

# 1

## Getting Started

### 1.1 Launching *Mathematica*

The first task you will face is opening your copy of *Mathematica* or Wolfram|One™. If you are using a desktop version of *Mathematica*, you are looking for an icon that looks something like this:



Just click it to launch. A window titled “Welcome to Wolfram Mathematica” will appear. In the top left corner there is a “New Document” button. Click it and an empty document window will appear. This is your *Mathematica notebook*, where you will carry out your work.

Cloud-based products such as *Mathematica Online* require only a web browser and an active account at WolframCloud.com. Once launched, the interface is nearly identical to that of the desktop version.

The remainder of this chapter is a quick tutorial that will enable you to get accustomed to the syntax and conventions of *Mathematica*, and demonstrate some of its many features.

### 1.2 The Basic Technique for Using *Mathematica*

A *Mathematica notebook* is an interactive environment. You type a command (such as  $2 + 2$ ) and instruct *Mathematica* to execute it. *Mathematica* responds with the answer on the next line. You then type another command, and so on. Each command you type will appear on the screen in a **boldface** font. *Mathematica*'s output will appear in a plain font.

#### 💡 Entering Input

After typing a command, you must enter it. To do this hold down the `SHIFT` key and then hit the `ENTER` key. Your keypad may say “Return” on this key, or “Enter,” or just have the `↵` symbol. Regardless, when we say the `ENTER` key we mean the key that takes you to a new line when you are typing.

Beware that you can't just hit the `ENTER` key. Hitting this key alone will insert a line break in your input. It will not process your input.

There is one exception to this rule: If you happen to have an extended keyboard with a numeric keypad on the right, you can use the `ENTER` key in the bottom right corner to enter your commands. But using `SHIFT+ENTER` on the main part of the keypad will still work. It is probably easiest to get used to using `SHIFT+ENTER` from the beginning since this will work on any keyboard.

### 1.3 The First Computation

For your first computation, type

```
2 + 2
```

then hit the `SHIFT+ENTER` combination.

```
In[1]:= 2 + 2  
Out[1]= 4
```



Notice the gray bar that appeared just below the output. This is the “Suggestions Bar.” *Mathematica* is trying to be helpful by providing a few suggestions related to your input that you may want to explore. Feel free to follow its suggestions to create and evaluate new inputs, or feel free to ignore them. A new Suggestions Bar will appear every time you enter a command. To prevent this from happening simply click on the small “×” button on the far right of the bar and *Mathematica* will turn off this feature for the current session.

When you enter your input, it is processed by an auxiliary program called the kernel. *Mathematica* really consists of two programs: the notebook front end where you type your commands and where output, graphics, and text are displayed, and the kernel, where your inputs are evaluated. Even though you will probably never interact directly with the kernel, it is useful to be aware of this structural dichotomy in *Mathematica*'s architecture. We will discuss some implications in the next chapter in Section 2.6.

### 1.4 Commands for Basic Arithmetic

*Mathematica* works much like a calculator for basic arithmetic. Just use the +, −, \*, and / keys on the keyboard for addition, subtraction, multiplication, and division. As an alternative to typing \*, you can multiply two numbers by leaving a space between them (the × symbol will automatically be inserted when you leave a space between two numbers). You can raise a number to a power using the ^ key. Use the dot (i.e., the period) to type a decimal point. Here are a few examples:

```
In[1]:= 17 - 1  
Out[1]= 16
```

```
In[2]:= 123 456 789 * 123 456 789
Out[2]= 15 241 578 750 190 521

In[3]:= 123 456 789 × 123 456 789
Out[3]= 15 241 578 750 190 521

In[4]:= 123 456 789 ^ 2
Out[4]= 15 241 578 750 190 521

In[5]:= 9.1 / 256.127
Out[5]= 0.0355292

In[6]:= 34 / 4
Out[6]=  $\frac{17}{2}$ 
```

This last line may seem strange at first. What you are witnessing is *Mathematica*'s propensity for providing exact answers. *Mathematica* treats decimal numbers as approximations, and will generally avoid them in the output if they are not present in the input. When *Mathematica* returns an expression with no decimals, you are assured that the answer is exact. Fractions are displayed in lowest terms.

