CHAPTER I

INTRODUCTION

1.1. Properties of energy levels

Any system of particles occupying a limited volume of space and in a state of dynamical equilibrium, must, according to the principles of quantum mechanics, have a total energy equal to one of a discrete set of values, the ‘energy levels’ of the system. Each of these energy levels corresponds either to a particular equilibrium (i.e. ‘stationary’) state of the system, or if the energy levels are degenerate, to a set of stationary states. These energies are eigen-values of the Hamiltonian operator representing the system of particles, and the stationary states are represented by the corresponding eigen-functions of the Hamiltonian. The levels are degenerate if a number of eigen-functions correspond to the same eigen-value. If the system is not completely in equilibrium but can be characterized by small, finite probabilities of making transitions to other states, not spatially restricted, then the energy levels of the system will not be exactly defined. Associated with the transition probabilities there will be finite level widths, Γ, such that if the system has a mean lifetime τ in the approximately stable state, then \( \Gamma \sim h/\tau \). Provided the transition probabilities, and therefore also the level widths, are sufficiently small the energy levels may still be regarded as discrete and the properties of the system will be determined, apart from possible degeneracy of the energy levels, by a knowledge of the particular energy level in which the system is formed. The wave functions describing any one of these approximately stationary states will be similar to those applicable to the exactly stationary states, i.e. the wave function will be an eigen-function of some Hamiltonian, \( \mathcal{H}_0 \), which represents the system accurately except for the small probability of transitions. These latter can be included by associating with each eigen-function of \( \mathcal{H}_0 \) additional quantities representing the probabilities of the various energetically possible transitions. \( \mathcal{H}_0 \) also determines the lowest, completely stationary state of the system so that we can
regard the energy-level spectrum of a system as discrete over the range of energies for which the state of the system can be described by a wave function corresponding to a single eigen-value of $H_0$, the Hamiltonian used to determine the ground state.

If the widths of the actual energy levels are comparable with the level separation, so that appreciable overlapping of levels occurs, then it is no longer possible to describe the state of the system in terms of the eigen-functions belonging to a single eigen-value of $H_0$; the description in terms of eigen-functions of $H_0$ involves a linear superposition of eigen-functions belonging to several different eigen-values of $H_0$. This superposition will involve definite phase relations between the component eigen-functions, and these phase differences, and hence also the various transition probabilities and other properties of the state, will be determined by the particular manner in which the system is formed. It is no longer possible to characterize the system by a description limited to the position and properties of the energy levels, and therefore the value of these concepts is considerably reduced.

In the present account of nuclear energy levels we shall deal principally with nuclear states which belong to that region of the energy-level spectrum in which the discrete nature of the level spectrum is still strongly in evidence, i.e. we shall be dealing with approximately stationary states with well-defined properties and which can be described in principle without any reference to a particular mode of formation. There is, of course, no sharp line of demarcation between those parts of the level spectrum in which states are discrete and those in which overlapping occurs. The extremes are easily recognized. For states of low energy for which the only energetically possible transitions are by electromagnetic radiation, the widths of the levels are extremely small and overlapping of levels can be completely ignored. For very high energies, the time during which all the constituent nucleons are together in a region of nuclear dimensions may be so small (i.e. only of the same order as the characteristic nuclear time, $10^{-21}$ sec.), that the general concept of a stationary state of the whole nucleus is irrelevant. For a limited range of intermediate energies, effectively discrete levels occur if the dissociation of one or more nucleons from the nuclear system is a sufficiently slow (i.e. improbable) process. Such levels
will be referred to as \textit{virtual} levels of the nucleus if the dissociation from the nucleus of one or more nucleons is energetically possible, and as \textit{bound} levels if the only transition possible is by electromagnetic radiation.

