CHAPTER I.

General View of the Method.

§ 1/1. IN figure 1,* let S. be the Sun; T, T′, T″, the places of the Earth at the three obſervations; C, C′, C″, the correſponding places of the Comet; P, P′, P″, the ſame places projected on the plane of the Ecliptic; t, c, p, the interſections of the Radii S T′, S C′, S P′, with the cords T T″, C C″, P P″. C I, is a right line parallel and equal to P P″, which meets P″ C″, (prolonged if neceſſary) in I. I ſuppoſe the obſervations ſo near one another, that the verſed ſines T′ t, C′ c, P′ p, are ſmall, compared to their Radii; yet far enough for the motion of the Comet in longitude not to be inſenſible, or ſo ſmall as to be materially affected by ſmall errors in obſervation. In general, an interval of from five to ten days between each obſervation, will be the beſt; but in ſome caſes, an interval of a day and an half, will not be too ſmall, and ſometimes, an interval of fifteen days, will not be too great. FIG. 1

§ 2/2. Obſervation gives the three longitudes of the Comet determined by the directions T P, T′ P′, T″ P″; and three

* In this figure, the triangular ſpace, P″ C″ R, muſt be conceived, as ſtanding at right angles to the plane P″ R S; and of courſe the angles, C P T, C P S, C′ P′ T′, C′ P′ S, and the reſt; to be right angles.

FIG. 1. latitudes P T C, P′ T′ C′, P″ T″ C″. The Ephemeris gives the three longitudes of the Earth, determined by the lines S T, S T′, S T″, with their diſtances from the Sun. The theory of Aſtronomy gives the excentricity of the Earth's orbit 0, 017 of the mean diſtance, and by the Newtonian theory of univerſal gravitation, we know that all bodies moving round the Sun deſcribe areas proportional to the times; and that the ſquare of the velocity in the Parabola, is reciprocally proportional to the diſtance of the Sun; being double the ſquare of the velocity in the circle, at equal diſtances from the Sun.

§ $\frac{2}{3}$. From theſe data, and the geometrical properties of the Parabola, we ſhall determine the dimenſions and poſition of the Parabola, deſcribed by the Comet, and the point in which it is in its orbit at a given time; its diſtance from the Sun, and the Earth; and its heliocentric longitude, and latitude.

§ $\frac{3}{4}$. The principal foundation of this method is the ſubſtitution of an equable and rectilinear motion of the Comet in the cord of its orbit, to its curvilinear and unequal one in its orbit.

§ $\frac{4}{5}$. We firſt find that when the areas are proportional to the times, though the velocity of motion in the curve is very unequal; the velocity of the interſection of the Radius Vector and the Cord is very nearly equable, in arches whoſe verſed ſine is ſmall, compared with the Radius Vector. Hence, by ſubſtituting the interſections of the Radius Vector of the Comet, and the Earth, at the ſecond obſervation, to the points of their orbits; we deduce the ratio of the curtate diſtances of the Comet from the Earth, to one another. One of theſe diſtances being aſſumed as known, gives the others; from theſe we find the Radii Vectores, (or true diſtances of the Comet from the Sun,) and the Cord of the Parabolic Orbit deſcribed by the Comet. Another Theorem gives the Ratio of the ſum of the Radii Vectores and the Cord to the motion of the Earth in its orbit, during the time elapſed between the firſt and laſt obſervation. The application of this Theorem to the Radii and Cord, deduced from the firſt aſſumed curtate diſtance from the Earth, gives the error of that poſition. FIG. 1.

§ $\frac{4}{6}$. The ſubſtitution of theſe interſections changes the longitude obſerved in the line T′ P′, into a longitude which would be determined by the line t p. But firſt it is eaſy to ſee that when the Comet is in conjunction with or in oppoſition to the Sun, theſe two lines fall into one; and I demonſtrate that when the Comet and Earth are equidiſtant from the Sun, they are parallel: in theſe circumſtances, which are not rare, no reduction for the ſubſtitution of one longitude for another, is neceſſary: and in poſitions not much different from them, the reduction is ſo ſmall as to be ſafely neglected. A very near approximation may indeed be often obtained, in poſitions far diſtant from theſe, without employing the reduction.

