

1 Integers

The first numbers you learn about are **whole numbers**, the numbers used for counting: 1, 2, 3, 4, 5, ..., ...

The whole number zero was only understood relatively recently in human history. The symbol 0 that is used to represent it is also a recent invention. The word 'zero' itself is of Arabic origin.

From the counting numbers, people developed the idea of **negative numbers**, which are used, for example, to indicate temperatures below zero on the Celsius scale.

In some countries, there may be high mountains and deep valleys. The height of a mountain is measured as a distance above sea level. This is the place where the land meets the sea. Sometimes the bottoms of valleys are so deep that they are described as 'below sea level'. This means that the distances are counted downwards from sea level. These can be written using negative numbers.

The lowest temperature ever recorded on the Earth's surface was -89°C , in Antarctica in 1983.

The lowest possible temperature is absolute zero, -273°C .

When you refer to a change in temperature, you must always describe it as a number of degrees. When you write 0°C , for example, you are describing the freezing point of water; 100°C is the boiling point of water. Written in this way, these are exact temperatures.

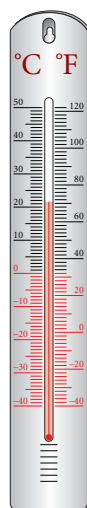
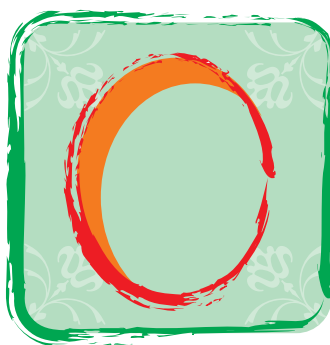
To distinguish them from negative numbers, the counting numbers are called **positive numbers**. Together, the positive (or counting) numbers, negative numbers and zero are called **integers**.

This unit is all about integers. You will learn how to add and subtract integers and you will study some of the properties of positive integers. You will explore other properties of numbers, and different types of number.

You should know multiplication facts up to 10×10 and the associated division facts.

For example, $6 \times 5 = 30$ means that $30 \div 6 = 5$ and $30 \div 5 = 6$.

This unit will remind you of these multiplication and division facts.



Key words

Make sure you learn and understand these key words:

whole number
 negative number
 positive number
 integer
 multiple
 common multiple
 lowest common multiple
 factor
 remainder
 common factor
 divisible
 prime number
 sieve of Eratosthenes
 product
 square number
 square root
 inverse

1.1 Using negative numbers

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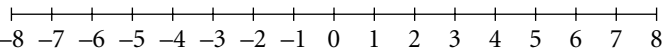
When you work with negative numbers, it can be useful to think in terms of temperature on the Celsius scale.

Water freezes at 0°C but the temperature in a freezer will be lower than that.

Recording temperatures below freezing is one very important use of negative numbers.

You can also use negative numbers to record other measures, such as depth below sea level or times before a particular event.

You can often show positive and negative numbers on a number line, with 0 in the centre.



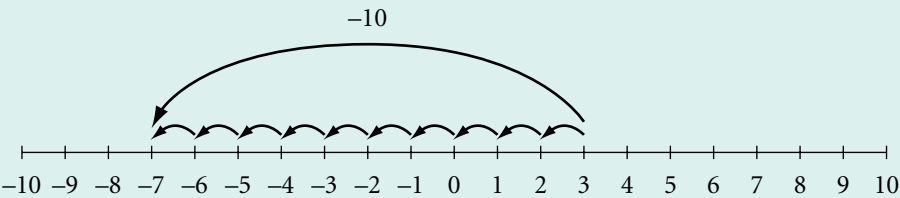
The number line helps you to put integers in order.
When the numbers 1, -1, 3, -4, 5, -6 are put in order, from lowest to highest, they are written as -6, -4, -1, 1, 3, 5.

Positive numbers go to the right.
Negative numbers go to the left.

Worked example 1.1

The temperature at midday was 3°C. By midnight it has fallen by 10 degrees.
What is the temperature at midnight?

The temperature at midday was 3°C. Use the number line to count 10 to the left from 3. Remember to count 0.



The temperature at midnight was -7°C.

You can write the calculation in Worked example 1.1 as a subtraction: $3 - 10 = -7$.
If the temperature at midnight was 10 degrees higher, you can write: $3 + 10 = 13$.

Exercise 1.1

- 1 Here are six temperatures, in degrees Celsius.
6 -10 5 -4 0 2
Write them in order, starting with the lowest.

Use the number line if you need to.

