Analysis of Aircraft Structures

Second Edition

As with the first edition, this textbook provides a clear introduction to the fundamental theory of structural analysis as applied to aircraft, spacecraft, automobiles, and ships. The emphasis is on the application of fundamental concepts of structural analysis that are employed in everyday engineering practice. All approximations are accompanied by a full explanation of their validity. Repetition is an important learning tool; therefore some redundancy is used to dispel misunderstanding. In this new edition, more topics, figures, examples, and exercises have been added. There is also a greater emphasis on the finite element method of analysis. Clarity remains the hallmark of this text.

Bruce K. Donaldson was first exposed to aircraft inertia loads when he was a carrier-based U.S. Navy antisubmarine pilot. He subsequently worked in the structural dynamics area at the Boeing Company and at the Beech Aircraft Corporation in Wichita, Kansas, before returning to school and embarking on an academic career in the area of structural analysis. He became a professor of aerospace engineering and then a professor of civil and environmental engineering at the University of Maryland. Professor Donaldson is the recipient of numerous teaching awards and has maintained industry contacts, working various summers at government agencies and for commercial enterprises, the last being Lockheed Martin in Fort Worth, Texas. He is the author of *Introduction to Structural Dynamics*, also published by Cambridge University Press.

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Analysis of Aircraft Structures

An Introduction

Second Edition

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TO MY WIFE, LOIS,
AND CHILDREN,
LEXA, SARA, AND KENNETH

Greek Alphabet

Forms	Name
$A\ \alpha$	alpha
$B\ \beta$	beta
$\Gamma\ \gamma$	gamma
$\Delta\ \delta$	delta
$E\ \epsilon$	epsilon
$Z\ \zeta$	zĕta
$H\ \eta$	ĕta
$\Theta\ \theta$	thĕta
$I\ \iota$	iota
$K\ \kappa$	kappa
$\Lambda\ \lambda$	lambda
$M\ \mu$	mu
$N\ \nu$	nu
$\Xi\ \xi$	xi
$O\ o$	omicron
$\Pi\ \pi$	pi
$P\ \rho$	rhŏ
$\Sigma\ \sigma$	sigma
$T\ \tau$	tau
$\Upsilon\ \upsilon$	upsilon
$\Phi\ \phi$	phi
$X\ \chi$	chi
$\Psi\ \psi$	psi
$\Omega\ \omega$	ŏmega

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Introduction to the Second Edition

In an attempt to improve the first edition, more topics, figures, examples, and exercises have been added. The author hopes all the old errors have been removed and few new errors have been introduced. The primary change has been a greater emphasis on preparing the student for a broad understanding of the finite element method of analysis. In the author's experience, various finite element method software packages are almost, if not totally, the only means of structural analysis used today in the aerospace industry and in the associated federal and state government agencies. The three chapters dealing with the finite element method of analysis, Chapters 16, 17, and 18, are hopefully just the right amount of exposure suitable for undergraduates.

The style of presentation has remained the same. Clarity rather than brevity has been the consistent goal. Hence, there is a purposeful use of extra words and sentences in order to try to assist the reader who is new to the material. This strategy of being wordy admittedly makes this textbook less useful as a reference for the instructor who already is quite familiar with the chosen material. Perhaps this wordiness will allow that instructor the luxury of being brief in his or her lectures, knowing that this textbook is available as a backup to those lectures.

The author would appreciate any suggestions or corrections. He can be reached at bkdonaldson@verizon.net.

B. D.

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Introduction to the First Edition

This text has a single purpose. That purpose is to provide clear instruction in the fundamental concepts of the theory of structural analysis as applied to vehicular structures such as aircraft, automobiles, ships, and spacecraft. To this end, the text offers explanations and applications of the fundamental concepts of structural analysis and indications of how those concepts are employed in everyday engineering practice. The text endeavors to foster in the reader the habit of asking questions until the reader is thoroughly clear on all important details within the scope of this text.

Three strategies are followed to achieve clarity with regard to the basic concepts of structural analysis. The first strategy is to be thoroughly logical within the scope of the presented material. No "assumptions" with regard to method of analysis are made anywhere in this textbook. All approximations are accompanied by a full explanation of their validity. The second strategy is to be repetitious and redundant. Repetition is an important learning tool, and redundancy dispels misunderstandings. The third strategy to obtain the goal of clarity is to limit the number of topics covered in detail in this text to only those that are essential to an introduction to modern structural analysis.