§ $\frac{5}{7}$. A ſhort and ſimple proceſs gives this reduction, which enables us, without ſenſible error, to ſubſtitute the motion of the interſection of the Radius Vector with the Cord, to the motion in the Arch. To find this reduction, we employ the verſed ſine of the Parabolic Arch, taken on the ſecond Radius Vector. This verſed ſine is found from the correſponding verſed ſine of the Arch of the Earth. This laſt is eaſily known, and is to the firſt, very nearly in the reciprocal ratio of the ſquares of the diſtances from the Sun; theſe two verſed ſines being nearly the meaſures of the effects of gravity on the two bodies.

§ $\frac{6}{8}$. Having, when neceſſary, made the reduction of the ſecond longitude as above, the two following Theorems give

FIG. 1. the Radii Vectores, and Parabolic Cord; and of courfe the figure and pofition of the Comet's orbit.

THEOREM FIRST.

The ratio of one of the curtate diftances of the Comet from the Earth, to any other curtate diftance; is compofed of the direct ratio of the intervals of time between the two obfervations belonging to thofe diftances, and the third obfervation; and of the reciprocal ratio of the motions in longitude anfwering to that time.

THEOREM SECOND.

§. $\frac{7}{9}$ If a be the fquare of twice the fpace which the Earth would pafs through by its mean motion in the interval of time between the firft and third obfervation;

b, the fum of the two extreme diftances of the Comet from the fun;

c, the Cord of the Arch of the Comet's motion between the firft and third obfervation: then $b\,c^2 = a$, when the Parabolic Arch is very fmall.

And when it is rather greater, a fmall correction is neceffary to the firft term, and the equation will ftand $b\,c^2 - \frac{c^4}{12\,b} = a$.

This correction is deduced from the fmall inequality found in the velocity of the interfection, when it is compared with the velocity of the Comet in the Parabolic Arch. The little inequality of the former is determined, and from thence the mean velocity of the interfection, and the point of the Arch in which the Comet has a velocity equal to this mean velocity, are found. The value of *a*, is eafily found by a conftant logarithm, and the double logarithm of the total time reduced into minutes. FIG. 1.

§ $\frac{8}{10}$. Let us now draw a circle of any fize, for the ecliptic, having the Sun in its centre, and having drawn three Radii for the longitudes of the Earth at the three obfervations, and fet off on thefe Radii the true diftances of the Earth from the Sun at each obfervation, from thefe points we draw three indefinite lines for the three obferved longitudes of the Comet; each of them making, with the Radius Vector of the Earth, an angle equal to the elongation of the Comet from the Sun, or the difference of their longitudes.

§ $\frac{8}{11}$. For the firft pofition we affume a curtate diftance of the Comet from the Earth at the fecond obfervation, and not to err very much in this affumption, we may form a guefs from the length and pofition of the Cord C C″, of the Parabolic Arch. If the diftance of the Comet from the Sun is twice that of the Earth, this Cord will be nearly equal to the Cord T T″, of the Arch of the Earth's Orbit. If the diftance of the Comet be half that of the Earth, the Parabolic Cord will be double that of the Earth in confequence of the theory of gravitation, (§ 2.) And as the length of the Parabolic Cord varies much flower than the diftances from the Sun, a near judgment of the diftances may be formed from the eftimated Cord. The Cord P P″, and diftance S P′, reduced to the plane of the Ecliptic, are always lefs than the true Cord C C″, and Radius Vector, S C′, of the real orbit, owing to its inclination to the plane of the Ecliptic: and this difference is eafily judged of from the latitudes, which raife the points of the orbit perpendicularly over the points of projection P, P″, to C, C″. FIG. 1 & 4.