1.1 Using negative numbers

2 Here are the midday temperatures, in degrees Celsius, of five cities on the same day.

Moscow	Tokyo	Berlin	Boston	Melbourne
−8	−4	5	−2	12

- a Which city was the warmest?
b Which city was the coldest?
c What is the difference between the temperatures of Berlin and Boston?

3 Draw a number line from −6 to 6. Write down the integer that is halfway between the two numbers in each pair below.

- a 1 and 5 b −5 and −1 c −1 and 5 d −5 and 1



4 Some frozen food is stored at −8 °C. During a power failure, the temperature increases by 3 degrees every minute. Copy and complete this table to show the temperature of the food.

Minutes passed	0	1	2	3	4
Temperature (°C)	−8				

- 5 During the day the temperature in Tom’s greenhouse increases from −4 °C to 5 °C. What is the rise in temperature?
6 The temperature this morning was −7 °C. This afternoon, the temperature dropped by 10 degrees. What is the new temperature?
7 Luigi recorded the temperature in his garden at different times of the same day.

Time	06 00	09 00	12 00	15 00	18 00	21 00
Temperature (°C)	−4	−1	5	7	1	−6

- a When was temperature the lowest?
b What was the difference in temperature between 06 00 and 12 00?
c What was the temperature difference between 09 00 and 21 00?
d At midnight the temperature was 5 degrees lower than it was at 21 00. What was the temperature at midnight?

8 Heights below sea level can be shown by using negative numbers.

- a What does it mean to say that the bottom of a valley is at −200 metres?
b A hill next to the valley in part a is 450 metres high. How far is the top of the hill above the bottom of the valley?

9 Work out the following additions.

- a −2 + 5 b −8 + 2 c −10 + 7
d −3 + 4 + 5 e −6 + 1 + 5 f −20 + 19

Think of temperatures going up.

10 Find the answers to these subtractions.

- a 4 − 6 b −4 − 6 c −8 − 7
d 6 − 7 − 3 e −4 − 3 − 3 f 10 − 25

Think of temperatures going down.

1.2 Adding and subtracting negative numbers

1.2 Adding and subtracting negative numbers

You have seen how to add or subtract a positive number by thinking of temperatures going up and down.

Examples: $-3 + 5 = 2$ $-3 - 5 = -8$

Suppose you want to add or subtract a negative number, for example, $-3 + -5$ or $-3 - -5$.

How can you do that?

You need to think about these in a different way.

To work out $-5 + -3$, start at 0 on a number line.

-5 means ‘move 5 to the left’ and -3 means ‘move 3 to the left’.

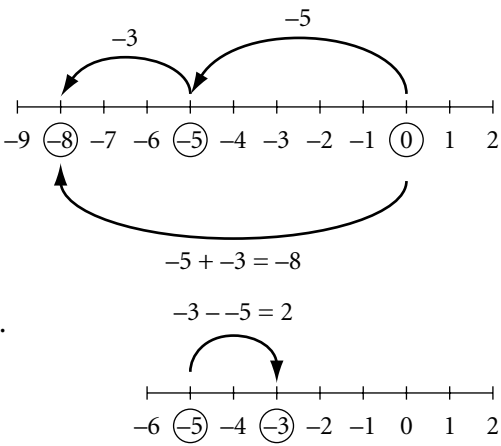
The result is ‘move 8 to the left’.

$-5 + -3 = -8$

To work out $-3 - -5$ you want the difference between -5 and -3 .

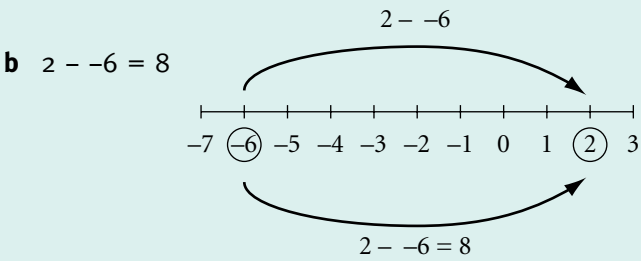
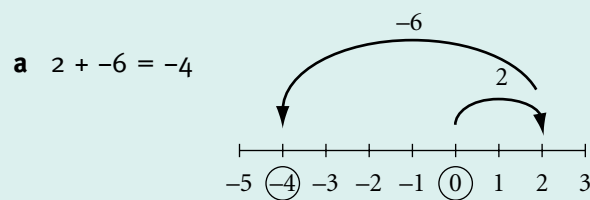
To go from -5 to -3 on a number line, move 2 to the right.