This text is meant to serve as a basis for, or a supplement to, a series of introductory courses for undergraduates. It may be necessary to justify beginning a course of undergraduate study with the mathematics-encumbered elements of the theory of elasticity. If so, note that for an engineer to properly use any method of structural engineering analysis, that engineer must understand both the breadth and the limitations of that method of analysis. Such knowledge is invariably based, at least in large part, upon a derivation of the method in question. A background in the elements of the theory of elasticity is essential to interpreting the choices made in a structural analysis derivation with respect to the applicability of the result. The challenge posed to undergraduates by the theory of elasticity is further justified by the fact that the need to achieve vehicular structural components that are as lightweight and as easy to manufacture as possible requires an accuracy of analysis that is only possible with sophisticated, modern methods of analysis. The theory of elasticity is the one basis of that sophistication.

The educational price to be paid for the more encompassing view of structural mechanics made possible by means of an introduction to the theory of elasticity and a more rigorous introduction to work and energy methods is the loss of time for repetition of structural analysis solution techniques. This loss of repetition quite often means that students quickly forget what structural mechanics theory and solution techniques they quickly learned in their undergraduate courses. This fact, and the additional, more advanced material scattered throughout the text, make this text also suitable as a basis for, or supplement to, first-year graduate courses.

The organization of the text is largely conventional. Part I provides an introduction to structural mechanics and a single, logical basis for proceeding through the remaining parts of the text. Part II combines the separate topics of Part I to offer a brief introduction to the theory of elasticity at as close to an elementary level as possible. Part III develops the strength of materials approach to elementary straight-beam theory. Part IV introduces work and energy principles, and Part V offers the unit load and finite element methods of analysis as two distinct types of work or energy methods. The development of the finite element method is limited to only that material which is suitable as an introduction to the topic. Part III and Parts IV plus V can be studied sequentially or concurrently, or their order can be interchanged if the reader/student has some knowledge of beam theory such as that normally obtained from a prior course in strength of materials. A full undergraduate use of Parts I and II together, and Part III, requires something more than one semester each, while Parts IV plus V, through Chapter 21, requires about one semester. Time pressure to meet this undergraduate semester schedule requires omitting some parts of this textbook and very quick treatment of the more advanced chapters that are heavy with theory, such as Chapter 15. Undergraduates, like the rest of us, often only develop an interest in the details of a theory after enjoying its applications. If this textbook is used to instruct junior-year students, then the teaching schedule may omit more topics (even most of some chapters such as Chapters 7 and 8) than would be proper for more advanced students. The purpose of Part VI is to make the textbook more useful for those curriculums that include (i) some plate theory or aeroelasticity and (ii) a more in-depth look at elastic stability than is incorporated in the last of the three beam bending chapters. Most advanced topics of interest to graduate students are located in sections that are identified by a double asterisk, or relegated to endnotes and the more difficult exercises, and Appendix A.

In order to use this text effectively, the reader is expected to have the knowledge base that is usually obtained by completing (i) three semesters of college-level analytical geometry and the calculus, one semester of linear algebra, and an additional course in ordinary differential equations; and (ii) courses of study concerning engineering statics and dynamics. The required mathematics cited above is often a stumbling block for students. It is good for all students to realize that this body of mathematics is the tool that makes modern structural engineering methods both possible and comprehensible. In other words, understanding mathematics makes engineering easy. This text uses only the above-cited body of mathematics to formulate the engineering concepts presented herein. Occasionally a more elegant mathematical tool will be mentioned but not used. The mention of a more elegant mathematical tool is made only in order to prepare the reader for future professional growth. When a mathematical tool at the edge of the reader's assumed knowledge base is used, a brief review of that technique is provided in the preface to the part of the textbook in which its use is first encountered. While highly desirable as further practice in summing forces and moments for structural bodies, a prior course in strength of materials is not strictly necessary. However, the absence of sufficient prior drill in summing forces and moments means that that drill would have to be inserted into this study program.

