Cambridge University Press 978-1-107-63857-0 — A Student's Manual for A First Course in General Relativity Robert B. Scott Index More Information

Index

∂_{α} , 120

absolute magnitude, 275 adiabatic index, 244 affine connection, 116 affine parameter contrast with non-affine parameter, 156 maintains magintude of tangent vector, 156 proper length along geodesic is one, 132 affine transformation defined, 17 apparent magnitude, 275 asymptotically flat spacetime, 250 atlas 123 basis vectors implicit ones in derivative along curve, 157-158 partial derivatives for, 120 Bianchi identities contracted 140 twice contracted, 140-141 binomial series, 295 bolometric apparent magnitude, 275 boundary conditions for stellar model, 233, 243-244 brightness of a receding star, 87-92 bucket experiment of Isaac Newton, 16 chart, 123 Christoffel symbols compatible with metric, 116-117 derive $\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}),$ 126–127 for diagonal metric, 120 for polar coordinates, 105 Kerr spacetime, 300-301 not a tensor, 119-120 post-Newtonian spherical rotating star, 299 proof $\Gamma^{\mu}_{\ \alpha\beta} = \Gamma^{\mu}_{\ \beta\alpha}$, 126 Rindler spacetime, 297 RW spacetime, 302 Schwarzschild spacetime, 298 static spherically symmetric spacetime, 145, 297 torsion free, 116-117 transformation of, 119-120 weak gravitational field, 162-163, 298 classical fluid mechanics continuity equation, 165-166

derive Euler's equation from $T^{i\beta}_{\ ;\beta} = 0, 163-165$ Euler's equation, 165 hydrostatic, 166-167 clock synchronization in stationary spacetime, 178 comma goes to semicolon rule, 160-161 Compton scattering, 37-39 conservation of four-momentum Compton scattering, 37-38 electron and positron annihilate cannot produce single particle, 33 physical interpretation, 44 practice applying, 32-33, 36-39 continuum hypothesis, 76-77 coordinate basis vectors components in coordinate basis, 25 coordinate curves, 112 coordinate system, 123 coordinate transformations 2D Cartesian to polar coordinates, 103-104 between Cartesian and spherical, 296 practice distinguishing degenerate and non-degenerate, 100-101 coordinates isotropic, 231 quasi-Cartesian, 231 cosmic microwave background follows black-body spectrum, 290-291 cosmic microwave radiation, 279-280 cosmic rays Compton scatter with CMB photons, 38-39 cosmological constant, negative case, 279 cosmological redshift, 290 as function of cosmic time, 274-275 derivation, 273-274, 283 covariant derivative applied twice to rank $\binom{1}{1}$ tensor, 152 compatible with the metric, 116-117 coordinate transformation of, 116 Leibniz rule, 142-143 tensor field, 116 Crab pulsar, dipole magnetic field strength of, 240 Cramer's formula for matrix inverse, 129, 295 curves through spacetime tangent vector, 101 versus paths, 101-103 Cyg X-1 X-ray spectra, 252

305

Cambridge University Press 978-1-107-63857-0 — A Student's Manual for A First Course in General Relativity Robert B. Scott Index

More Information

306	ndex
determinant	expanding universe
definition using cotactors, 127	balloon model, 268
gradient of, $12/-128$	order of magnitude of various quantities, 283
of the metric is a scalar density, 117	recession speed proportial to distance, 268
dispersion of a wave, 224	
divergence	first law of thermodynamics, 284
of a vector in polar coordinates, 107	flat spacetime
of a vector in polar coordinates, derivation,	vanishing Riemann tensor is necessary and
129–130	sufficient condition, 136
of a vector in spherical coordinates, 157	fluid equation
of a vector in spherical coordinates, derivation,	derived from first law of thermodynamics,
129–130	284–285
Doppler shift, 266	four-acceleration
angle of no shift, 34–35, 45	defined, 182
calculated from four-momentum conservation.	orthogonal to three-velocity, 44-45
34–35	practice calculating, 30–31
dot product	relation to geodesic equation, 182
in GR 177	transforms like a 4-vector, 182
of two one-forms 57–58	four-momentum
dust 70	conserved components 169
dust, 79	contrast with three-momentum 78–79
earliest time of universe we can explore.	practice interpreting 31_32 45_46
281	four valacity
Einstein clock synchronization 21	defined 20
in Robertson–Walker universe 282–283	difference between two 4 valuation not a
Firstein field equations	4 subsition 45
analytical solution 245	4-velocity, 45
constraint equations, no second time derivatives	normalization, $41-42$, 177
	relation with three-velocity, 41
dimensional consistency of 204, 207	sum of or difference between two is not a
for vocume groaptime, 224	tour-velocity, 45
of linearized theory 102, 102	Fourier transform
Di linearized theory, 192–193	to solve Laplace's equation, 210–211
Einstein tensor	Friedmann equation, 277
on 3D subspace, be careful, 2/1	Frobenius' theorem, 291
proof it is divergence free, 141	future directed
RW spacetime, 302	defined, 46
static spherically symmetric spacetime, 242–243,	
298	Gödel universe, 291
symmetry, 156–157	gauge condition
weak gravitational field, 299	Lorenz gauge, 193
Einstein universe, 285–288	gauge transformation
energy conditions, 96-97	Maxwell's equations, 193
energy of a particle	relation to Killing's equation, 190
contrast between $-p_0$ and locally measured,	Gauss' law
227–228	in curved spacetime, 158-159
energy at infinity, $-p_0$, 242	used to solve Poisson's equation, 186-187
gravitational potential energy, 227–228	geodesic
not p^0 in GR, 242	deviation, 154–155
photon, 241	deviation due to gravitational wave. 218–219
equation of state	equation derived from four-acceleration. 182
$p = p(\rho, S)$ and static star 229	equation for. 154
Buchdahl for interior of stars $231-232$	proof of extremal length 133 151–152
dark energy 278	spacelike doesn't change along it 132
narticles of uniform speed 85 87	geodesic sphere 286
particles of uniform spece, $03-07$	geometric series 00
photon gas, $0.5-0/$	geometric vertes, 99
Stellar Hodel, $244-245$	geometric units, 244
	VICIDAL DASIS 1 1 1