\section*{1.2. Phenomenological and theoretical viewpoints}

The study of nuclear energy levels can be regarded from two viewpoints which for brevity we call the \textit{phenomenological} and \textit{theoretical}. From the first of these viewpoints the essential problem is the \textit{experimental} determination of the energies and transition probabilities of individual nuclear energy levels or the deduction of these essential quantities from particular experimental data. Once established, such a set of parameters can be used to interpret, correlate and predict a range of nuclear phenomena, particularly in the field of nuclear disintegrations and scattering. This approach to the problem is of great practical importance. Its interest from the theoretical point of view is twofold. In the first place it indicates the validity of theoretical principles well established in other branches of physics, albeit for a limited interpretation of nuclear phenomena. Secondly, it reduces a large amount of experimental material to a minimum number of empirically determined quantities which form a suitable basis for comparison with theory. The ‘theory’ is now concerned not with predictions about nuclear processes \textit{from} a knowledge of nuclear levels but rather with the \textit{prediction} of the level positions and properties themselves, or more generally with the problem of the interaction of nucleons and the structure of nuclei. Whether or not the normal quantum-mechanical methods for dealing with systems of many particles, and in particular the construction of a Hamiltonian which is a function of the co-ordinates of nucleons and some assumed laws of interaction between them, are in fact justifiable for interpretation of nuclear phenomena is best judged by the results of such attempts. In the discussion that follows we shall assume that this procedure is justifiable (or at least that no better procedure is known), and that the shortcomings of the theoretical treatments are attributable to imprecise knowledge of the interactions between nucleons and the difficulties inherent in the application of the usual quantum-mechanical methods to complicated nuclear systems.
INTRODUCTION

The phenomenological approach has been developed extensively and successfully, especially since Bohr first indicated the significance of the many-body nature of nuclear processes. The formal development of the Dispersion Formula for nuclear processes and the application of the principles of statistical mechanics have provided a clear indication of the general validity and scope of the phenomenological method and confirmed the view that the nucleus must be treated as a complex, many-body system. But from the theoretical point of view the emphasis on the many-body aspect, justified by the success of the developments it has engendered, only serves to stress the difficulties that are involved in any attempts to formulate a theory of nuclear structure comparable in scope, for example, with that of atomic or molecular structure.

1.3. Spectrum of levels

The dominant feature governing nuclear structure and the interaction of nucleons with nuclei is the short range of the forces between nucleons. As a result of this property of the forces, the nucleus forms an essentially closed structure, i.e. an impinging nucleon cannot pass through the nucleus without interacting strongly with many of the constituent nucleons; in fact, in general it must be regarded as interacting with the nucleus as a whole. If it penetrates to the surface of the nucleus it will, normally, form a ‘compound’ nucleus which will exist for a time large in comparison with that taken for the nucleon to traverse a distance of the order of nuclear dimensions; indeed, the incident nucleon, or any other, will not escape from the system until a chance redistribution concentrates sufficient energy on a single nucleon (or group) sufficient for it to dissociate from the remainder. Such a system, in an approximately stationary state, will have a well-defined energy, and if the energy is not too large the energy levels will be practically discrete, as is, in fact, found to be the case experimentally. Thus the short-range nature of nuclear forces and the many-body nature of the nucleus can explain the occurrence of discrete virtual levels of nuclei. It also suggests the type of energy-level spectrum to be expected. A fixed potential field of the short-range type, e.g. a deep

\* For a characteristic kinetic energy of the nucleon of 5 MeV. and a medium-size nucleus this time is of the order $10^{-21}$ sec.
INTRODUCTION

rectangular potential well, gives rise to a set of energy levels quite different from those associated with, for example, the long-range Coulomb forces of an atom (fig. 1a, b). Characteristic of the short-range forces is the fact that the spacing of the levels does not approach zero asymptotically as the energy approaches zero (as in the case of the Coulomb potential). Thus for a short-range force there is a finite number of bound levels. Of course the actual energy levels of the nucleus cannot be represented by the levels of a single particle in a fixed potential well; in fact, the decrease in level spacing with increasing energy will be much more rapid for a system of particles than for a single particle, but the essential feature, a finite number of bound levels, remains.

When the excitation energy of the nucleus exceeds that required to dissociate from it a nucleon (or group of nucleons) the energy levels may still be very narrow and the spectrum of levels not essentially different from the spectrum at excitation energies just less than the dissociation energy. For a nucleus, then, in marked contrast with the energy levels of optical spectra, one would not expect any change in character of the level spectrum above and below the dissociation limit.