FIG. 4. The pofition of the reduced Cord muft be fuch, that its ends muft coincide with the lines of the two extreme obferved longitudes of the Comet, and be cut in the ratio of the times elapfed between the obfervations, in a point which muft be very near the line of the fecond longitude; fo as to leave on the Radius Vector, which cuts it in this ratio, a very fmall fpace for the verfed fine.

§ $\overline{12}$ (9). Set off therefore on a ftrait edge of a piece of paper three points of the Cord of the Earth's Arch T, t, T″, Fig. 4.* and move it on the lines of the obferved longitudes of the Comet, backwards and forwards, keeping the point p near the line of the fecond longitude, and between it and the Sun, to leave on the oppofite fide, the fmall fpace anfwering to the verfed fine of the Parabolic Arch; and fo that the diftances of the two points G, G′, from the two lines of extreme longitudes, fhall be very nearly in the ratio of the intervals p G, p G′, going beyond the lines of longitudes with refpect to p, or keeping within them, according as by the circumftances mentioned in the laft paragraph, we may judge the reduced cord lefs or greater than the Earth's. A little habit will direct to a near guefs at this pofition. When the daily motion of the Comet in longitude is not very fmall, by moving the edge a very little, the part intercepted between the lines of extreme longitudes will change much, and the limits of probability of the real length of the cord, will lie in a narrow compafs. By this firft fuppofition, we fhall in general not be very far from the truth.

§ $\overline{13}$ (10). But at all events, having affumed a firft pofition, and having the point S, we have alfo the curtate diftance from the

* The points T t T″ fo transferred and applied to the lines of elongation of the Comet, are reprefented in Fig. 4. by G p G′, in the fituation chofen for them after the confiderations mentioned in the text.

Sun; and by means of the angles of latitudes, prepared apart, as hereafter defcribed, the elevation of the Comet from the plane of the Ecliptic for this pofition, and its whole diftance from the Sun are eafily found; and are the elements for the reduction of the fecond longitude. This reduction being made, by a fhort calculation, we fhall deduce from the fecond curtate diftance from the Earth, the two extreme ones, which will give the two curtate diftances from the Sun; and by the angles of latitudes, the elevations from the plane of the Ecliptic will be found. From hence will be found the firft, and third, whole diftances from the Sun; and the length of the cord, which give the values of b, c, in the formula $b c^2 - \frac{c^4}{12 b} = a$. If the value of the formula comes out different from a, known before, the fecond curtate diftance muft be changed, and in general diminifhed, if the value is greater; and increafed, if fmaller, than a. The errors of the formula in two pofitions, will by a fimple proportion give the true pofition, or near it; and very often the fecond pofition will be fo near the truth, as to give the orbit of the Comet near enough to find its apparent motion for the reft of its appearance. FIG. 4.

§ $\frac{1}{14}$. The elements of the orbit will be eafily found, as we fhall hereafter fhew.

§ $\frac{14}{15}$. If more precifion is defired, after two or three conftructed pofitions, trigonometry may be called in. And the folution of three oblique angled, and three right angled, plane triangles, will give the diftances from the Sun, and the Cord, which anfwer exactly to the pofition; and having obtained $b c^2 - \frac{c^4}{12 b} = a$; a fimple trigonometrical calculation, will give all the elements of the orbit.

CHAPTER II.

On the Motion of the Point of Interſection of the Radius Vector and Cord.

FIG. 1. § 1. [16] IN the orbit of the Comet C C′ C″, Figure 1. let cords be drawn joining C C′, and C′ C″; the ſegments intercepted between theſe cords, and the arches of the orbit, will be very ſmall, when compared with the areas of the ſectors C S C′, C′ S C″; the ratios of theſe ſectors will therefore be ſenſibly the ſame as thoſe of the triangles C S C′, C′ S C″, which only differ from them by the ſegments; now the areas of the ſectors are as the times; and the areas of the triangles as C c, to c G″; which are the baſes of the triangles C S c, c S′C″, having a common termination at S; and of the triangles C C′ c, c C′ C″, having a common termination at C′; therefore the ſegments C c, c C″, of the cord C C″, are ſenſibly as the times in which they are moved through by the interſection of the Radius Vector S C′. Therefore the motion of the point of interſection is ſenſibly uniform.