## 1.5 Input and Output

You've surely noticed that *Mathematica* is keeping close tabs on your work. Each time you enter an expression, *Mathematica* gives it a name such as In[1]:=, In[2]:=, In[3]:= . The corresponding output comes with the labels Out[1]=, Out[2]=, Out[3]= . At this point, it is enough to observe that these labels will appear all by themselves each time you enter a command, and it's okay:

```
In[1]:=  $\left(\frac{1}{2}\right)^6$ 
Out[1]=  $\frac{1}{64}$ 
```

You've surely noticed something else too (you'll need to be running a live session for this): those brackets along the right margin of your notebook window. Each input and output is written into a *cell*, whose scope is shown by the nearest bracket directly across from the respective input or output text. Cells containing input are called *input cells*. Cells containing output are called *output cells*. The brackets delimiting cells are called *cell brackets*. Each input–output pair is in turn grouped with a larger bracket immediately to the right of the cell brackets. These brackets may in turn be grouped together by a still larger bracket, and so on. These extra brackets are called *grouping brackets*.

At this point, it's really enough just to know these brackets are there and to make the distinction between the innermost (or smallest, or leftmost) brackets which delimit individual cells and the others which are used for grouping. If you are curious about what good can possibly come of them, try

positioning the tip of your cursor arrow anywhere on a grouping bracket and double-click. You will *close the group* determined by that bracket. In the case of the bracket delimiting an input–output pair, this will have the effect of hiding the output completely (handy if the output runs over several pages). Double-click again to open the group. This feature is useful when you have created a long, complex document and need a means of managing it. Alternately, you can double-click on any output cell bracket to *reverse-close* the group. This has the effect of hiding the input code and displaying only the output.

Since brackets are really only useful in a live *Mathematica* session, they will not, by default, show when you print a notebook. Additional details about brackets and cells will be provided in Section 2.2.

One last bit of terminology is in order. When you hit the `SHIFT+ENTER` combination after typing an input cell, you are *entering the cell*. You'll be seeing this phrase quite a bit in the future.

## 1.6 The Basic Math Assistant Palette


There may already be a narrow, light-gray window full of mathematical symbols along the side of your screen. If so, you are looking at one of *Mathematica*'s palettes, and chances are that it is the Basic Math Assistant palette. If you see no such window, go to the Palettes dropdown menu at the top of your screen and select Basic Math Assistant to open it.





The Basic Math Assistant palette

This palette is currently available only on desktop versions of *Mathematica*. If you are using *Mathematica* Online, look in the menu for `Insert`  $\triangleright$  `Special Characters`. This will open a palette with a simpler interface that allows you to insert special symbols one character at a time. To simulate many of the buttons on the Basic Math Assistant palette, online users can take advantage of *command completion* and *command templates*; these are discussed in Section 1.11.

The Basic Math Assistant palette is a handy tool. You will use it to help typeset your *Mathematica* input so that it looks like traditional mathematical notation. The palette will also help you construct your inputs easily by providing a template with correct syntax for you to fill in. And if you need to submit a technical paper it will help you to typeset your notebook into a beautifully formatted document that contains all your *Mathematica* computations.


The top section of the Basic Math Assistant palette is called Calculator. If this section is not open, click on the  to open it. You can toggle between the Basic and Advanced calculators by hitting the tabs with those labels. We will start with the Basic Calculator and discuss other sections of this palette in Section 1.10.




To type an exponential expression such as  $17^{19}$ , use the  button in the middle of the Calculator. To do this, first type 17 into your *Mathematica* notebook, then highlight it with your mouse. Next, push the  palette button with your mouse. The exponent structure shown on that button will be pasted into your notebook, with the 17 in the position of the black square on the palette button (the black square is called the *selection placeholder*). The text insertion point will move to the placeholder in the exponent position. Your input cell will look like this:



17<sup>■</sup>

Now type the value of the exponent, in this case 19, into the placeholder, then enter the cell:

```
In[1]:= 1719
Out[1]= 239 072 435 685 151 324 847 153
```

△ Another way to accomplish the same thing is this: First hit the palette button, then type 17 into the first placeholder. Next hit the  key to move to the second placeholder (in the exponent position). Now type 19 and enter the cell. This procedure is perhaps a bit more intuitive, but it can occasionally get you into trouble if you are not careful with grouping. For instance, if you want to

enter  $(1 + x)^8$ , and the first thing you do is push the  button on the palette, then you must type  $(1 + x)$  with parentheses, then , then 8. By contrast, you could type  $1 + x$  with or without parentheses and highlight the expression with your mouse, then hit the  palette button, and then type 8. If needed, the parentheses are added automatically when this procedure is followed.