Reference numbers are enclosed within brackets, and endnotes are enclosed within parentheses. The matrix and determinant symbols used are

$[\ ]$ is a square or rectangular matrix
$\{\ \}$ is a column matrix, often called a vector
$\lfloor\ \rfloor$ is a row matrix, also sometimes called a vector
$|\ |$ is a determinant
$[\ ]^{\mathbf{t}}$ is the transpose of the original matrix
$[\ ]^{-1}$ is the inverse of the original matrix

A ■ is used to indicate the end of an example problem.

Finally, since this text is almost exclusively devoted to explaining the methods needed for the analysis of the structural portion of whatever system is under study, it is appropriate to mention briefly the other activities that are also essential parts of the process of creating a land, marine, or flight vehicle. Analysis is but one part of the continuous product production chain – marketing to design to analysis to manufacturing to testing, and back to marketing. The above activities are somewhat ordered in time as stated. However, for any complex product, each activity is linked with every other activity with considerable overlap. In particular, the people responsible for the design and analysis activities work interactively in order to achieve better products. Expert analysis is an essential part of any vehicular design today.

List of Repeated Engineering Symbols

a	The vector of acceleration
a, b, c	Lengths; with subscripts, series coefficients
a	Distance aft from midchord to the elastic axis as a fraction of the semichord length, b
a	An arbitrary parameter
b	Width (Parts II–VI) or depth (Part I) of a rectangular beam cross-section; y axis intercept of a straight line; semichord length, $\frac{1}{2}c$
c	Airfoil chord length, $2b$
c_{mn}	Double Fourier series coefficients used in the solution for the twisting of a uniform beam with a rectangular cross-section
c_{nx}, c_{ny}, c_{nz}	The three direction cosines for the normal to a plane; that is, the respective cosines of the angles between the normal direction and the Cartesian coordinate axes; entirely equivalent to lower-case ν with single coordinate subscripts
c_{xz}	Typical coordinate rotation direction cosine, which, in this case, is the cosine of the angle between the (rotated) x^* Cartesian coordinate axis and the (original) z Cartesian coordinate axis. The first subscript always indicates the rotated coordinate system axis, while the second subscript always indicates the original coordinate system axis. See the next entry.
$[c]$	Three by three matrix of direction cosines of a rotated coordinate system arranged so that the first element subscript indicates the row number and the second element subscript indicates the column number
d_0, D_0	A rectilinear deflection of a structural support causing the structure to deform
e	A distance from some reference point; that is, an eccentricity; chord length distances, dimensional or nondimensional
e_y, e_z	Rectilinear distances to the beam cross-section shear center from an arbitrarily chosen moment center; these distances respectively parallel the y and z coordinate axes, and they are without a sign convention
f	Vibratory frequencies in units of cycles per second = Hz
$f, \mathscr{f}$	Arbitrary function
f_0	A magnitude associated with an applied force per unit length of beam axis, or an applied force per unit area in the case of a plate analysis
f_x, f_y, f_z	Externally applied force components per unit length applied along the beam centroidal axis in the respective coordinate directions of the subscripts, or similar forces per unit area applied at a plate midplane
f_{ij}	The i, j flexibility coefficient; sometimes called the i, j flexibility influence coefficient
$[f]$	Flexibility influence coefficient matrix; that is, the array of f_{ij}
g	Acceleration of gravity; damping factor; arbitrary function