§ 2. [17] The diviſion of the cord by the point of interſection in the ratios of the times, will be almoſt perfectly exact near the middle, where the triangles will be equal, and the ſegments nearly ſo. There is a point in which the ratio is exact, but without ſeeking this point, it is obvious, that in arches whoſe verſed ſine is ſmall when compared with the Radius Vector, as we ſuppoſe in theſe reſearches; when the intervals of time are nearly equal, the two ſegments which we neglect, ever very ſmall, and in this caſe nearly equal, will leave an inſenſible error in this ratio.

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§ 3. As in all orbits deſcribed by central forces, the areas are in the direct ratio of the times; it follows that the theorem above ſtated, holds good in them all. Therefore in the Cord T T″, of the Earth's orbit; the Segments T t, t T″, will be ſenſibly in the ratio of the times. FIG. 1. & 4.

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§ 4. But it will be uſeful to determine the little inequality, which exiſts in the velocity of the point of interſection of the Radius Vector, and the Cord; as by this means greater arches may be made uſe of. This correction is the $\frac{c^4}{12b}$ in the formula given in the laſt chapter. Let then C A C′ (figure 2) be the arch of a curve, the areas of whoſe ſectors terminated in S, are in the ratio of the times; conceive two Radii Vectores S A, S a, infinitely near each other, and meeting the Cord C C′, in B, b; and the line S G, perpendicular to the Cord; then the triangle B S b, and the Sector A S a, may be conſidered as ſimilar, or as two ſectors of circles; therefore the firſt will be to the ſecond as SB^2, to SA^2; and as the firſt is $= Bb \times \frac{1}{2} SG$, the ſecond will be $= \frac{\frac{1}{2} SG \times SA^2 \times Bb}{SB^2}$. Now the velocity of the point of interſection B, is as the ſpace B b, divided by the time employed to paſs over it; which is proportional to the Sector A S a. The ratio therefore of the velocity ſought, will be found by dividing B b, by the value of the Sector, which gives $\frac{2 SB^2}{SG \times SA^2}$, and as 2, and S G, are conſtant; the velocity will vary as the fraction $\frac{SB^2}{SA^2}$. FIG. 2.

FIG. 2.

§ 4.[21] If the arch is deſcribed by forces tending to the centre S, it will be ever concave towards that point, and if the arch is not large, it will have a tangent K D K′, parallel to the Cord C C′, and beyond it, ſo that the Radius Vector terminating at the point of contact D, ſhall cut the Cord C C′, in a point F, and will cut other parallel lines A A′ in points I.

The velocity in queſtion, will be greateſt at the points C, and C′; and will go on continually diminiſhing from C to F, where it will be at its minimum; and then increaſe again from F to C′; but this inequality, will be ever ſmall when the verſed ſine D F, is ſmall, if compared to the Radius S D: for we ever have $\frac{SF}{SI^2} = \frac{SB^2}{SA^2}$; and as S F, is conſtant, the velocity of the inter-ſection B, will be in the reciprocal ratio of the ſquare of S I; which will be the leaſt when the points A, A′, coincide with C, C′, and the point I, falls on F; and will be greater, in proportion as A, A′, approach to D; the line S I, having a maximum, when A, A′, fall on D; when it is equal to S D. as the change of the line S I, never exceeds the verſed ſine D F, its variation, and the variation of the velocity, which is reciprocally proportional to it, will be ever ſmall.

§ 5.[22] There will be a mean velocity, with which in the ſame time, the ſame Cord would be deſcribed with an uniform velocity: we ſhall now find the point Q, of the parabolic orbit, in which the Comet has a velocity, equal to that mean velocity. The length of the Radius S Q, is the principal object of this re-ſearch; as from that, will be deduced the correction of the formula, given in Chapter 1, § 9.

This length will be firſt found as compared with the Radius S D, and the verſed ſine D F; from whence we ſhall find its