

Introduction

This book is aimed primarily at providing a reference point for the common graphs, definitions and equations that are part of the FRCA syllabus. In certain situations, for example the viva sections of the examinations, a clear structure to your answer will help you to appear more confident and ordered in your response. To enable you to do this, you should have a list of rules to hand which you can apply to any situation.

Graphs

Any graph should be constructed in a logical fashion. Often it is the best-known curves that candidates draw most poorly in their rush to put the relationship down on paper. The oxyhaemoglobin dissociation curve is a good example. In the rush to prove what they know about the subject as a whole, candidates often supply a poorly thought out sigmoid-type curve that passes through none of the traditional reference points when considered in more detail. Such an approach will not impress the examiner, despite a sound knowledge of the topic as a whole. Remembering the following order may help you to get off to a better start.

Size

It is important to draw a large diagram to avoid getting it cluttered. There will always be plenty of paper supplied so don't be afraid to use it all. It will make the examiner's job that much easier as well as yours.

Axes

Draw straight, perpendicular axes and label them with the name of the variable and its units before doing anything else. If common values are known for the particular variable then mark on a sensible range, for example 0–300 mmHg for blood pressure. Remember that logarithmic scales do not extend to zero as zero is an impossible result of a logarithmic function. In addition, if there are important reference points they should be marked both on the axis and where two variables intersect on the plot area, for example 75% saturation corresponding to 5.3 kPa for the venous point on the oxyhaemoglobin dissociation curve. Do all of this before considering a curve and do not be afraid to talk out loud as you do so – it avoids uncomfortable silences, focuses your thoughts and shows logic.

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Beginning of a curve

Consider where a curve actually starts on the graph you are drawing. Does it begin at the origin or does it cross the y axis at some other point? If so, is there a specific value at which it crosses the y axis and why is that the case? Some curves do not come into contact with either axis, for example exponentials and some physiological autoregulation curves. If this is the case, then you should demonstrate this fact and be ready to explain why it is so. Consider what happens to the slope of a curve at its extremes. It is not uncommon for a curve to flatten out at high or low values, and you should indicate this if it is the case.

Middle section

The middle section of a curve may cross some important points as previously marked on the graph. Make sure that the curve does, in fact, cross these points rather than just come close to them or you lose the purpose of marking them on in the first place. Always try to think what the relationship between the two variables is. Is it a straight line, an exponential or otherwise and is your curve representing this accurately?

End of a curve

If the end of a curve crosses one of the axes then draw this on as accurately as possible. If it does not reach an axis then say so and consider what the curve will look like at this extreme.

Other points

Avoid the temptation to overly annotate your graphs but do mark on any important points or regions, for example segments representing zero and first-order kinetics on the Michaelis–Menten graph.

Definitions

When giving a definition, the aim is to *accurately* describe the principle in question in as few words as possible. The neatness with which your definition appears will affect how well considered your answer as a whole comes across. Definitions may or may not include units.

Definitions containing units

Always think about what units, if any, are associated with the item you are trying to describe. For example, you know that the units for clearance are $\text{ml} \cdot \text{min}^{-1}$ and so your definition must include a statement about both volume (ml) and time (min).

When you are clear about what you are describing, it should be presented as succinctly as possible in a format such as

'x' is the **volume** of plasma ...

'y' is the **pressure** found when ...

'z' is the **time** taken for ...

Clearance ($\text{ml} \cdot \text{min}^{-1}$) is the volume (**ml**) of plasma from which a drug is completely removed per unit time (**min**)

Pressure ($\text{N} \cdot \text{m}^{-2}$) describes the result of a force (**N**) being applied over a given area (**m²**).

You can always finish your definition by offering the units to the examiner if you are sure of them.

Definitions without units

If there are no units involved, think about what process you are being asked to define. It may be a ratio, an effect, a phenomenon, etc.

Reynold's number is a **dimensionless number** ...

The blood:gas partition coefficient is the **ratio** of ...

The second gas effect is the **phenomenon** by which ...

Conditions

Think about any conditions that must apply. Are the measurements taken at standard temperature and pressure (STP) or at the prevailing temperature and pressure?

