JOHN NAPIER AND THE INVENTION OF LOGARITHMS, 1614

In the present year there will be held a celebration, under the auspices of the Royal Society of Edinburgh, of the tercentenary of one of the great events in the history of Science, the publication of John Napier's "Mirifici Logarithmorum Canonis Descriptio," a work which embodies one of the very greatest scientific discoveries that the world has seen. The invention of Logarithms not only marks an advance of the first importance in Mathematical Science, but as providing a great labour-saving instrument for the use of all those who have occasion to carry out extensive numerical calculations it can be compared in importance only with the great Indian invention of our system of numeration.

It is almost always extremely instructive to study in detail the form in which a great discovery or invention was presented by its originator, and to trace in detail the mode in which the fundamental ideas connected with the discovery shaped themselves in his mind, even when, and just because, later developments or simplifications may have so transformed the underlying principles, and still more the practice, of the invention, that we have become accustomed to look at the matter from a point of view, at least superficially, very different from the original one of the discoverer. The case of logarithms is very far from being an exception to this rule; accordingly I propose to give an account, as concise as may be, of the conception of a logarithm in the mind of Napier, and of the methods by which he actually constructed his table of logarithms.

In order fully to appreciate the nature of the difficulties of the task accomplished by the genius of John Napier, some effort of imagination is

required, to be expended in realizing the narrowness of the means available in the early part of the seventeenth century for the calculation of the tables, at a time, before the invention of the Differential and Integral Calculus, when calculation by means of infinite series had not yet been invented. Napier's conception of a logarithm involved a perfectly clear apprehension of the nature and consequences of a certain functional relationship, at a time when no general conception of such a relationship had been formulated, or existed in the minds of Mathematicians, and before the intuitional aspect of that relationship had been clarified by means of the great invention of coordinate geometry made later in the century by René Descartes. A modern Mathematician regards the logarithmic function as the inverse of an exponential function; and it may seem to us, familiar as we all are with the use of operations involving indices, that the conception of a logarithm would present itself in that connection as a

fairly obvious one. We must however remember that, at the time of Napier, the notion of an index, in its generality, was no part of the stock of ideas of a Mathematician, and that the exponential notation was not yet in use.

Summary of the life of Napier.

I must content myself with giving an exceedingly brief account of the external facts of the life of Napier*.

John Napier†, the eighth Napier of Merchiston, usually described as Baron, or Fear, of Merchiston, was born at Merchiston near Edinburgh in 1550, when his father Archibald Napier was little more than sixteen years old. John Napier matriculated at St Andrews in 1563, but did not stay there

* For a full account of the life and activities of Napier the "Memoirs of John Napier of Merchiston" by Mark Napier, published in 1834, may be consulted.

† The name Napier was spelled in various ways, several of which were used by John Napier; thus we find Napeir, Nepair, Nepeir, Neper, Nepper, Naper, Napare, Naipper.

sufficiently long to graduate, as he departed previous to 1566 in order to pursue his studies on the Continent, whence he returned to Merchiston in or before 1571. His first marriage, by which he had one son Archibald who was raised to the peerage in 1627 as Lord Napier, and one daughter, took place in 1572. A few years after the death of his wife in 1579, he married again. By his second marriage he had five sons and five daughters; the second son, Robert, was his literary executor. The invasion of the Spanish Armada in 1588 led Napier, as an ardent Protestant, to take a considerable part in Church politics. In January $159\frac{3}{4}$ he published his first work "A plaine discovery of the whole Revelation of St John." This book is regarded as of considerable importance in the history of Scottish theological literature, as it contained a method of interpretation much in advance of the age; it passed through several editions in English, French, German, and Dutch.

In July 1594, Napier entered into a curious contract with a turbulent baron, Robert Logan of Restalrig, who had just been outlawed. In this contract, which appears to shew that John Napier was not free from the prevalent belief in Magic, he agreed to endeavour to discover a treasure supposed to lie hidden in Logan's dwelling-place, Fast Castle. Napier was to receive a third part of the treasure when found, in consideration that "the said Jhone sall do his utter & exact diligens to serche & sik out, and be al craft & ingyne that he dow, to tempt, trye, and find out the sam, and be the grace of God, ather sall find the sam, or than mak it suir that na sik thing hes been thair; sa far as his utter trawell diligens and ingyne may reach."

In a document dated June 7, 1596, Napier gave an account of some secret inventions he had made which were "proffitabill & necessary in theis dayes for the defence of this Iland & withstanding of strangers enemies of God's truth &

relegion." His activities in this direction were no doubt stimulated by the fear of the generally expected invasion by Philip of Spain. It is interesting to note them, in view of the military tastes of many of his descendants. The inventions consisted of a mirror for burning the enemies' ships at any distance, of a piece of artillery capable of destroying everything round an arc of a circle, and of a round metal chariot so constructed that its occupants could move it rapidly and easily, while firing out through small holes in it. Napier's practical bent of mind was also exhibited in the attention he paid to agriculture, especially on the Merchiston estate, where the land was tilled by a system of manuring with salt.

There is evidence that Mathematics occupied Napier's attention from an early age. From a MS. that was first published in 1839 under the title "De Arte Logistica" it appears that his investigations in Arithmetic and Algebra had led him to a consideration of the imaginary roots of

equations, and to a general method for the extraction of roots of numbers of all degrees. But, led probably by the circumstances of the time, he put aside this work in order to devote himself to the discovery of means of diminishing the labour involved in numerical computations. The second half of the sixteenth century was the time in which the Mathematicians of the Continent devoted a great deal of attention to the calculation of tables of natural trigonometrical functions. The most prominent name in this connection is that of Georg Joachim Rheticus, the great computer whose work has never been superseded, and the final result of whose labours is embodied in the table of natural sines for every ten seconds to fifteen places of decimals, published by Pitiscus in 1613, the year before the publication by Napier of the discovery which was destined to revolutionize all the methods of computation, and to substitute the use of logarithmic for that of natural trigonometrical functions. It was in the early

years of the seventeenth century that Johannes Kepler was engaged in the prodigious task of discovering, and verifying by numerical calculation, the laws of the motion of the planets. In this age of numerical calculation then Napier occupied himself with the invention of methods for the diminution of the labour therein involved. He himself states in his "Rabdologia," to which reference will presently be made, that the canon of logarithms is "a me longo tempore elaboratum." It appears from a letter of Kepler that a Scotsman, probably Thomas Craig, a friend of the Napier family, gave the astronomer Tycho Brahe in the year 1594 hopes that an important simplification in the processes of arithmetic would become available. There is strong evidence that Napier communicated his hopes to Craig twenty years before the publication of the Canon.

The "Descriptio," of which an account will be given presently, was as stated at the outset

published in 1614. About the same time Napier devised several mechanical aids for the performance of multiplications and divisions and for the extraction of square and cube roots. He published an account of these inventions in 1617 in his "Rabdologia," as he says, "for the sake of those who may prefer to work with the natural numbers." The method which Napier calls Rabdologia consists of calculation of multiplications and divisions by means of a set of rods, usually called "Napier's bones." In 1617, immediately after the publication of the "Rabdologia," Napier died.

The "Descriptio" did not contain an account of the methods by which the "wonderful canon" was constructed. In an "Admonitio" printed at the end of Chapter II, Napier explains that he prefers, before publishing the method of construction, to await the opinion of the learned world on the canon; he says "For I expect the judgement & censure of learned men hereupon, before