A Student’s Manual for *A First Course in General Relativity*

This comprehensive student manual has been designed to accompany the leading textbook by Bernard Schutz, *A First Course in General Relativity*, and uses detailed solutions, cross-referenced to several introductory and more advanced textbooks, to enable self-learners, undergraduates, and postgraduates to master general relativity through problem solving. The perfect accompaniment to Schutz’s textbook, this manual guides the reader step-by-step through over 200 exercises, with clear easy-to-follow derivations. It provides detailed solutions to almost half of Schutz’s exercises, and includes 125 brand-new supplementary problems that address the subtle points of each chapter. It includes a comprehensive index and collects useful mathematical results, such as transformation matrices and Christoffel symbols for commonly studied spacetimes, in an appendix. Supported by an online table categorizing exercises, a Maple worksheet, and an instructors’ manual, this text provides an invaluable resource for all students and instructors using Schutz’s textbook.

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Preface

General relativity is a beautiful theory, our standard theory of gravity, and an essential component of the working knowledge of the theoretical physicist, cosmologist, and astrophysicist. It has the reputation of being difficult but Bernard Schutz, with his groundbreaking textbook, *A First Course in General Relativity* (first edition published in 1984, current edition in 2009), demonstrated that GR is actually quite accessible to the undergraduate physics student. With this solution manual I hope that GR, using Schutz's textbook as a main resource and perhaps one or two complementary texts (see recommendations at the end of this preface), is accessible to all "technically minded self-learners" e.g. the retired engineer with some time to devote to a dormant interest, a philosopher of physics with a serious interest in deep understanding of the subject, the mathematics undergraduate who wants to become comfortable with the language of the physicist, etc.

You can do it too!

I'm speaking with some experience when I say that an engineer can learn GR and in particular starting with Schutz's textbook. My bachelor's and master's degrees are in engineering and I started learning GR on my own when my academic career had gained enough momentum that I could afford a bit of time to study a new area in my free time. I must admit it wasn't always easy. I personally found the explanations of mathematics in the excellent textbook by Misner, Thorne and Wheeler (1973) more confusing then helpful. (In retrospect I'm at a loss to explain why; in no way do I blame the authors.) Soon two children arrived miraculously in our household, free time became an oxymoron, but with the constant reward I found from beavering away at Schutz's exercises I continued to learn GR, albeit slowly and with screaming (not always my own) interruptions. In his autobiography John A. Wheeler explains that he started learning GR in the 1940s when he finally got the chance to teach the subject. Similarly the real breakthrough for me came when I was offered the possibility to teach the subject to third-year undergraduate students at the Université de Bretagné Occidentale in Brest, France. Suddenly my hobby became my day job, fear of humiliation became my motivation, and most significantly I was forced to view the subject from the student's point of view. I also had to learn French. I can honestly state with no exaggeration that, even with Canadian high-school French instruction and years living in Montreal, it was much harder to learn the local language than to learn GR! *Vraiment!*

Is it better to start with a popular level book?

Popular level books are for people who want a superficial overview. There's nothing wrong with that if that's your ultimate goal. But if you want to really understand GR, skip the popular books for now. By the way, the French word for popular level books is *vulgarisation*. If they're good then they'll be at least as difficult, probably much more difficult, to understand then the real thing. Why? Because the author is obliged to explain mathematical concepts in an artificial language, divorced from the logical and precise language in which the ideas were developed. Popularizers are forced to come up with creative analogies that are insightful if you already understand the idea but are always somewhat misleading. If you made it through your college calculus class and basic linear algebra course then you'll find learning the tools of GR a natural extension of these ideas. Once you understand what the metric on a Riemann manifold is (say after working through Chapter 6 of Schutz), then you can easily and completely understand the Robertson–Walker metric that explains the expansion of the Universe. You won't need bread pudding analogies but if you want to create them yourself you'll be free to do so; just be sure to explain to your listener all the caveats.

Using this book

Suppose you'd like to learn some to play chess, or guitar, or ice hockey. You go to the library, or amazon.co.uk, or a bookstore and find an instruction manual. Reasonable start, but you wouldn't expect that after reading the manual you would be a chess master, awesome guitar player, or hockey star. You would have to practice first, a lot, and learn from your experience. Learning physics takes practice too and the initial practice comes from doing exercises. So don't make the mistake of reading this solution manual like a novel or recipe book thinking you've bypassed the practice sessions. Schutz's *A First Course in General Relativity* has a lot of exercises (338 in fact), many more than most of the other textbooks at that level. Do them!