$-3 - -5 = 2$



Worked example 1.2

Work these out. **a** $2 + -6$ **b** $2 - -6$



Exercise 1.2

- 1 Work these out. ☐

a $-3 + 4$

b $3 + -6$

c $-5 + -5$

d $-2 + 9$
- 2 Work these out. ☐

a $3 - 7$

b $4 - -1$

c $2 - -4$

d $-5 - 8$
- 3 Work these out. ☐

a $3 + 5$

b $-3 + 5$

c $3 + -5$

d $-3 + -5$
- 4 Work these out. ☐

a $4 - 6$

b $4 - -6$

c $-4 - 6$

d $-4 - -6$
- 5 a Work these out.

i $3 + -5$

ii $-5 + 3$

iii $-2 + -8$

iv $-8 + -2$

b If $\blacktriangle$ and $\blacktriangledown$ are two integers, is it always true that $\blacktriangle + \blacktriangledown = \blacktriangledown + \blacktriangle$?
Give a reason for your answer.
- 6 a Work these out.

i $5 - -2$

ii $-2 - 5$

iii $-4 - -3$

iv $-3 - -4$

b If $\blacktriangle$ and $\blacktriangledown$ are two integers, what can you say about $\blacktriangle - \blacktriangledown$ and $\blacktriangledown - \blacktriangle$?

1.3 Multiples

Look at this sequence. $1 \times 3 = 3$ $2 \times 3 = 6$ $3 \times 3 = 9$ $4 \times 3 = 12 \dots, \dots$

The numbers 3, 6, 9, 12, 15, ... are the **multiples** of 3.

The multiples of 7 are 7, 14, 21, 28, ..., ...

The multiples of 25 are 25, 50, 75, ..., ...

Make sure you know your multiplication facts up to 10×10 or further.

You can use these to recognise multiples up to at least 100.

The dots ... mean that the pattern continues.

Worked example 1.3

What numbers less than 100 are multiples of both 6 and 8?

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, ..., ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, ..., ...

Multiples of both are 24, 48, 72, 96, ..., ...

The first number in both lists is 24.

These are all multiples of 24.

Notice that 24, 48, 72 and 96 are **common multiples** of 6 and 8. They are multiples of both 6 and 8.

24 is the smallest number that is a multiple of both 6 and 8. It is the **lowest common multiple** of 6 and 8.

Exercise 1.3

- Write down the first six multiples of 7.
- List the first four multiples of each of these numbers.
a 5 **b** 9 **c** 10 **d** 30 **e** 11
- Find the fourth multiple of each of these numbers.
a 6 **b** 12 **c** 21 **d** 15 **e** 32
- 35 is a multiple of 1 and of 35 and of two other numbers. What are the other two numbers?
- The 17th multiple of 8 is 136.
a What is the 18th multiple of 8? **b** What is the 16th multiple of 8?
- a** Write down four common multiples of 2 and 3.
b Write down four common multiples of 4 and 5.
- Find the lowest common multiple for each pair of numbers.
a 4 and 6 **b** 5 and 6 **c** 6 and 9 **d** 4 and 10 **e** 9 and 11

Remember to start with 7.



- 8** Ying was planning how to seat guests at a dinner. There were between 50 and 100 people coming. Ying noticed that they could be seated with 8 people to a table and no seats left empty. She also noticed that they could be seated with 12 people to a table with no seats left empty. How many people were coming?



- 9** Mia has a large bag of sweets.



If I share the sweets equally among 2, 3, 4, 5 or 6 people there will always be 1 sweet left over.

What is the smallest number of sweets there could be in the bag?

1.4 Factors and tests for divisibility

1.4 Factors and tests for divisibility

A **factor** of a whole number divides into it without a **remainder**. This means that 1 is a factor of every number. Every number is a factor of itself.

2, 3 and 12 are factors of 24. 5 and 7 are not factors of 24.

3 is a factor of 24

24 is a multiple of 3

These two statements go together.

$24 \div 2 = 12$ $24 \div 3 = 8$
 $24 \div 12 = 2$
 $24 \div 5 = 4$ remainder 1
 $24 \div 7 = 3$ remainder 4

Worked example 1.4

Work out all the factors of 40.