The triple point of water is the temperature at which all three phases are in equilibrium at **611.73 Pa. It occurs at 0.01 °C.**

There is no need to mention a condition if it does not affect the calculation. For example, there is no need to mention ambient pressure when defining saturated vapour pressure (SVP) as only temperature will alter the SVP of a volatile.

Those definitions with clearly associated units will need to be given in a clear and specific way; those without units can often be 'padded' a little if you are not entirely sure.

Equations

Most equations need only be learned well enough to understand the components which make up the formula such as in

$$V = IR$$

where V is voltage, I is current and R is resistance.

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There are, however, some equations that deserve a greater understanding of their derivation. These include,

- The Bohr equation
- The Shunt equation
- The Henderson–Hasselbach equation

These equations are fully derived in this book with step by step explanations of the mathematics involved. It is unlikely that the result of your examination will hinge on whether or not you can successfully derive these equations from first principles, but a knowledge of how to do it will make things clearer in your own mind.

If you are asked to derive an equation, remember four things.

1. Don't panic!
2. Write the end equation down **first** so that the examiners know you know it.
3. State the first principles, for example the Bohr equation considers a single tidal exhalation comprising both dead space and alveolar gas.
4. Attempt to derive the equation.

If you find yourself going blank or taking a wrong turn midway through then do not be afraid to tell the examiners that you cannot remember and would they mind moving on. No one will mark you down for this as you have already supplied them with the equation and the viva will move on in a different direction.

Section 1

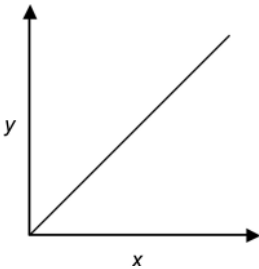
Mathematical principles

Mathematical relationships

Mathematical relationships tend not to be tested as stand-alone topics but an understanding of them will enable you to answer other topics with more authority.

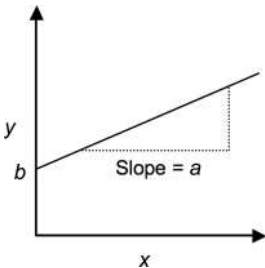
Linear relationships

$y = x$



Draw and label the axes as shown. Plot the line so that it passes through the origin (the point at which both x and y are zero) and the value of y is equal to the value of x at every point. The slope when drawn correctly should be at 45° if the scales on both axes are the same.

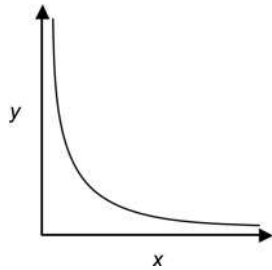
$y = ax + b$



This line should cross the y axis at a value of b because when x is 0, y must be $0 + b$. The slope of the graph is given by the multiplier a . For example, when the equation states that $y = 2x$, then y will be 4 when x is 2, and 8 when x is 4, etc. The slope of the line will, therefore, be twice as steep as that of the line given by $y = 1x$.

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Hyperbolic relationships ($y = k/x$)

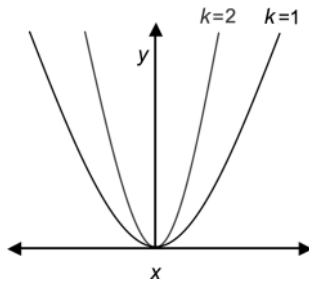


This curve describes any inverse relationship. The commonest value for the constant, k , in anaesthetics is 1, which gives rise to a curve known as a rectangular hyperbola. The line never crosses the x or the y axis and is described as asymptotic to them (see definition below). Boyle's law is a good example (volume = $1/\text{pressure}$). This curve looks very similar to an exponential decline but they are entirely different in mathematical terms so be sure about which one you are describing.

Asymptote

A curve that continually approaches a given line but does not meet it at any distance.

Parabolic relationships ($y = kx^2$)



These curves describe the relationship $y = x^2$ and so there can be no negative value for y . The value for a constant ' k ' alters the slope of the curve in the same way as ' a ' does in the equation $y = ax + b$. The curve crosses the y axis at zero unless the equation is written $y = kx^2 + b$, in which case the whole curve is shifted upwards and it crosses at the value of ' b '.

