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Mathematical Theory of Electricity and Magnetism

Sir James Jeans (1877–1946) is regarded as one of the founders of British cosmology, and was the first to suggest (in 1928) the steady state theory, which assumes a continuous creation of matter in the universe. He made many major contributions over a wide area of mathematical physics, but was also well known as an accessible writer for the non-specialist. This well-known treatise, first published in 1908, covers the topics in electromagnetic theory required by every non-specialist physicist at that time. It provides the relevant mathematical analysis and was therefore useful to those with only limited mathematical knowledge, as well as to more advanced physicists, engineers and applied mathematicians. It includes a large number of examples, and provides an interesting snapshot of the state of the discipline a century ago.
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Mathematical Theory of Electricity and Magnetism

James Jeans
THE MATHEMATICAL THEORY
OF
ELECTRICITY AND MAGNETISM
THE
MATHEMATICAL THEORY
OF
ELECTRICITY AND
MAGNETISM

BY
SIR JAMES JEANS

FIFTH EDITION

CAMBRIDGE
AT THE UNIVERSITY PRESS
1963
PREFACE

[to the first edition]

There is a certain well-defined range in Electromagnetic Theory, which every student of physics may be expected to have covered, with more or less of thoroughness, before proceeding to the study of special branches of developments of the subject. The present book is intended to give the mathematical theory of this range of electromagnetism, together with the mathematical analysis required in its treatment.

The range is very approximately that of Maxwell's original Treatise, but the present book is in many respects more elementary than that of Maxwell. Maxwell's Treatise was written for the fully-equipped mathematician: the present book is written more especially for the student, and for the physicist of limited mathematical attainments.

The questions of mathematical analysis which are treated in the text have been inserted in the places where they are first needed for the development of the physical theory, in the belief that, in many cases, the mathematical and physical theories illuminate one another by being studied simultaneously. For example, brief sketches of the theories of spherical, zonal and ellipsoidal harmonics are given in the chapter on Special Problems in Electrostatics, interwoven with the study of harmonic potentials and electrical applications: Stokes' Theorem is similarly given in connection with the magnetic vector-potential, and so on. One result of this arrangement is to destroy, at least in appearance, the balance of the amounts of space allotted to the different parts of the subject. For instance, more than half the book appears to be devoted to Electrostatics, but this space will, perhaps, not seem excessive when it is noticed how many of the pages in the Electrostatic part of the book are devoted to non-electrical subjects in applied mathematics (potential-theory, theory of stress, etc.), or in pure mathematics (Green's Theorem, harmonic analysis, complex variable, Fourier's series, conjugate functions, curvilinear coordinates, etc.).

A number of examples, taken mainly from the usual Cambridge examination papers, are inserted. These may provide problems for the mathematical student, but it is hoped that they may also form a sort of compendium of results for the physicist, shewing what types of problem admit of exact mathematical solution.

It is again a pleasure to record my thanks to the officials of the University Press for their unfailing vigilance and help during the printing of the book.

J. H. JEANS.

Princeton,
December, 1907.
Preface

[TO THE SECOND EDITION]

The second Edition will be found to differ only very slightly from the first in all except the last few chapters. The chapter on Electromagnetic Theory of Light has, however, been largely rewritten and considerably amplified, and two new chapters appear in the present edition, on the Motion of Electrons and on the General Equations of the Electromagnetic Field. These last chapters attempt to give an introduction to the more recent developments of the subject. They do not aim at anything like completeness of treatment, even in the small parts of the subjects with which they deal, but it is hoped they will form a useful introduction to more complete and specialised works and monographs.

J. H. JEANS.

Cambridge,
August, 1911.

[TO THE THIRD EDITION]

In preparing a third Edition I have made only a few changes in the latter chapters, which were necessary to bring the book up to date.

J. H. JEANS.

London,
November, 1914.

[TO THE FOURTH EDITION]

It will be found that the main changes in the fourth Edition consist in a rearrangement of the later chapters and the addition of a wholly new chapter on the Theory of Relativity. It need hardly be said that no attempt is made to give a full account of the Theory; I have tried to present its broad outlines in the simplest possible way, and in striving after simplicity I have intentionally omitted all elaboration and detail. It is hoped that the new chapter will provide a suitable introduction to the Theory of Relativity for the student who approaches the subject for the first time, equipped with such knowledge of general electrical theory as can be gained from the rest of the book.

J. H. JEANS.

Dorking,
December, 1919.
Preface

[TO THE FIFTH EDITION]

In preparing a Fifth Edition I have introduced the changes that seemed to be called for by the now established position of the new theories of relativity and quanta. I have not attempted any detailed account of the theory of quanta but have added a chapter on “The Electrical Structure of Matter” which will introduce the reader to this theory.

It is a pleasure to record my thanks to friends and correspondents who have helped me by making suggestions and pointing out errors and misprints in earlier editions. My thanks are especially due to Dr A. Russell, F.R.S., Dr Harold Jeffreys, F.R.S., Professor E. P. Adams, Dr R. E. Baynes, Mr L. A. Pass and Dr H. L. Curtis.

Dorking,
March, 1925.

J. H. JEANS.
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