Agent-Based Models in Economics

In contrast to mainstream economics, complexity theory conceives the economy as a complex system of heterogeneous interacting agents characterised by limited information and bounded rationality. Agent-based models (ABMs) are the analytical and computational tools developed by the proponents of this emerging methodology. Aimed at students and scholars of contemporary economics, this book includes a comprehensive toolkit for agent-based computational economics, now quickly becoming the new way to study evolving economic systems. Leading scholars in the field explain how ABMs can be applied fruitfully to many real-world economic examples, and represent a great advancement over mainstream approaches. The essays discuss the methodological bases of agent-based approaches and demonstrate step-by-step how to build, simulate and analyse ABMs, and how to validate their outputs empirically using the data. The contributors also present a wide set of model applications to key economic topics, including the business cycle, labour markets and economic growth.

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Agent-Based Models in Economics
A Toolkit

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To our heterogeneous most relevant ones.
An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: What does happen is that the opponents gradually die out.

(Max Plank)
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Preface

As Schumpeter pointed out long ago, conceptual frameworks, models and policy prescriptions are embedded in the economist’s ‘preanalytic vision’ of the economy. And preanalytic visions have been, and still are, very different in the profession.

Nowadays the majority of the profession embraces the neoclassical **approach** to economic behaviour, according to which agents are endowed with substantial rationality, adopt optimal rules and interact indirectly through the price vector on markets which are continuously in equilibrium. This approach has been extraordinarily fruitful, as it has allowed economists to build models that can be solved analytically and yield clear-cut policy implications. The obvious case in point is Walras’s theory of General Equilibrium, beautifully outlined in his *Elements d’Economie Politique*, and elegantly extended and refined by Arrow and Debreu. Moreover, the approach has been remarkably flexible. Appropriately designed variants of the neoclassical approach have been applied to economies characterised by imperfect competition, imperfect information, strategic interaction, and heterogeneous agents. The most insightful of these theoretical developments have been incorporated in micro-founded macroeconomic models of the **New Neoclassical Synthesis** that have been all the rage during the years of the Great Moderation.

However, the capability of the neoclassical approach to encompass and explain all the complex details of economic life has reached a limit. For instance, it is now abundantly clear that the neoclassical approach is not well-suited to describe the Global Financial Crisis and the Great Recession. In models that follow the New Neoclassical Synthesis, in fact, a great recession may be explained only by a large aggregate negative shock, whose probability is extremely low (i.e., it is an extreme and rare event). This mechanism does not clarify much of the crisis and does not help to devise appropriate remedies.
Preface

The current predicament, both in the real world and in the public debate, resembles the early 1930s. The way out of the Great Depression required a new economic theory and the Second World War. Luckily, in order to escape the current predicament, we can dispense at least with the latter. We still need, however, to reshape the way in which we think about the economy.

For several years now, a complexity approach has been developed which conceives the economy as a complex system of heterogeneous interacting agents characterised by limited information and bounded rationality. In this view, a ‘crisis’ is a macroscopic phenomenon which spontaneously emerges from the web of microscopic interactions. Agent-Based Models (ABMs) are the analytical and computational tools necessary to explore the properties of a complex economy.

Agent-based macroeconomics is still in its infancy, but it is undoubtedly a very promising line of research. So far only a small minority in the economic profession has adopted this approach. This may be due to the wait-and-see attitude of those who want to see the approach well established in the profession before embracing it. The hesitation, however, may also come from methodological conservatism. For instance, while in other disciplines the explanatory power of computer simulations is increasingly recognized, most economists remain dismissive of any scientific work that is not based on strict mathematical proof. With the passing of years, however, agent-based (AB) tools have been refined. This book is a guide to the main issues which an interested reader may encounter when approaching this field. We hope this will help in nudging a new generation of curious minds to explore the fascinating field of complexity.

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1 The unemployment rate, which peaked at 1/4 of the labour force during the Great Depression, went back to the long-run ‘normal’ of around 1/20 only after the end of the war. The huge increase in government spending due to the war effort helped to absorb the unemployment generated by the Great Depression.

2 A recent intriguing line of research aims at providing analytical solutions to multi-agent systems adopting the apparatus of statistical mechanics, e.g., the Fokker-Planck equations. See, for instance, M. Aoki (2011), Di Guilmi (2016).