THE HYPERCIRCLE IN MATHEMATICAL PHYSICS

THE HYPERCIRCLE IN MATHEMATICAL PHYSICS

A METHOD FOR THE APPROXIMATE SOLUTION OF BOUNDARY VALUE PROBLEMS

BY

J. L. SYNGE, Sc.D., F.R.S.

Senior Professor, School of Theoretical Physics
Dublin Institute for Advanced Studies

CAMBRIDGE
AT THE UNIVERSITY PRESS
1957

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107666559

First published 1957
First paperback edition 2011

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-66655-9 Paperback

CONTENTS

PREFACE

This book describes a technique for the approximate solution of certain boundary value problems of mathematical physics. This technique involves concepts of function-space. These are developed *ab initio*, so that no special knowledge beyond the calculus is required on the part of the reader. Thus it is an elementary book in a mathematical sense, but the arguments are, I hope, mathematically exact, although I have tried to avoid the rather bleak axiomatics which repel mathematical physicists and engineers for whom the book is intended and for whom function-space will remain a means to an end and not an end in itself.

The book has been ten years in the making, during which time I have given isolated lectures, and in some cases short courses, on various aspects of the subject in a number of places: Princeton University, Massachusetts Institute of Technology, University of Leeds, Harvard University, Brown University, Carnegie Institute of Technology, Institute for Fluid Dynamics and Applied Mathematics (University of Maryland), Severi Jubilee Celebration (Rome, 1950), St Andrews Mathematical Colloquium (1951), University of Trieste, Henri Poincaré Colloquium (Paris, 1954), and the Dublin Institute for Advanced Studies. The experience so gained has been valuable because it brought home to me how reluctant mathematicians and physicists are in this age of analysis to use geometrical intuition as a guide, particularly when the geometry is that of a multi-dimensional space, or (worse) a space with an infinity of dimensions. Since this intuitional approach seems to me the first essential (for suggestion, not for proof), I have developed it very slowly in the early part of the book in order to establish a common understanding with the reader.

Outside this special field of application my knowledge of function-space is slight, and I am much indebted to Dr F. Smithies, mathematical adviser of the Cambridge University Press, whose kindly advice has kept me from wandering too far from current usages.

As stated in the Introduction, Professor W. Prager was joint originator of the hypercircle method in 1946; my thanks are due to him not only for the stimulating collaboration at that time but also for reading most of the manuscript of this book. Professor A. J. McConnell, Provost of Trinity College, Dublin, has taken an active interest in the work, his approach through variational

principles (see Chapter 5) throwing light on the range of applicability of the method. Critical discussions with Professors J. B. Diaz and A. Weinstein have helped me very much, and Dr J. McMahon and Mr V. G. Hart, while Scholars at the Dublin Institute for Advanced Studies, gave me assistance in the preparation of the manuscript; the computations of Chapter 5 were done by Mr Hart, who also wrote more than a third of the text of that chapter.

Mr Hart has given invaluable assistance in proof-reading. In a book of this sort, a complete absence of errors is not to be expected; the most we can hope is that they are few and venial. Our task has been greatly lightened by the efficiency of the Cambridge University Press in handling a rather difficult job.

DUBLIN J.L.S.

August 1956