CALCULATING INSTRUMENTS AND MACHINES
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PREFACE

In the early fall of 1948 I visited the University of Illinois at the invitation of Dean Louis N. Ridenour, Dean of the Graduate School, to give a short series of lectures on calculating instruments and machines. These lectures, with only minor modifications, form the content of this book.

These lectures were to be devoted primarily to recent developments in the subject. But Dean Ridenour invited me to give some attention to its historical side, and I have been glad to do so if only to pay tribute to the remarkable vision and foresight, as it now seems, of two pioneers of thought in this field, Charles Babbage and Lord Kelvin.

Like the lectures, this book is intended as a general introduction to those who have no specialised knowledge in the subject, not as a detailed account for those already expert in it. To anyone who is, or has been, engaged in development work on any of the equipment mentioned, or on similar projects, this account will probably appear sketchy and inadequate; and probably no one group will consider that its own contribution to the subject is adequately represented. All I can hope is that this survey will be a useful introduction to the subject for those to whom it is primarily addressed.

In a series of lectures such as this, it seems appropriate for the lecturer to draw on his own first-hand experience, and to follow his own bias of interest, to a greater degree than would be suitable in a formal text-book, and this I have done, particularly in Chapter 3 and parts of Chapters 7, 8, and 9. If I am thought to have given too much prominence in Chapter 7 to two particular machines, the Harvard “Mark I Calculator” and the Eniac, I must explain that this prominence is deliberate; not because I happen to be better acquainted with these machines than with some others, but because I regard them as being outstanding steps in the development of automatic general-purpose machines, the one as the first practical realisation of such a machine and the other as the first electronic digital machine.

The subjects of calculating instruments (analogue machines) and calculating machines (digital machines) are here treated almost entirely separately, and the reader who is mainly interested in the latter can omit Chapters 2, 3, and 4 without missing anything important to the later argument. I have regarded desk machines and standard punched-card equipment as outside the scope of these lectures; the only digital machines with which I have been concerned are the automatic general-purpose machines.

In this field particularly, the subject is a live one, in which vigorous development is taking place. Since these lectures were given, a simple form of machine using Professor Williams’ form of electrostatic storage (p. 96) has been put into operation at the University of Manchester (ref. 117).
Also quite recently the EDSAC at the Mathematical Laboratory of Cambridge University (p. 97) has been completed and proved to work satisfactorily, confirming that for this purpose a storage based on the use of mercury delay lines (p. 95) is a practicable project, and reports of satisfactory tests of a machine using the same form of storage have come from the United States. There is little doubt that the next few years will see further substantial developments in this field.

I wish to express my thanks to the Institute of Electrical Engineers for permission to draw on some of the material of my Kelvin Lecture to the Institution (ref. 90) for parts of Chapters 3 and 4; to the Metropolitan-Vickers Electrical Co. Ltd., to the Director of the Mathematical Laboratory, Cambridge University, and to Professor S. H. Caldwell for the photographs of differential analysers; to Professor H. H. Aiken for the photographs of the Harvard Mark I and Mark II Calculators; and to the U. S. War Department for the photographs of the Eniac.

Last, but not least, I am glad to take this opportunity to thank Dean Ridenour for the opportunity of giving these lectures at the University of Illinois, and to express my warm appreciation of the hospitality and friendly kindness I met there. I also wish to thank Dr. Wilbur Schramm and the staff of the University of Illinois Press for their cooperation in the production of these lectures in book form.

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PREFACE TO THE ENGLISH EDITION

I wish to thank the Cambridge University Press for undertaking the publication of this book in England, and the University of Illinois Press for agreeing to this course. I have taken the opportunity of this re-publication to make some corrections in the text and some additions to the list of references.

D. R. H.

May, 1950
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