$1 \times 40 = 40$ Start with 1. Then try 2, 3, 4, ... 1 and 40 are both factors.
 $2 \times 20 = 40$ 2 and 20 are both factors.
 $4 \times 10 = 40$ 3 is not a factor. $40 \div 3$ has a remainder. 4 and 10 are factors.
 $5 \times 8 = 40$ 6 and 7 are not factors. $40 \div 6$ and $40 \div 7$ have remainders. 5 and 8 are factors.
You can stop now. You don't need to try 8 because it is already in the list of factors.
The factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

1 is a factor of every whole number.

A **common factor** of two numbers is a factor of both of them.

You don't have to list factors in order but it is neater if you do.

The factors of 24 are ①, ②, 3, ④, 6, ⑧, 12, 24.

$1 \times 24 = 24$ $2 \times 12 = 24$ $3 \times 8 = 24$ $4 \times 6 = 24$

The factors of 40 are ①, ②, ④, 5, ⑧, 10, 20, 40.

$1 \times 40 = 40$ $2 \times 20 = 40$ $4 \times 10 = 40$ $5 \times 8 = 40$

1, 2, 4 and 8 are common factors of 24 and 40.

Tests for divisibility

If one number is **divisible** by another number, there is no remainder when you divide the first by the second. These tests will help you decide whether numbers are divisible by other numbers.

Divisible by 2 A number is divisible by 2 if its last digit is 0, 2, 4, 6 or 8. That means that 2 is a factor of the number.

Divisible by 3 Add the digits. If the sum is divisible by 3, so is the original number.

Example Is 6786 divisible by 3? The sum of the digits is $6 + 7 + 8 + 6 = 27$ and then $2 + 7 = 9$. This is a multiple of 3 and so therefore 6786 is also a multiple of 3.

Divisible by 4 A number is divisible by 4 if its last two digits form a number that is divisible by 4.

Example 3726 is not a multiple of 4 because 26 is not.

Divisible by 5 A number is divisible by 5 if the last digit is 0 or 5.

Divisible by 6 A number is divisible by 6 if it is divisible by 2 and by 3. Use the tests given above.

1.4 Factors and tests for divisibility

- Divisible by 7** There is no simple test for 7. Sorry!
- Divisible by 8** A number is divisible by 8 if its last three digits form a number that is divisible by 8.
Example 17 816 is divisible by 8 because 816 is. $816 \div 8 = 102$ with no remainder.
- Divisible by 9** Add the digits. If the sum is divisible by 9, so is the original number. This is similar to the test for divisibility by 3.
Example The number 6786, used for divisibility by 3, is also divisible by 9.
- Divisibility by 10 or 100** Multiples of 10 end with 0. Multiples of 100 end with 00.

Exercise 1.4

- 1 The number 18 has six factors. Two of these factors are 1 and 18. Find the other four.
- 2 Find all the factors of each of each number.
a 10 **b** 28 **c** 27 **d** 44
e 11 **f** 30 **g** 16 **h** 32
- 3 The number 95 has four factors. What are they?
- 4 One of the numbers in the box is different from the rest. Which one, and why?
- 5 The numbers 4 and 9 both have exactly three factors. Find two more numbers that have exactly three factors.
- 6 Find the common factors of each pair of numbers.
a 6 and 10 **b** 20 and 25 **c** 8 and 15
d 8 and 24 **e** 12 and 18 **f** 20 and 50
- 7 There is one number less than 30 that has eight factors. There is one number less than 50 that has ten factors. Find these two numbers.

13 17 21 23 29

Think about the factors of 4 and 9.

- 8 **a** Find a number with four factors, all of which are odd numbers.
b Find a number with six factors, all of which are odd numbers.
- 9 Use a divisibility test to decide which of the numbers in the box:
a is a multiple of 3 **b** is a multiple of 6
c is a multiple of 9 **d** has 5 as a factor.
- 10 **a** Which of the numbers in the box:
i is a multiple of 10 **ii** has 2 as a factor
iii has 4 as a factor **iv** is a multiple of 8?
b If the sequence continues, what will be the first multiple of 100?

421 222 594 12 345 67 554

55 808 55 810 55 812
55 814 55 816 55 818

1.5 Prime numbers

1.5 Prime numbers

You have seen that some numbers have just two factors.
The factors of 11 are 1 and 11. The factors of 23 are 1 and 23.
Numbers that have just two factors are called **prime numbers** or just **primes**.
The factors of a prime are 1 and the number itself. If it has any other factors it is not a prime number.
There are eight prime numbers less than 20:

2, 3, 5, 7, 11, 13, 17, 19

1 is not a prime number. It only has one factor and prime numbers always have exactly two factors.
All the prime numbers, except 2, are odd numbers.
9 is not a prime number because $9 = 3 \times 3$. 15 is not a prime number because $15 = 3 \times 5$.

The sieve of Eratosthenes

One way to find prime numbers is to use the **sieve of Eratosthenes**.

- 1 Write the counting numbers up to 100 or more.
- 2 Cross out 1.
- 3 Put a box around the next number that you have not crossed out (2) and then cross out all the multiples of that number (4, 6, 8, 10, 12, ..., ...)
You are left with 2 3 5 7 9 11 13 15
- 4 Put a box around the next number that you have not crossed off (3) and then cross out all the multiples of that number that you have not crossed out already (9, 15, 21, ..., ...)
You are left with 2 3 5 7 11 13 17 19
- 5 Continue in this way (next put a box around 5 and cross out multiples of 5) and you will be left with a list of the prime numbers.

Eratosthenes was born in 276 BC, in a country that is modern-day Libya. He was the first person to calculate the circumference of the Earth.

Did you know that very large prime numbers are used to provide secure encoding for sensitive information, such as credit card numbers, on the internet?

Worked example 1.5

Find all the prime factors of 30.

2 is a factor because 30 is even.
3 is a factor.
5 is a factor because the last digit of 30 is 0.
The prime factors are 2, 3 and 5.

You only need to check the prime numbers.
 $2 \times 15 = 30$
 $3 \times 10 = 30$
 $5 \times 6 = 30$
6 is in our list of factors (5×6) so you do not need to try any prime number above 6.

1.5 Prime numbers

Exercise 1.5

- 1 There are two prime numbers between 20 and 30. What are they?
- 2 Write down the prime numbers between 30 and 40. How many are there?
- 3 How many prime numbers are there between 90 and 100?
- 4 Find the prime factors of each number.
 - a 10 b 15 c 25
 - d 28 e 45 f 70
- 5 a Find a sequence of five consecutive numbers, none of which is prime.
b Can you find a sequence of seven such numbers?
- 6 Look at this table.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

- a i Where are the multiples of 3? ii Where are the multiples of 6?
 - b In one column all the numbers are prime numbers. Which column is this?
 - c Add more rows to the table. Does the column identified in part b still contain only prime numbers?
- 7 Each of the numbers in this box is the **product** of two prime numbers.

226	321	305	133
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Find the two prime numbers in each case.



- 8 Hassan thinks he has discovered a way to find prime numbers. Investigate whether Hassan is correct.



I start with 11 and then add 2, then 4, then 6 and so on.
The answer is a prime number every time.

Numbers such as 1, 2, 3, 4, 5 are consecutive. 2, 4, 6, 8, 10 are consecutive even numbers.

The product is the result of multiplying numbers.

11 11 + 2 = 13
13 13 + 4 = 17
17 17 + 6 = 23...

- 9 a Find two different prime numbers that add up to:
 - i 18 ii 26 iii 30.
b How many different pairs can you find for each of the numbers in part a?

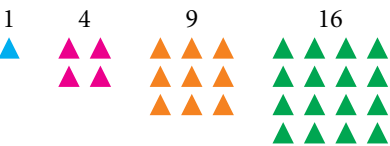
1.6 Squares and square roots

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$1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$ $4 \times 4 = 16$ $5 \times 5 = 25$

The numbers 1, 4, 9, 16, 25, 36, ... are called **square numbers**.

Look at this pattern.



You can see why they are called square numbers.
The next picture would have 5 rows of 5 symbols, totalling 25 altogether, so the fifth square number is 25.
The square of 5 is 25 and the square of 7 is 49.
You can write that as $5^2 = 25$ and $7^2 = 49$.
Read this as ‘5 squared is 25’ and ‘7 squared is 49’.
You can also say that the **square root** of 25 is 5 and the square root of 49 is 7.
The symbol for square root is $\sqrt{}$.
 $\sqrt{25} = 5$ and $\sqrt{49} = 7$

$\sqrt{25} = 5$

 means

$5^2 = 25$

Be careful: 3^2 means 3×3 , not 3×2 .

Adding and subtracting, and multiplying and dividing, are pairs of **inverse** operations. One is the ‘opposite’ of the other.
Squaring and finding the square root are also inverse operations.

Exercise 1.6

- Write down the first ten square numbers.
- Find 15^2 and 20^2 .
- List all the square numbers in each range.
a 100 to 200 **b** 200 to 300 **c** 300 to 400
- Find the missing number in each case.
a $3^2 + 4^2 = \square^2$ **b** $8^2 + 6^2 = \square^2$
c $12^2 + 5^2 = \square^2$ **d** $8^2 + 15^2 = \square^2$
- Find two square numbers that add up to 20^2 .
- The numbers in the box are square numbers.
a How many factors does each of these numbers have?
b Is it true that a square number always has an odd number of factors? Give a reason for your answer.
- Find:
a the 20th square number **b** the 30th square number. **c** the 50th square number.

16 25 36 49 81 100

1.6 Squares and square roots



8 Write down the number that is the same as each of these.

- a $\sqrt{81}$ b $\sqrt{36}$ c $\sqrt{1}$ d $\sqrt{49}$ e $\sqrt{144}$
f $\sqrt{256}$ g $\sqrt{361}$ h $\sqrt{196}$ i $\sqrt{29 + 35}$ j $\sqrt{12^2 + 16^2}$

The square root sign is like a pair of brackets. You must complete the calculation inside it, before finding the square root.

9 Find the value of each number.

- a i $(\sqrt{36})^2$ ii $(\sqrt{196})^2$ iii $\sqrt{5^2}$ iv $\sqrt{16^2}$
b Try to write down a rule to generalise this result.

10 Find three square numbers that add up to 125. There are two ways to do this.



11 Say whether each of these statements about square numbers is always true, sometimes true or never true.

- a The last digit is 5. b The last digit is 7.
c The last digit is a square number. d The last digit is not 3 or 8.

Summary

You should now know that:

- ★ Integers can be put in order on a number line.
- ★ Positive and negative numbers can be added and subtracted.
- ★ Every positive integer has multiples and factors.
- ★ Two integers may have common factors.
- ★ Prime numbers have exactly two factors.
- ★ There are simple tests for divisibility by 2, 3, 4, 5, 6, 8, 9, 10 and 100.
- ★ 7^2 means '7 squared' and $\sqrt{49}$ means 'the square root of 49', and that these are inverse operations.
- ★ The sieve of Eratosthenes can be used to find prime numbers.

You should be able to:

- ★ Recognise negative numbers as positions on a number line.
- ★ Order, add and subtract negative numbers in context.
- ★ Recognise multiples, factors, common factors and primes, all less than 100.
- ★ Use simple tests of divisibility.
- ★ Find the lowest common multiple in simple cases.
- ★ Use the sieve of Eratosthenes for generating primes.
- ★ Recognise squares of whole numbers to at least 20×20 and the corresponding square roots.
- ★ Use the notation 7^2 and $\sqrt{49}$.
- ★ Consolidate the rapid recall of multiplication facts to 10×10 and associated division facts.
- ★ Know and apply tests of divisibility by 2, 3, 4, 5, 6, 8, 9, 10 and 100.
- ★ Use inverse operations to simplify calculations with whole numbers.
- ★ Recognise mathematical properties, patterns and relationships, generalising in simple cases.

End of unit review

End of unit review

- 1 Here are the midday temperatures one Monday, in degrees Celsius, in four cities.

Astana	Wellington	Kuala Lumpur	Kiev
−10	6	18	−4

- a Which city is the coldest?
b What is the temperature difference between Kuala Lumpur and Kiev?
c What is the temperature difference between Kiev and Astana?
- 2 At 9 p.m. the temperature in Kurt’s garden was -2°C .
During the night the temperature went down 5 degrees and then it went up 10 degrees by midday the next day.
What was the temperature at midday in Kurt’s garden?
- 3 Work these out.
a $6 - 11$ b $-5 - 4$ c $-8 + 6$ d $-3 + 18$
- 4 Work these out.
a $-7 + -8$ b $6 - -9$ c $-10 - -8$ d $5 + -12$
- 5 Write down the first three multiples of each number.
a 8 b 11 c 20
- 6 Find the lowest common multiple of each pair of numbers.
a 6 and 9 b 6 and 10 c 6 and 11 d 6 and 12
- 7 List the factors of each number.
a 25 b 26 c 27 d 28 e 29
- 8 Find the common factors of each pair of numbers.
a 18 and 27 b 24 and 30 c 26 and 32
- 9 Look at the numbers in the box. From these numbers, write down:
a a multiple of 5
b a multiple of 6
c a multiple of 3 that is not a multiple of 9.
- 26 153 26 154 26 155 26 156 26 157
-  10 There is just one prime number between 110 and 120.
What is it?
- 11 Find the factors of 60 that are prime numbers.
-  12 a What is the smallest number that is a product of three different prime numbers?
b The number 1001 is the product of three prime numbers. One of them is 13.
What are the other two?