If you don't understand what some of the palette buttons do, don't fret. Just stick with the ones that you know for now. For instance, you can take a cube root like this: Type a number and highlight it with the mouse, then push the  button on the Basic Math Assistant palette, then hit the  key, and finally type 3. Now enter the cell:

```
In[2]:=  $\sqrt[3]{50\,653}$ 
Out[2]= 37
```

This is equivalent to raising 50 653 to the power 1/3:

```
In[3]:=  $50\,653^{1/3}$ 
Out[3]= 37
```

And of course we can easily check the answer to either calculation:

```
In[4]:=  $37^3$ 
Out[4]= 50\,653
```

You may have noticed the Classroom Assistant palette and the Writing Assistant palette options at the top of the Palettes menu. The Classroom Assistant palette is quite similar to the Basic Math Assistant palette, but it also contains a Navigation section and a Keyboard section, as well as all the sections of the Writing Assistant palette.

#### Using Palette Buttons

Speaking in general terms, all palette buttons that contain a solid black placeholder  $\blacksquare$  are used in this way:

- Type an expression into a *Mathematica* notebook.
- Highlight all or part of the expression with your mouse (by dragging across the expression).
- Push a palette button. The structure on the face of the button is pasted into your notebook, with the highlighted text appearing in the position of the solid black square.
- If there are more placeholders in the structure, use the `TAB` key or forward arrow (or move the cursor with your mouse) to move from one to the next.

## 1.7 Decimal In, Decimal Out

Sometimes you don't want exact answers. Sometimes you want decimals. For instance, how big is this number? It's hard to get a grasp of its magnitude when it's expressed as a fraction:

```
In[1]:=  $\frac{17^{19}}{19^{17}}$ 
Out[1]=  $\frac{239\,072\,435\,685\,151\,324\,847\,153}{5\,480\,386\,857\,784\,802\,185\,939}$ 
```

And what about this?

```
In[2]:=  $\sqrt[3]{59\,875}$ 
Out[2]=  $5 \times 479^{1/3}$ 
```

*Mathematica* tells us that the answer is 5 times the cube root of 479 (remember that raising a number to the power  $1/3$  is the same as taking its cube root). The output is exact, but again it is difficult to grasp the magnitude of this number. How can we get a nice decimal approximation, like a calculator would produce?

If any of the numbers in your input is in decimal form, *Mathematica* regards it as approximate. It responds by providing an approximate answer, that is, a decimal answer. It is handy to remember this:

```
In[3]:=  $\frac{17.0^{19}}{19^{17}}$ 
Out[3]= 43.6233
```

```
In[4]:=  $\sqrt[3]{59875.0}$ 
Out[4]= 39.1215
```

A slightly quicker way to accomplish this is to type a decimal point after a number with nothing after it. That is, *Mathematica* regards “17.0” and “17.” as the same quantity. This is important for understanding *Mathematica*’s output:

```
In[5]:=  $\sqrt[3]{59875.}$ 
Out[5]= 39.1215
```

```
In[6]:=  $\frac{30.}{2}$ 
Out[6]= 15.
```

Note the decimal point in the output. Since the input was only “approximate,” so too is the output. Get in the habit of using exact or decimal numbers in your input according to the type of answer, exact or approximate, you wish to obtain. Adding a decimal point to any single number in your input will cause *Mathematica* to provide an approximate (i.e., decimal) output. A detailed discussion on approximate numbers can be found in Section 8.3.

## 1.8 Use Parentheses to Group Terms

Use ordinary parentheses ( ) to group terms in algebraic expressions. This is very important, especially with division, multiplication, and exponentiation. Being a computer program, *Mathematica* takes what you say quite literally; operations are performed in a definite order, and you need to make sure that it is the order you intend. Get in the habit of making a mental check for appropriate parentheses before entering each command. Here are some examples. Can you see what *Mathematica* does in the absence of parentheses?

In[1]:=  $3 * (4 + 1)$

Out[1]= 15

In[2]:=  $3 * 4 + 1$

Out[2]= 13

In[3]:=  $(-3)^2$

Out[3]= 9

In[4]:=  $-3^2$

Out[4]= -9

In[5]:=  $(3 + 1) / 2$

Out[5]= 2

In[6]:=  $3 + 1 / 2$

Out[6]=  $\frac{7}{2}$

The last pair of examples shows one benefit of using the Basic Math Assistant palette instead of typing from the keyboard. With the typesetting capability afforded by the palette there is no need for grouping parentheses, and no chance for ambiguity:

In[7]:=  $\frac{3 + 1}{2}$

Out[7]= 2

In[8]:=  $3 + \frac{1}{2}$

Out[8]=  $\frac{7}{2}$

The lesson here is that the order in which *Mathematica* performs operations in the absence of parentheses may not be what you intend. When in doubt, add parentheses.