h	Depth of rectangular beam cross-section; plate thickness
$h(t)$	Downward deflection of an airfoil
$[h]$	Elastic stability matrix defined in Chapter 23, which has as a factor the unknown value of the compressive load that causes buckling
i	Square root of -1
i, j, k	Integer indices
k	Spring constant/factor [units of force over length]; plate buckling coefficient defined in Example 22.4
k_{ij}	The i, j element of a stiffness coefficient matrix. It represents the value of the ith generalized external force required by a unit value of the jth generalized deflection (i.e., generalized coordinate) when all other generalized deflections are zero
$[k]$	Element stiffness coefficient matrix whose square size depends upon the number of generalized coordinates required, perhaps approximately, to uniquely specify the structural element deflections
$[\tilde{k}]$	Element stiffness matrix reduced in size by the application of the global or system boundary conditions to the element degrees of freedom
l	Length of beam finite element, usually a fraction of the total beam length; beam length
m	A discrete mass; slope of a straight line
m_y, m_z	Applied beam moments per unit length
m, n	Integers or integer indices, sometimes maximum values for the indices i, j, k
n, s	Respective normal and tangential orthogonal coordinates at any point along a line on a flat surface. When the curve is a closed curve, s is positive counterclockwise, and n is positive in the outer normal direction
p	Pressure on the surface of a structural body
q	Shear flow; that is, the product of (i) the shearing stress on the surface of the beam cross-section in the direction of the centerline of the thin skin and (ii) the thickness of the thin skin
$\mathscr{q}$	Fluid dynamic pressure equal to one-half the fluid mass density multiplied by the fluid velocity squared
q_i	The ith generalized coordinate
$\{q\}$	Column matrix of generalized coordinates; specifically, without modification, the element degrees of freedom of a finite element; with a ˆ, the structural system degrees of freedom; with a ˜, the DOF vector reduced by use of the system boundary conditions (later the tildes are dropped from use)
r	Radial coordinate of a cylindrical or spherical coordinate system, or a moment arm for shear flows and shearing stresses about an arbitrary moment center
$\mathbf{r}$	Position vector from the coordinate origin to the geometric point under consideration
$[r(\theta)]$	A coordinate rotation matrix for strains; see Eq. (6.15)
s	Arc length coordinate, usually with an arbitrary origin; also see statement regarding n, s
$\text{stp}(x - x_0)$	Heaviside (unit) step function; see Section 11.4
t	Thickness; the addition of an asterisk indicates a modulus weighted thickness, $(G/G_0)t$, in the case of beam torsion

t, t_0	Time, as a variable and as a parameter
u, v, w	Orthogonal displacement components of a material point within or on the surface of a structural body, hence a function of all three spatial coordinates, and perhaps time also
u, v, w	Deflections of points along the beam axis, hence a function only of x and perhaps time also; similarly for a plate midplane, as functions of x, y and perhaps t
$\tilde{u}, \tilde{v}, \tilde{w}$	Known orthogonal components of displacements on the surface of a structural body; that is, displacement boundary conditions
u_i, v_i, w_i	Finite element DOF; modified with ˆ , structural system DOF; modified with ˜, structural system degrees of freedom retained after application of system boundary conditions
x, y, z	Cartesian coordinates that locate material points before or after deformation
y, z	Coordinates of a beam cross-section originating at the centroid
y_0, z_0	Coordinates for a beam cross-section with an arbitrarily selected origin
A	Cross-sectional area of a beam
$\hat{A}$	Area enclosed within the centerline of a thin, closed beam cross-section
A_x, A_y, A_z	Areas perpendicular to the x, y, z axes, respectively
$\{A\}$	$[E]\{\alpha\}$, a vector associated with thermal strains and the resulting stresses
$[\mathscr{A}]$	One of several aerodynamic matrices in Chapter 23
$[A]$, $[B]$	Arbitrary square matrices
A_i, B_i, C_i, D_i	Constants of integration
$\mathbf{A}$, $\mathbf{B}$	Arbitrary vectors
ALS	Abbreviation for "actual load system"; the system of externally applied loads and the corresponding internal stress resultants induced by the applied loads as used by the unit load method for calculating unknown or known deflections
B	With coordinate subscripts, the body force per unit mass in the indicated coordinate direction
$[B]$	Coefficient matrix for finite element degrees of freedom in the expression for the intraelement strains
BC	Abbreviation for "boundary condition"
$[C]$	Damping coefficient matrix used to represent velocity-dependent forces
C_l	Lift coefficient; an empirical factor, which, when multiplied by the fluid dynamic pressure and a relevant area, yields the lift component of the total aerodynamic force [nondimensional]
$C_{l\alpha}$	Lift curve slope; that is, the slope of the straight-line portion of the curve on the plot of lift coefficient (C_l) versus angle of attack (α)
C_m	Aerodynamic moment coefficient; that is, an empirical factor, which, when multiplied by the fluid dynamic pressure, an appropriate area, and an appropriate moment arm, yields the moment acting upon an airfoil at the aerodynamic center
℄	Centerline of symmetry
D	Diameter
D	Plate bending stiffness coefficient equal to $Eh^3/[12(1-\nu^2)]$
$\mathbf{D}$	The total displacement vector whose components are u, v, and w
DOF	Abbreviation for "degree(s) of freedom"