1.4. Level spectrum and structure

The energy-level spectrum of a system is characteristic not only of the forces between the constituents but of the structure of the
system. Thus for an atomic system with a single predominant centre of force and mass it is possible to consider, with success, that each electron moves in a fixed static field, and consequently can be characterized by its individual energy level. The atom can be excited so that an electron in, say, the $K$ shell, is moved into any one of the vacant outer orbits, i.e. a lightly bound level. In this way the energy-level spectrum of the whole atom can be regarded as a repetition, with varying energy intervals, of the basic pattern of the optical level spectrum, this repetition being characteristic of the shell structure of the atom (fig. 1c). Similar grouping of levels, corresponding to regularities in physical structure are exhibited by molecular spectra. In the case of nuclei, however, no such well-defined structure has been discovered, nor perhaps should it be expected for a system comprising many similar strongly interacting particles with no single strong centre of force. The nuclear level spectrum would be expected to show a progressive reduction in level spacing as the excitation energy is decreased, but no marked grouping into characteristic energy regions has been observed. Incidentally, it should be noticed that the discrete virtual levels of an atom (i.e. levels with sufficient energy to dissociate an electron) are well defined in energy because of the weak interaction between individual electrons, whereas virtual levels of the nucleus owe their discrete nature to the strong interaction between the many constituent nucleons. This difference is illustrated by the difference in physical properties of states of the two types. An atom excited by ejection of a $K$ electron will normally lose energy by electromagnetic radiation; a nucleus with excitation energy greatly exceeding the dissociation limit will normally eject one or more nucleons.

15. Scope of investigations

Apart from the theoretically significant differences between the spectra of nuclei and of atoms and molecules, technical differences in available experimental procedures lead to a different emphasis in studying spectra of the two types. The most outstanding feature of the study of atomic spectra is the great precision of the measurement, particularly in the optical region, and the relative simplicity of the methods of exciting complete atomic spectra. Nuclear spectra cannot be excited so easily, nor can the radiation be measured with
INTRODUCTION

comparable precision or resolution. More emphasis is therefore placed on a study of the properties of individual nuclear levels, particularly in the case of virtual levels where nucleon dissociation as well as electromagnetic radiation can occur. The limiting excitation energy for bound levels varies from zero to about 10 MeV. For light nuclei (Z < 10) the number of bound levels appears sufficiently small for the determination of their individual excitation energies and properties to be possible. For heavier nuclei the absolute positions of the lower levels, perhaps up to 4 or 5 MeV. excitation energy, may be significant and experimentally ascertainable with a reasonable expenditure of effort. For higher energies, including those of the virtual levels, the density, grouping, transition probabilities and other characteristics of the levels, rather than their exact location, are the properties that form the subject of investigations.
CHAPTER II

EXCITATION OF BOUND STATES

2. EXCITATION IN SCATTERING AND IN NUCLEAR TRANSMUTATIONS\(^{(1,2)}\)

Although the transition in the energy-level spectrum of nuclei between bound and virtual excited states is, in most respects, a smooth one, we shall, nevertheless, classify the experimental procedures employed according to the type of nuclear level about which they provide information. In many cases, particularly in the study of nuclear transmutations by bombardment with nucleons, information about both types of level may be obtained more or less simultaneously, but the particular conditions of an experiment must usually be arranged differently if emphasis is placed on information of one type or the other. This classification is particularly convenient where we are concerned with the scope of the various experimental methods.

The energy of even the first excited state of most nuclei is some \(10^4\) eV, or more in excess of the stable* ground state. It is therefore impractical to excite such levels by thermal means, at least with temperatures of a few thousand degrees available in the laboratory.† Excitation of nuclei by irradiation with beams of high-energy electrons or electromagnetic radiation, analogous to the methods used in the study of atomic spectra, have been successfully employed; but, owing to the strong interaction between such radiations and the extranuclear structure and their relatively weak interactions with the nucleus itself, and also on account of the technical problems of controlling the energy of the incident radiation and detecting the process of excitation (or de-excitation), such methods have not played such a predominant role in the study of

* Throughout this book reference to the ground state of a nucleus will be to the state of lowest energy which does not emit electromagnetic radiation. It may either be stable in the usual sense, or \(\beta\)-radioactive.

† Even if in some instances the first excited state happened to be only a few electron volts above the ground state, the 'lifetime' of such an excited nucleus would be so long (small radiation width) that the detection of the feeble radiation would be very difficult.
nuclear spectra as their counterparts in atomic spectra. We shall return to a more detailed discussion of such methods after dealing with the two most widely used procedures for examining bound nuclear states, viz. (a) the excitation by collision with a nucleon, or group of nucleons, in which either a disintegration occurs leaving the residual nucleus in an excited state or the colliding nucleon is scattered inelastically, (b) examination of the $\beta$-spectrum and subsequent $\gamma$-radiation from $\beta$-radioactive nuclei.

