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Unit 1

Number, set notation and language

CORE CURRICULUM

Learning outcomes

By the end of this unit you should be able to understand and use:

- natural numbers, integers, prime numbers, common factors and multiples
- rational and irrational numbers, real numbers
- number sequences
- generalisation of number patterns using simple algebraic statements, e.g. *n*th term

1.01 Numbers

Natural numbers

These are the counting numbers: 1, 2, 3, 4, …

Integers

These are positive or negative whole numbers, e.g. −5, 3, 25. If the number contains a fraction part or a decimal point, then it cannot be an integer. For example, the numbers 4.2 and $\frac{1}{2}$ are not integers.

Prime numbers

Numbers that can only be divided by themselves, e.g. 2, 3, 5, 7, 11, 13, are prime numbers. Note that 1 is not considered prime and 2 is the only even prime number.

Progress check

1.01 Explain the difference between natural numbers and integers.

1.02 Give two examples of prime numbers between 20 and 30.

1.02 Factors

A number is a factor of another number if it divides exactly into that number without leaving a remainder. For example, the factors of 6 are 1, 2, 3, 6; the factors of 15 are 1, 3, 5, 15.

To find the factors of a number quickly, find which numbers were multiplied together to give that number. For example, the products which give 8 are 1×8 or 2×4, so the factors of 8 are 1, 2, 4, 8.

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Prime factors

A prime factor is a prime number that is also a factor of another number. For example, the prime factors of 24 are 2 and 3, since $2 \times 2 \times 2 \times 2 \times 3 = 24$.

Highest Common Factor (HCF)

This is the highest factor which is common to a group of numbers.

1.01 Worked example

Find the HCF of the numbers 6, 8 and 12.

Factors of 6	=	1, **2**, 3, 6
Factors of 8	=	1, **2**, 4, 8
Factors of 12	=	1, **2**, 3, 4, 6, 12

As the number 2 is a the highest factor of the three numbers, HCF = 2.

1.03 Multiples

These are the 'times table' of a number. For example, multiples of 4 are 4, 8, 12, 16, …; multiples of 9 are 9, 18, 27, 36, …

Lowest Common Multiple (LCM)

This is the lowest multiple which is common to a group of numbers. It is found by listing all the multiples of a group of numbers and finding the lowest number which is common to each set of multiples.

TIP

Recurring decimals can be displayed in a variety of ways. One way of representing a repeating decimal is by placing a dot over the recurring figure:

$5.3\dot{4}$

Another common way of indicating that a decimal is recurring is by placing a horizontal line over the figure which repeats:

$5.3\overline{4}$

1.02 Worked example

Find the Lowest Common Multiple of the numbers 2, 3 and 9.

Multiples of 2 are	2, 4, 6, 8, 10, 12, 14, 16, **18**, 20, …
Multiples of 3 are	3, 6, 9, 12, 15, **18**, 21, 24, …
Multiples of 9 are	9, **18**, 27, 36, …

The number 18 is the lowest number that occurs as a multiple of each of the numbers, so the LCM is 18.

Progress check

1.03 Find the HCF and LCM of the numbers 24 and 36.

1.04 Rational and irrational numbers

Rational numbers

Rational numbers are numbers that can be shown as fractions, they either terminate or have repeating digits, for example $\frac{3}{4}$, 4.333, 5.343434…, etc.

Note that recurring decimals are rational.

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Irrational numbers

An irrational number cannot be expressed as a fraction, e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π. Since these numbers never terminate, we cannot possibly show them as fractions. The square root of any odd number apart from the square numbers is irrational. (Try them on your calculator, you will find that they do not terminate.) Also, any decimal number which neither repeats nor terminates is irrational.

For more information on square numbers look up special number sequences at the end of this unit.

> **Progress check**
>
> 1.04 In general, square roots of which type of numbers are always rational?

1.05 Number sequences

A number sequence is a set of numbers that follow a certain pattern, for example:

1, 3, 5, 7, … Here the pattern is either 'consecutive odd numbers' or 'add 2'.
1, 3, 9, 27, … The pattern is '3 × previous number'.

The pattern could be add, subtract, multiply or divide. To make it easier to find the pattern, remember that for a number to get bigger, you generally have to use the add or multiply operation. If the number gets smaller, then it will usually be the subtract or divide operation.

Sometimes the pattern uses more than one operation, for example:

1, 3, 7, 15, 31, … Here the pattern is 'multiply the previous number by 2 and then add 1'.

The *n*th term

For certain number sequences it is necessary, and indeed very useful, to find a general formula for the number sequence.

Consider the number sequence 4, 7, 10, 13, 16, 19, …

We can see that the sequence is 'add 3 to the previous number', but what if we wanted the 50th number in the sequence?