$\%D$	Percent fatigue damage as represented by the Palmgren–Miner rule of Eq. (5.2), with 100-percent damage indicating a 50-percent probability of fracture failure as a result of a disordered sequence of loading cycles
$[D]$	The dynamic matrix; see Eq. (23.18)
E	Young's modulus, the modulus of elasticity; coordinate subscripts indicate the direction in which this material property is measured; the lack of a subscript indicates an isotropic material
E_0	Reference value for Young's modulus, chosen arbitrarily in order to produce more natural units for quantities useful in beam bending analyses
$[E]$	The material stiffness matrix that, when postmultiplied by the strain vector, often provides the stress vector
F	Applied force, often with numerical subscripts
$\mathbf{F}$	A force vector
F, G, H	Arbitrary functions
FEM	Abbreviation for "finite element method"; see Chapter 17 and subsequent chapters
G	Shear modulus, the modulus of rigidity; double coordinate subscripts indicate that the shear modulus is referred to the material axes of an orthotropic material
G_0	Reference value for the shear modulus, akin to E_0
GDE	Abbreviation for "governing differential equation"
$[H]$	The assembled elastic stability matrix, see Example 23.1
I	An integral
$\bar{I}$	Moments or product of inertia about the sub-area centroid
I_x, I_y, I_z	Mass moments of inertia about the axis indicated by the subscript
I_{yy}, I_{zz}	Area moments of inertia for a beam cross-section about the y and z axes, respectively, see Eqs. (9.6)
I_{yz}	Area product of inertia for a beam cross-section, see Eqs. (9.6)
J	The St. Venant constant for uniform beam torsion, which is calculated on the basis of the classification of the type of beam cross-section; an asterisk indicates that J is modulus weighted
K	Torsional spring constant or factor; units of force length, with various subscripts; in-plane plate stiffness coefficient
K_t	Stress concentration factor; the t subscript indicates that the factor is theoretical
$[K]$, $[\tilde{K}]$	Structural stiffness matrix assembled from element stiffness matrices, after the application of the global or system boundary conditions; the tilde is omitted in later equations when it is clear that the stiffness matrix is associated with only the unknown degrees of freedom
$[\hat{K}]$	Singular structural stiffness matrix assembled from the element stiffness matrices, before the application of the global boundary conditions
L	Beam length; or length, in general
$\mathscr{L}$	Lift force; that is, the component of the net aerodynamic force acting upon an airfoil or other solid body perpendicular to the direction of the motion of the body or the fluid stream. (The drag is the force component in the direction of the fluid stream.) For an airfoil, the lift force acts at the airfoil aerodynamic center.
M	Often with subscripts 0, 1, 2, . . . , bending or twisting moment applied to beam or other structures
$\mathscr{M}$	Aerodynamic moment at the airfoil aerodynamic center