For the purpose of studying bound nuclear states, the type of nuclear collision that is most useful can usually be represented by the scheme

$$A(Z, N) + P(p, x) \rightarrow C(Z + p, N + x)$$
$$\rightarrow B(Z + p - q, N + x - y) + Q(q, y),$$  \hspace{1cm} (2.1)

where the symbols in parentheses denote the numbers of protons and neutrons in the respective nuclei. In most cases of interest, where moderate collision energies ($W_P$) are used, $Q$ will be either a simple particle or a stable group ($\alpha$-particle), and the possibility of $Q$ being produced in an excited state will not arise; this possibility will be ignored unless stated specifically to the contrary. The nucleus $B$, on the other hand, will not in general be produced in its ground state. Denoting by $E_P$, $E_Q$ the binding energies of the particles $P, Q$ in the compound nucleus $C$, then $E_{\text{max}}$, the maximum energy available for excitation of $B$, will be equal to $E_P - E_Q + W_P$, corresponding to zero energy of emission of particles $Q$, and it is energetically possible for all levels with excitation energies less than $E_{\text{max}}$ to be excited. The actual number of nuclei $B$ produced in the individual excited states will depend on the relative probability of the compound nucleus $C$ emitting a particle $Q$ with the corresponding kinetic energy, these different modes of disintegration of $C$ being in mutual competition. Since the compound nucleus $C$ is produced in a state which is unstable with respect to emission of nucleons (the emission of both $P$ and $Q$ at least are possible) its lifetime will normally be very short ($\ll 10^{-18}$ sec.), and the width of the states in which $C$ is formed will be correspondingly large ($\gg 1$ eV.). In general, the energy of excitation $E'$ of $C$ will be very

* All kinetic energies $W$ refer to measurements made in a co-ordinate frame for which the centre of gravity of the whole system is at rest.
high \( (E' = E_B + W_Q) \), so that the density of levels will be large, and it is usually the case that the compound nucleus \( C \) must be regarded as being formed in a large number of broad overlapping states.

From statistical-mechanical considerations of the relation between the emission of \( Q \) from \( C \) with energy \( W_Q \) and the inverse process, i.e. formation of \( C \) by collision of \( Q \) with energy \( W_Q \) and the appropriate excited nucleus \( B \), it can be shown that the probability per unit time, \( \Gamma(W_Q) \), of the former process is related to the cross-section for formation in the latter, \( \sigma(W_Q) \), by the relation (5)

\[
\Gamma(W_Q) = \left( m \sigma(W_Q) W_Q \right) / (\pi \hbar^2 \rho_C(E')) \tag{2.2}
\]

where \( \rho_C(E') \) is the density of levels of the compound nucleus \( C \) at excitation energy \( E' \) and \( m \) is the reduced mass of \( Q \) in the system \( B + Q \). The probability \( \Gamma \) is expressed here, as generally, in energy units, i.e. \( \Gamma = \hbar / \tau \), where \( \tau \) is the (partial) lifetime for the particular process.

If the distance between the levels of \( B \), corresponding to the relevant range of energies \( W_Q \), is much smaller than the experimental resolution (determined by the homogeneity of particles \( Q \) and the target thickness), then the relative number of particles \( Q \) produced in a given energy interval is the significant quantity rather than the number corresponding to a particular level of \( B \). Denoting by \( \rho_B(E) \) the density of levels of the product nucleus \( B \) with energy \( E(=W_{\max.} - W_Q) \), then from (2.2) we obtain for the distribution in energy of particles \( Q \) from a compound nucleus \( C \) with energy \( E' \), the expression

\[
N(W_Q) dW_Q = \alpha(E') \sigma(W_Q) W_Q (\rho_B(W_{\max.} - W_Q)) \tag{2.3}
\]

where \( \alpha(E') \) depends on \( E' \) but not on \( W_Q \).

If we define a quantity \( S \) by \( S_B(E) = \kappa \log \rho_B(E) \) and \( T \) by \( (\partial S / \partial E)_T = 1 / T(E) \), then if \( W_Q \ll W_{\max.} \), we can write

\[
S_B(E) = S_B(W_{\max.}) - (\partial S / \partial E)W_{\max.} W_Q = S(W_{\max.}) - W_Q / T_B(W_{\max.}) \tag{2.4}
\]

substituting in (2.3) we get

\[
N(W_Q) dW_Q \sim \alpha(E') \sigma(W_Q) \exp \left[ -W_Q / \kappa T_B(W_{\max.}) \right] dW_Q. \tag{2.5}
\]