Cambridge University Press 978-1-107-63857-0 — A Student's Manual for A First Course in General Relativity Robert B. Scott Index

More Information

307	Index		
global inertial frame impossible in presence of gravity, 1	5 Killing 4 Conserved quantities, 172–173, 181		
alabally marallal yeatan field 120, 121	de Sitter, 170		
gravitational redshift 98 251–252 26	66 Kerr 176		
gravitational redshift, 98, 251–252, 20 argument from energy conservation between two arbitrary radii in Schr spacetime, 242 in uniform gravitational field, 118 perpetual motion if violated, 98 static spherically symmetric spaceti surface redshift, 232–233 group Lorentz, 66, 174 Pairceré 174	30 Kerr, 176 1, 98–100 linear combination of Killing vectors is Killing, 17 warzschild Minkowski, 173–174 Robertson–Walker, 176–177, 183–185 Schwarzschild, 175–176 set of all Killing vector fields forms a linear space, 174–175 symmetry and Killing vector fields, 173–174 Kronecker delta coordinate transformation of, 117–118		
group velocity of a wave 224	Leibniz rule, 74–75, 142, 149		
history of the universe, 279 hopeful/frustrated experimentalist, 16 245, 288–290	implies form of covariant derivative, 117 Lie bracket, 148–151 Lie derivative, 149 arbitrary rank tensor, 151		

not geodesic motion when $p \neq 0, 291$ Hubble parameter as a function of redshift, 275 Hubble's law, 268 anisotropic, 269-270 apparently superluminal speeds, 284 hydrostatic equilibrium, relativistic and classical, 166 hyperbolae asymptotics, 7 hypersurface induced metric on, 159, 180 orthogonal and Frobenius' theorem, 291 ideal clock defined, 178 impact parameter, 247-248 index raising and lowering generalizes to curvilinear coordinates, 115-116 indices practice identifying free and dummy, 23-24 inertial reference frame, experiment to construct, 15 integrability condition, 130-131 interval of SR proof it can be written as a symmetric quadratic form, 4 intrinsic derivative, 155 isometry, 170 isotropic coordinates, 231 Jacobi's formula, 129 Jacobian, 100 Kepler's third law, 179 Kerr Killing fields, 176 Christoffel symbols, 300-301 massive particle orbits, 259-262 metric, 169

of scalar, 156 Lipschitz condition, 215 local inertial frame, 179 caution about using equations only valid in LIF, 155 - 156transformation matrix to, 123-126 look-back time, 274 Lorentz contraction derivation using Lorentz transformations, 11-13 formula for, 11 "moving rods contract", 12 small velocity approximation, 10-11 Lorentz group, 66-69 Lorentz scalar, 72 Lorentz transformations analogy with rotation, 78 argument that must be affine, 17-18 as coordinate transformations with constant coefficients, 52-53 formulae in standard configuration, 11 in general units for which $c \neq 1, 20-21$ interpreting matrix, finding inverse, 28 inverse property, 53 isotropy in the derivation, 18 matrix form for standard configuration, 14-15 not standard configuration, 41-44 of basis vectors, 27, 39-40 small velocity limit, 21 symmetry of matrix, 41 transform straight lines into straight lines, 17 luminosity, 88 manifold, 121 does not include boundary, 122 must be mapped continously to \mathbb{R}^n , 122 matter equation, 277

matter-dominated universe, 278

photon orbits, 266

Cambridge University Press 978-1-107-63857-0 — A Student's Manual for A First Course in General Relativity Robert B. Scott Index