Note also that only round brackets can be used for the purpose of grouping terms. *Mathematica* reserves different meanings for square brackets and curly brackets, so never use them to group terms.

## 1.9 Three Well-Known Constants

*Mathematica* has several built-in constants. The three most commonly used are  $\pi$ , the ratio of the circumference to the diameter of a circle (approximately 3.14);  $e$ , the base of the natural logarithm (approximately 2.72); and  $i$ , the imaginary number whose square is  $-1$ . You can find each of these constants near the top of the Basic Calculator section of the Basic Math Assistant palette.



```
In[1]:=  $\pi$ 
Out[1]=  $\pi$ 

In[2]:=  $\pi + 0.$ 
Out[2]= 3.14159
```

Again, note *Mathematica*'s propensity for exact answers. You will often use  $\pi$  to indicate the radian measure of an angle to be input into a trigonometric function. There are examples in the next section.

It is possible to enter each of these three constants directly from the keyboard, as well. You can type `ESC p ESC` for  $\pi$ , `ESC ee ESC` for  $e$ , and `ESC i ESC` for  $i$ .

- △ You can also type `Pi` for  $\pi$ , `E` for  $e$ , and `I` for  $i$ . The capitalization is important. These do not look as nice, but it illustrates an important point: It is possible to type any *Mathematica* input using only the characters from an ordinary keyboard. That is, every formatted mathematical expression that can be input into *Mathematica* has an equivalent expression constructed using only characters from the keyboard. Indeed, versions 1 and 2 of *Mathematica* used only such expressions. These days, the keyboard, or `InputForm`, of an expression is used when you include a *Mathematica* input or output in an email message (say, to a friend or to your professor). If you copy a formatted expression such as  $\pi^{1/3}$  from *Mathematica* and paste it into an email or text editor, you'll find that it becomes `Pi^(1/3)` (or just  `$\pi^{1/3}$`  if the editor has the  $\pi$  symbol available). The point is that it is exceedingly simple to include formatted *Mathematica* expressions in plain text environments. Note that you can display any input cell in `InputForm` from within *Mathematica* by clicking on its cell bracket to select it, and going to the Cell menu and choosing `ConvertTo > InputForm`.


## 1.10 Mathematica Commands from Palettes

*Mathematica* contains thousands of *commands*. Commands provide a means for instructing *Mathematica* to perform all sorts of tasks, from computing the logarithm of a number, to simplifying an algebraic expression, to solving an equation, to plotting a function. *Mathematica*'s commands are more numerous, more flexible, and more powerful than those available in any hand-held calculator, and in many ways they are easier to use.


Commands can be typed from the keyboard, but if you are using a desktop version of the software it is easiest to start by using the palette buttons. The Basic Commands section of the Basic Math Assistant palette contains many of the commands you are likely to use. If this section of the palette is not open, click on the  $\blacktriangleright$  to open it. At the top of this section there is a bar with seven tabs. If you hover over one of these tabs with your mouse, the name of its sub-palette will appear. The seven sub-palettes are shown in Table 1.1.

Icon	Sub-palette Name	Topics
$\sqrt{x}$	Mathematical Constants and Functions	Trigonometric, exponential, and logarithmic functions
$y = x$	Algebra Commands	Factoring, expanding, and simplifying algebraic expressions
$d \int \Sigma$	Calculus Commands	Computing derivatives, integrals, sums, and limits
$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$	Matrix Commands	Entering matrices and linear algebra computations
List	Table, List, Vector Commands	Creating tables and working with data
2D	2D Plot Commands	Creating two-dimensional graphs
3D	3D Plot Commands	Creating three-dimensional graphs


**Table 1.1 Basic Commands.** The seven tabs in the Basic Commands section of the Basic Math Assistant palette.

We will start with the Mathematical Constants and Functions sub-palette. This appears when you click on the  tab.

### Numerical Approximation and Scientific Notation

The first command we will introduce is `N`. It is the first button under Numeric Functions, just below the Mathematical Constants buttons. Click on the button labeled  and the following will appear in your notebook at the position of the cursor:

`N[expr]`

Now type an expression, such as  $5/3$ , and it will be inserted where *expr* appears in the input. Then enter the cell by hitting . By default, the approximation will have six significant digits:

```
In[1]:= N[5 / 3]
Out[1]= 1.66667
```

The palette is not strictly necessary here; if you type the input above directly from the keyboard, the result is the same. The point here is that the palette provides easy access to many commonly used commands. As you use the palette and gradually become familiar with these commands, you may eventually find it is simpler to type them directly from the keyboard.

`N` is a command you are likely to use frequently. Very large or very small numbers will be put in scientific notation: