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LIQUID HELIUM

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PREFACE

The liquid helium problem can be conveniently divided into two parts, a problem in statistical mechanics and a problem in hydrodynamics. The problem in statistical mechanics is essentially a many body problem, in which the interactions between the atoms cannot be ignored and the symmetry of the wave-functions plays a dominant role. An analogous situation is encountered in heavy nuclei and has received so much attention in recent years that it is very close to being solved. In the case of liquid He^4 , Landau has proposed a scheme of elementary excitations (or spectrum of energy levels) which is able to explain the thermodynamic properties, and Feynman has derived a wave-function which gives this scheme of excitations. We are therefore far advanced in our understanding of the thermodynamic nature of the liquid, and, although it is still necessary to justify our ideas by a rigorous solution of the wave-mechanical problem of a large number of interacting helium atoms, there is every indication that such a solution will soon be forthcoming. On the other hand, the detailed behaviour of the liquid in the immediate vicinity of the λ -point is still not understood, and the indications are that some very difficult mathematical problems are involved, but that their solution will lead to a better understanding of co-operative phenomena in general.

An interesting aspect of the situation is that He^3 differs from He^4 mainly because He^3 atoms require antisymmetric wave-functions. Experimentally, liquid He^3 shows none of the unusual properties of liquid He^4 and therefore presents a separate problem of its own.

A thorough understanding of the statistical mechanics of liquid He^4 is not in itself sufficient to solve the hydrodynamical problem of superfluidity. The Onsager-Feynman concept of quantized vortex lines promises to provide the additional basic idea which is needed, but there is still much more experimental and theoretical work to be done on the details.

In these circumstances, this book has a double purpose: first, to attempt a detailed survey of the experimental results for the

benefit of active research workers in the field; secondly, to present an exposition of the current state of the theory, in the belief that the ultimate solution will be along similar lines. An attempt has been made to give an exhaustive survey of all work in the field, but a certain amount of selection is inevitable and the author apologizes in advance for his personal prejudices. In discussing the theory, emphasis has been placed on physical significance rather than mathematical details. Consideration was given to the possibility of including a compilation of the numerical data presently available on the various properties of liquid helium, but it became clear that there are so many uncertainties in the exact numerical values that such a compilation might be more misleading than helpful. However, adequate references have been given to enable the reader to refer to the original sources and form his own opinion of the relative reliability of the conflicting measurements.

I should like to express my gratitude to a large number of my colleagues for many helpful discussions and for informing me of their results in advance of publication. To name them all would occupy much space and would involve the risk of an inadvertent omission. I am particularly indebted to Dr D. Shoenberg for his helpful editorial comments; to Drs H. B. Callen, H. E. Hall and W. F. Vinen for pointing out several errors and obscurities; to the National Science Foundation for a grant which enabled me to continue work in the field during the later stages of preparing this work; and especially to my wife, who gave invaluable assistance with the many tedious chores concomitant with authorship.

K. R. A.

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