More Information

308	Index

Maxwell's equations, 92-95, 193 MCLIRF, 242 metric and Newtonian potential, 167-168 determinant not an invariant scalar, 117 Euclidean space, polar coordinates, 108 $g^{\alpha}_{\ \beta} \equiv \delta^{\alpha}_{\ \beta}, 158$ Gödel universe, 291 gradient of determinant, 128-129 induced on hypersurface, 159, 179-180 Minkowski spacetime, spherical coordinates, 169-170 physical meaning, 178, 179 post-Newtonian spherical rotating star, 196 - 200static spherically symmetric spacetime, 144 three-sphere, 144 two-sphere, 141, 268 Minkowski inside a spherical shell of uniform matter, 245 Killing fields, 173-174 metric in spherical coordinates, 169-170, 227 moment of inertia of uniform ball, 239 natural units practice converting from, 2 practice converting to, 1 neutron star equation of state, 245 slowly rotating, 239 Newton's first law invariant under Lorentz transformations for a velocity boost, 16-17 Newton's second law from divergence of stress-energy tensor, 80-81 Newtonian limit, 168, 194-195, 264 Newtonian potential for a point particle, 186-187 of order velocity squared, 185 relation to metric, 167-168, 263 non-affine parameter magnitude of tangent vector changes, 156 ODE existence and uniques of solutions, 215 one-form associated with a vector 116 normal to a surface, defined, 53 one-form basis dual basis not simply the one-forms associated with vector basis, 115 not dual to coordinate vector basis \vec{e}_{α} , 51 transformation under a change of basis, 51-52 one-forms contraction with vector is frame invariant, 50-51

determined by contraction with four independent vectors, 49

linear independence of, 49-50 set of all forms a vector space, 47-49 orbit circular, in Schwarzschild spacetime, 179, 249-250 circular, Keplerian, 179, 188-189 circular, post-Newtonian effects of rotating star, 196-202, 208-209 massive particle orbits around Kerr black hole, 259-262 photon orbits around Kerr black hole, 266 radial, in Schwarzschild spacetime, 265 $O(\epsilon^n), 10$ orthogonal matrix defined, 58 Eucldian space Cartesian to Cartesian coordinate transformation, 58-59 parallel transport, effect on vector dot product, 131 particle flux divergence, 160 Peano existence theorem, 215 phase speed of a wave, 224 photon gas equation of state derivation, 85-87 stress-energy tensor, 183 Planck temperature, 281-282 polarization depends upon coordinate orientation, 219 post-Newtonian effect, 196-202 post-Newtonian spherical rotating star Christoffel symbols, 299 metric, 196-200 Pound-Rebka-Snider experiment, 118 power series solution to ODE system, 233, 243 pressure gradient force per unit volume, 83 projection operator, 64-66 proper area, 148 proper distance, 179 effect of gravitational wave upon, 215-217 in Schwarzschild metric, 180-181, 225 is an affine parameter along a geodesic, 132 not radar distance, 225 proper length, 11 proper time, relation to coordinate time, 179 proper volume, 148, 243 pseudo-Cartesian coordinates, 52 pulsar dipole magnetic field strength, 240 quadratic form, definition, 5 quality factor Q of an oscillator, 220-221 quantum gravity, 281-282 radar distance, 217 not proper distance, 225 radiation density constant, 280 radiation dominated universe, 277

radiation dominated universe, 277 radiation energy density for black-body spectrum, 277

radiation-dominated era, 280

Cambridge University Press 978-1-107-63857-0 — A Student's Manual for A First Course in General Relativity Robert B. Scott Index

More Information

Index

rank $\binom{0}{2}$ tensor basis, deepen understanding of, 54–55 rank $\binom{1}{1}$ tensor as map from vectors to vectors and one-forms to one-forms, 56-57 not just a matrix, 61-62 trace is a scalar, 72 rank $\binom{2}{0}$ tensor, symmetry uneffected by lower indices, 62-63rank $\binom{M}{N}$ tensors basis for, 60-61 form a vector space, 59-60 reciprocity principle, 89 redshift parameter, 273 redshift-magnitude relation, 275-276 rest mass temperature dependence, 35-36 Ricci scalar relation to stress-energy tensor trace, 183 static spherically symmetric spacetime, 154 within a centrifuge, 181 within a neutron star, 245 Ricci tensor proof it is symmetric, 139 RW spacetime, 302 static spherically symmetric spacetime, 154, 297 weak gravitational field, 299 Riemann tensor $R_{\alpha\alpha\mu\nu} = R_{\alpha\beta\mu\mu} = 0,151$ to first order in metric perturbation, gauge invariant, 189 - 190careful which equation you use, 155 diagonal elements, 151 for gravitational wave in TT gauge, 217-218 independent components in 2D, 135 independent components in 4D spacetime, 217-218 independent contractions, 138-139 static spherically symmetric spacetime, 144-147 two-sphere, 141-142 vanishing is necessary and sufficient condition for flat spacetime, 136 zero in polar coordinates in Euclidean plane, 135 Rindler spacetime, 111-115, 297 Christoffel symbols, 297 metric, 297 Robertson-Walker Killing fields, 176-177, 185 Christoffel symbols, 302 Einstein tensor, 302 metric, 169 metric derivation, 271 metric in $x^{\mu} = (t, \chi, \theta, \phi)$ coordinates, 283 metric, flat space, 172 radially moving photon, 273, 283-284 Ricci tensor, 302 rotation sign convention of, 219 symmetry, 170 ruler distance, 179