Some advice to GR students and self-learners in particular:

1 Don't give up. It's normal to not understand something the first time you read it. Make note of what you don't understand, try to articulate your question to yourself as clearly as possible, then press on.
2 Read several textbooks. See additional resources for suggestions.
3 Work the exercises yourself, using this solution manual as a guide to get you over the hurdles and to verify your answers.

You'll probably find that sometimes you can find a simpler solution to the exercise then the solution I have offered. Your solution is probably fine. I have followed the idea that the exercises are designed to teach you to use the new mathematical techniques of differential geometry and tensor calculus (the "big machinery") in a simple setting where you might guess the answer or find it easily with simpler tools. For example you certainly don't need the Minkowski space metric tensor to know that the unit basis vector $\vec{e}_x$ is orthogonal to $\vec{e}_y$. But when you learn that in fact the component of the metric $\eta_{xy} = \vec{e}_x \cdot \vec{e}_y$, it's nice to notice that of course $\eta_{xy} = 0$.

You'll also find that I explain the solution steps in more detail than you really need. I certainly don't mean to insult you! My aim was to be complete, to spell it all out. I endeavor to explain the steps with brief comments to the right of most equation lines that anticipate and answer your question: "what did he do to get this line from the previous line?" If you find it too easy, read it quickly!

To distinguish between references to my equations and those in Schutz's book I use the form Schutz Eq. $(n.m)$ for his equations and eqn.$(n.m)$ for equations in this book. If you see something like

$$\begin{aligned}\bar{t} &= (t - vx)\left(1 - v^2\right)^{-1/2} && \text{used Schutz Eq. (1.12)}\\ &= (t - vx)\left(1 + \frac{1}{2}v^2 + O(v^4)\right), && \text{used eqn.(B.2)} \qquad (0.1)\end{aligned}$$

this means that the first line follows from Eq. (1.12) in Schutz's textbook, while the RHS of the second line used the equation eqn.(B.2), which in this case is found in Appendix B of the book you're holding. Some of you might not have seen $O(x^2)$ before; look in Table A.2 because it's an equation symbol. For abbreviations and acronyms in the text, like "RHS" in the sentence before the previous one, look in Table A.1.

From time to time I make reference to an accompanying Maple™ worksheet. This is available for free download from the Cambridge University Press website. Please also visit the authors website for this book at http://stockage.univ-brest.fr/~scott/Books/Schutz/index_schutz.html.

Additional resources

There are many good introductory resources for learning GR and throughout this manual you'll find references to them. Eric Poisson (Poisson, 2004, preface) recommends you read Schutz's textbook to get started, then Misner, Thorne, and Wheeler's mammoth tome (Misner et al., 1973) for breadth, and finally Robert Wald's monograph (Wald, 1984) for rigor. It would be hard to improve upon that advice. I suggest if you have time and find you can read Misner, Thorne, and Wheeler (1973) straight off you could even skip Schutz and this solution manual. Otherwise I agree with Eric, start here. But to complement Schutz's book I recommend books at a similar level, for example either Hobson et al. (2006) or Rindler (2006). The first is similar to Schutz's book but at times may be a bit more challenging to the reader. Rindler is a bit weaker on tensor analysis, but great for geometrical and physical insight. Sean Carroll (2004) has a flair for clear explanation and has covered a lot of the material in Wald (1984) in a more concrete fashion.

If you find you are struggling with Schutz's book you are probably missing some basic background. The most important background is a working knowledge of basic differential calculus, for which there are countless good begining university level books. If you have this but your math skills need polishing, you could work through the first six chapters of Felder and Felder (2014) concurrently with Schutz and this solution manual. After completing a good number of Schutz's exercises you'll be ready for advanced books (Misner et al., 1973; Hawking and Ellis, 1973; Wald, 1984; Poisson, 2004) and can even read some of the literature, especially *American Journal of Physics*, *European Journal of Physics*, and *Foundations of Physics* articles.

Thanks

I would especially like to thank Gary Felder who read carefully the first six chapters of this textbook and offered valuable suggestions for improvement. Jean-Philippe Nicolas, Jose Luis Jaramillo, Richard Tweed, and Fred Taylor also had helpful input. I dedicate this book to Dr. Donald Taylor, who was my first instructor in relativity, my first physics supervisor, and the first to encourage me in a career in physics.

If you find any errors, or have suggestions for learning GR, you can first check this book's website: http://stockage.univ-brest.fr/~scott/Books/Schutz/index_schutz.html and, if it is not already there, contact the author via email: robert.scott@univ-brest.fr.