (see pp. 36–37 of [21]). In the end of the day, the mathematics of quantum field theory must be clear and consistent by the standards of Boltzmann for a theory to become acceptable as an image of nature. We hence adopt the following, even more far-reaching postulate.

*First Metaphysical Postulate:* A mathematical image of nature must be rigorously consistent; mathematical elegance is an integral part of any appealing image of nature.

I would like to remind the reader that the metaphysical postulates should be read with benevolence, in particular, if they involve subjective judgments. If someone really doesn’t know what *appealing* means, the word may be replaced by *acceptable*. If the word *acceptable* is unacceptable, it may be omitted. But it would be disappointing to give up the idea that we all recognize mathematical elegance when we encounter it. Let’s try to approach this with the same acceptive attitude that makes us visit an art museum.

The belief in the universal harmony of nature reflected in mathematical elegance, or even reflecting mathematical elegance, is in the tradition of Plato and Pythagoras. “The latter took mathematics as the foundation of reality and the universe as fundamentally mathematical in its structure. It was assumed that observable phenomena must conform to the mathematical structures, and that the mathematical structures should have implications for further observations and for counterfactual inferences which went beyond what were given” (see p. xvii of [22]). Note that the reliability and truth of mathematical images depends on the idea of “uniformity of nature,” that is, the idea that the succession of natural events is determined by immutable laws.

According to Dworkin [23], the intrinsic beauty and sublimity of the universe belong to the characteristics of a religion without god. With or without (a personalized) god, these properties of the universe should be reflected in the elegance of the mathematical image.

Mathematical theories and concepts are most reliably introduced within the axiomatic approach. All objects are characterized by properties. The emphasis on mathematical images hence suggests to build ontology on properties. Some advantages of the mathematical formulation of physical theories for philosophical considerations have been emphasized by Auyang (see p. 7 of [24]): “Since physical theories are mathematical, their conceptual structures are more clearly exhibited. This greatly helps the philosophical task of uncovering presuppositions.”

Our first metaphysical postulate covers several of the six metaphysical requirements formulated by Margenau in chapter 5 of [7]: (a) logical fertility (“natural