scalar density, 117 scale factor evolution equation, 277-278 $R \propto t^{1/2}$ in radiation-dominated era, 280 $R \propto t^{2/3}$ in matter-dominated era, 278 Schwarzschild Killing fields, 175-176 relation between r and radius, 180-181 Christoffel symbols, 298 metric, 169, 230 metric in isotropic coordinates, 231 orbit equation, 264 radial orbit, 265 twin paradox, 264-265 spacelike vector, sum of any two is spacelike, 29-30 spacetime diagrams geometric construction, 16 practice drawing, 2 tips for constructing and interpreting, 18-22 speed of light invariant under Lorentz transformations for a velocity boost, 16 speed of sound less than c for causality, 232 spherical coordinates, 296 derive vector divergence formula, 129-130 Euclidean metric, 129 vector divergence, 157 spherical shell, 245 spherically symmetric spactime defined, 170 static fluid defined, 228 hydrostatic balance, 166-167 static spacetime defined, 166 implied by static matter field, 167 static spherically symmetric spacetime Christoffel symbols, 145, 297 Einstein tensor, 242-243, 298 metric, 144 Ricci scalar, 154 Ricci tensor, 154, 297 Riemann tensor, 144-147 static star fluid must have vanishing spatial four-velocity, 228-229 stationary spacetime defined, 178 stellar model boundary conditions at center, 243-244 Buchdahl interior solution, β parameter of, 243 construct with two variable EOS, 229 numerical solution of, 236-238 power series near center, 233-236 stress-energy tensor divergence of, 80, 177-178 for a perfect fluid, 81-82 for dust, 79-80 inequalities for Newtonian system, 193-194

Cambridge University Press 978-1-107-63857-0 — A Student's Manual for A First Course in General Relativity Robert B. Scott Index

More Information

310 In	Index		
strass anargy tansor (aant)	torsion tonsor 117		
linear in arguments 70	transformation		
nhoton gas 183	active 118 170		
physical argument it is a tensor 70	general Lorentz 174		
random non-colliding particles 85–87	passive 118		
spatial components of divergence trivial in RW 285	Poincaré 174		
Sun	transverse_traceless 211_215		
angular momentum 237	applicable to a general superposition of plan		
collapse to size of neutron star, 239	waves 214		
symmetry	does not permit static solutions 214–215		
conserved quantity 170	gauge degrees of freedom 212–213		
Killing vector 170	solve for gauge transformation variables		
maximally symmetric manifold 173	213_214		
maximally symmetric mannola, 175	twin paradox		
tangant vestor 120	in Kerr spacetime 252–253		
tangent vector to spacetime curve 101	not related to acceleration 264–267		
tangent vector to spacetime eurve, 101	two-sphere metric 268		
contraction 72	two sphere metric, 200		
density 117	uniform acceleration, 111-115		
differentiation 74	unit timelike vector, definition, 64		
tensor differentiation obeys Leibniz (product) rule			
63 64	vacuum spacetime, 224		
three-sphere 285_288	vector		
definition 143	associated with a one-form, 116		
metric 144	unchanged under passive transformation, 118		
proper area 148	velocity-addition law		
volume 285	derivation using Lorentz transformations, 29		
time dilation	formula for, 11		
derivation using Lorentz transformations 11–13	small velocity approximation, 10–11		
effect on half-life of pion 10	using velocity parameter $V \equiv \arctan(v)$,		
gravitational 251–252	13-14		
"moving clocks run slowly." 12	weak energy condition 96–97		
orbiting clock in Schwarzschild spacetime 251	weak gravitational field		
self-consistency of using spacetime diagram	Christoffel symbols, 162–163, 298		
7–9. 22	Einstein tensor. 299		
small velocity approximation, 10	inverse metric, 161–162		
time of decoupling 270, 280	line element 161		
time of decoubling, $2/9-280$			