Exponential relationships and logarithms

Exponential

A condition where the rate of change of a variable at any point in time is proportional to the value of the variable at that time.

or

A function whereby the x variable becomes the exponent of the equation $y = e^x$.

We are normally used to x being represented in equations as the *base* unit (i.e. $y = x^2$). In the exponential function, it becomes the exponent ($y = e^x$), which conveys some very particular properties.

Euler's number

Represents the numerical value 2.71828 and is the base of natural logarithms. Represented by the symbol 'e'.

Logarithms

The power (x) to which a base must be raised in order to produce the number given as for the equation $x = \log_{\text{base}}(\text{number})$.

The base can be any number, common numbers are 10, 2 and e (2.71828). $\text{Log}_{10}(100)$ is, therefore, the power to which 10 must be raised to produce the number 100; for $10^2 = 100$, therefore, the answer is $x = 2$. Log_{10} is usually written as \log whereas \log_e is usually written \ln .

Rules of logarithms

Multiplication becomes addition

$$\log(xy) = \log(x) + \log(y)$$

Division becomes subtraction

$$\log(x/y) = \log(x) - \log(y)$$

Reciprocal becomes negative

$$\log(1/x) = -\log(x)$$

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Power becomes multiplication

$$\log(x^n) = n \cdot \log(x)$$

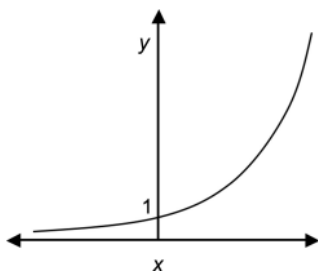
Any log of its own base is one

$$\log_{10}(10) = 1 \text{ and } \ln(e) = 1$$

Any log of 1 is zero because n^0 always equals 1

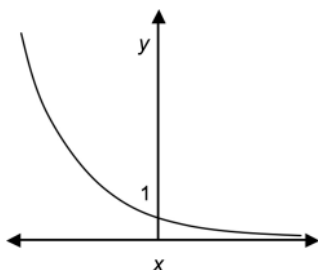
$$\log_{10}(1) = 0 \text{ and } \ln(1) = 0$$

Basic positive exponential ($y = e^x$)



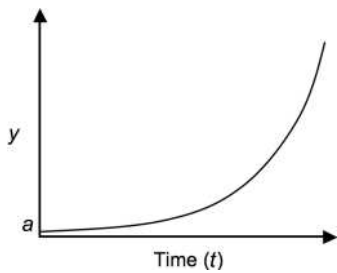
The curve is asymptotic to the x axis. At negative values of x , the slope is shallow but the gradient increases sharply when x is positive. The curve intercepts the y axis at 1 because any number to the power 0 (as in e^0) equals 1. Most importantly, the value of y at any point equals the slope of the graph at that point.

Basic negative exponential ($y = e^{-x}$)



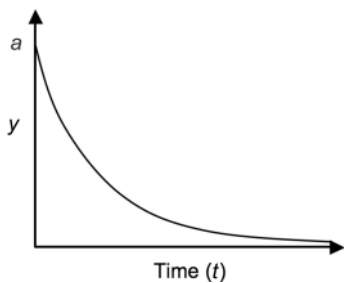
The x axis is again an asymptote and the line crosses the y axis at 1. This time the curve climbs to infinity as x becomes more negative. This is because $-x$ is now becoming more positive. The curve is simply a mirror image, around the y axis, of the positive exponential curve seen above.

Clinical tear-away positive exponential ($y = a.e^{kt}$)



The curve crosses y axis at value of a . It tends towards infinity as value of t increases. This is clearly not a sustainable physiological process but could be seen in the early stages of bacterial replication where y equals number of bacteria.

Physiological negative exponential ($y = a.e^{-kt}$)



The curve crosses the y axis at a value of a . It declines exponentially as t increases. The line is asymptotic to the x axis. This curve is seen in physiological processes such as drug elimination and lung volume during passive expiration.

Physiological build-up negative exponential ($y = a - b.e^{-kt}$)