$M_t(x)$	Internal beam twisting moment at the beam cross-section identified by the coordinate x
M_x, M_y	When functions of x, y: plate bending moments about the y, x axes, per unit length along the y, x axes, respectively; units of [force]
M_y, M_z	When functions of x only: internal beam bending moments about the y and z axes, respectively, at the beam cross-section identified by the coordinate x
$M_{xy}(x, y)$	Plate twisting moment about the x axis per unit length in the y coordinate direction; units of [force]
$M_{yx}(x, y)$	Plate twisting moment about the y axis per unit length in the x coordinate direction; units of [force]
$[M]$	Mass matrix in the system matrix equations of motion, which, when postmultiplied by the negative of the acceleration vector, produces the system inertia forces
N, N_0	Applied beam axial force, or, more generally, a force normal to a specified plane in space
$N(x)$	Internal beam axial force acting at the beam cross-sectional centroid, positive in tension
N_i	The number of fatigue cycles to failure for a single specified mean and alternating stress; or, when a function of one or more spatial coordinates, the deflection shape function associated with the ith structural element generalized coordinate
N_x, N_y	Internal or externally applied tensile or compressive forces that act in the indicated coordinate direction per unit length in the other coordinate direction and that lie within the midplane of a membrane or plate; units of [force/length]; or, more generally, a force acting on a plane in the coordinate subscript direction
N_{xy}, N_{yx}	Internal or externally applied shearing forces per unit length that lie within the midplane of a membrane or plate, and that follow the same sign convention as do shearing stresses; units of [force/length]
O, P, Q, R, S	Points in the volume of the structural body
P	Replacement symbol for $N(x)$ when the axial force in the beam is negative, that is, a compressive beam loading
Q_i	The ith generalized force, which is determined by use of the virtual work expression in conjunction with a variation of the ith generalized coordinate
$Q_y(s), Q_z(s)$	First moments of those portions of the beam cross-sectional area that lie behind the point s on the thin-beam centerline
Q_x, Q_y	Plate internal shear forces acting in the thickness direction, per unit length in the y, x directions, respectively; units of [force/length]
$\{Q\}$	Column matrix of generalized forces
R	A fixed radius; a force or moment reaction at a support
R	With two coordinate subscripts, ratios of inertia defined by Eqs. (10.6)
$[R(\theta)]$	A coordinate rotation matrix for stresses; see Eq. (6.14)
S	Beam–column slenderness ratio; an airfoil planform area
S_1, S_2	Surface areas for a structural body; subscript 1 indicates an area where forces/tractions, rather than deflections, are prescribed, and subscript 2 indicates the opposite
$[S]$	The material compliance matrix, which, when postmultiplied by the stress vector, produces the strain vector; see Eq. (6.1)

T	Either (i) kinetic energy; or (ii) a torque; or (iii), when modified by subscripts, and so on, a specified temperature change
T_x, T_y, T_z	Surface tractions (forces per unit area) acting upon the surface of the structural body in the x, y, and z directions, respectively
$\{T\}$	The vector of surface tractions
$[T]$	A rectangular transformation matrix between two sets of generalized coordinates
$\mathcal{T}$	A twisting moment of aerodynamic origin
U	Total strain energy of a structural body
U_0	Strain energy density; that is, the strain energy per unit volume
ULM	Abbreviation for the "unit load method"; see Chapters 20 and 21
ULS	Abbreviation for a "unit load system"; a system of self-equilibrated virtual forces and virtual moments used by the ULM to calculate known or unknown actual deflections
Vol.	Volume
V	Potential function for body forces per unit volume; see Eq. (8.10); with an upper bar, a similar potential function in plate theory
V	Potential energy of the externally applied loads that are the surface tractions and the body forces
V_y, V_z	Internal shear forces acting upon a beam cross-section in the indicated coordinate directions
V_x, V_y	Kirchhoff plate shearing forces that are important to plate boundary conditions. They are a combination of the ordinary plate shearing forces per unit length and derivatives of the twisting moment; see Fig. 22.3
W	Total work, that is, the work done by both the external and internal forces acting upon a structure
W_{ex}	Work done by the external forces acting upon a structure
W_{in}	Work done by the internal forces acting upon a structure
X, Y	x/a, y/b, respectively, nondimensional Cartesian coordinates used in certain finite element derivations
Y, Z	The principal centroidal coordinates of a beam cross-section (used only when a clear distinction is useful)
α	An angle, sometimes specifically the angular coordinate of the cylindrical coordinate system when θ has another meaning in the same analysis; an airfoil angle of attack; or a nondimensional factor
α	The coefficient of thermal expansion; coordinate subscripts indicate the material direction for which this material property is measured
$\{\alpha\}$	The 6×1 vector of the coefficients of thermal expansion for each of the three coordinate directions, plus three zeros; or the corresponding 3×1 plane stress vector
α, β	Factors, depending upon the aspect ratio of the rectangular beam cross-section, that arise from the calculation of the maximum shear stress and the St. Venant constant for uniform torsion, respectively
β	An angle, or a nondimensional or other parameter
γ	A nondimensional factor for the beam cross-sectional stiffness for shearing deformation γGA; see Section 20.5
γ	An engineering shearing strain, usually with two different coordinate subscripts; that is, the change in the original right angle defined by the two subscripted coordinates as a result of structural deformation