This would mean us continuing the sequence up to the 50th value, which would be very time consuming.

A quicker method is to find a general formula for any value of n and then substitute 50 to find its corresponding value. The following examples show the steps involved.

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1.03 Worked example

Find the *n*th term and hence the 50th term of the number sequence 4, 7, 10, 13, 16, 19, …

We can see that you add 3 to the previous number. To find a formula for the *n*th term, follow the steps below:

Step 1 Construct a table and include a difference row.

n	1	2	3	4	5	6
Sequence	4	7	10	13	16	19
1st difference	3	3	3	3	3	

Step 2 Look at the table to see where the differences remain constant.

We can see that the differences are always 3 in the (n) row; this means that the formula involves $3n$. If we then add 1 we get the sequence number, as shown below.

When $n = 1$:
$3 \times (1) + 1 = 4$

When $n = 2$:
$3 \times (2) + 1 = 7$

Step 3 Form a general *n*th term formula and check:

Knowing that we have to multiply n by 3 and then add 1:

$$n\text{th term} = 3n + 1$$

This formula is extremely powerful as we can now find the corresponding term in the sequence for any value of n. To find the 50th term in the sequence:

Using nth term $= 3n + 1$ when $n = 50$:

$$3(50) + 1 = 151$$

Therefore the 50th term in the sequence will be 151.
This is a much quicker method than extending the sequence up to $n = 50$.

Sometimes, however, we have sequences where the first difference row is not constant, so we have to continue the difference rows, as shown in the following example.

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TIP

Some more complicated sequences will require a third difference row (n^3) for the differences to be constant, so we have to manipulate n^3 to get the final formula.

Some special sequences

TIP

The counting numbers squared.

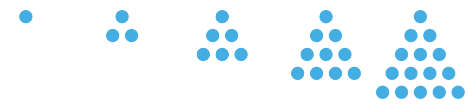
Square numbers	1	4	9	16	25	...
	(1^2)	(2^2)	(3^2)	(4^2)	(5^2)	...

The counting numbers cubed.

Cubed numbers	1	8	27	64	125	...
	(1^3)	(2^3)	(3^3)	(4^3)	(5^3)	...

Each number can be shown as a triangle, or simply add an extra number each time.

Triangular numbers	1	3	6	10	15	...



Progress check

1.05 Find the nth term and 20th term of 5, 8, 11, 14, 17,...

TERMS

- **Natural numbers**
  The positive counting numbers, 1, 2, 3, etc.
- **Integers**
  Whole numbers, including zero.
- **Prime numbers**
  Only divide by themselves and 1.
- **Factors**
  Numbers that divide into other numbers exactly.
- **Multiples**
  Times table of numbers.
- **LCM**
  Lowest Common Multiple of two or more numbers.
- **HCF**
  Highest Common Factor of two or more numbers.
- **Rational numbers**
  Numbers that terminate or have same recurring digit(s).
- **Irrational numbers**
  Numbers that exhibit continuous random digits.
- **Sequence**
  A set of numbers that exhibit a definite pattern.
- ***n*th term**
  A general formula for a number sequence.

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Exam-style questions

1.01 Write down the next two prime numbers after 53. (2)

1.02 For the set of numbers 2, -2.3, $\sqrt{5}$, π, 5, 0.3333…, write down which are:
- a integers
- b rational numbers
- c prime numbers (3)

1.03 What is the sum of the 2nd square number and the 3rd triangular number? (2)

EXTENDED CURRICULUM

Learning outcomes

By the end of this unit you should be able to understand and use:

- set notation such as $n(A)$, $\in$, $\notin$, ξ, A', $\cap$, $\cup$, $\subseteq$
- Venn diagrams and appropriate shading of well-defined regions
- calculations involving missing regions in sets
- exponential sequences and combinations.

1.06 Sets

Definition of a set

A set is a collection of objects, numbers, ideas, etc. The different objects, numbers, ideas and so on in the set are called the **elements** or **members** of the set.

1.04 Worked example

Set A contains the even numbers from 1 to 10 inclusive. Write this as a set.

The elements of this set will be 2, 4, 6, 8, 10, so we write:
$A = \{2, 4, 6, 8, 10\}$

1.05 Worked example

Set B contains the prime numbers between 10 and 20 inclusive. Write this as a set.

The elements of this set will be 11, 13, 17 and 19, so:
$B = \{11, 13, 17, 19\}$

Progress check

1.06 Set C contains the triangular numbers between 1 and 20 inclusive. Write as a set.

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The number of elements in a set

The number of elements in set A is denoted n(A), and is found by counting the number of elements in the set.

1.06 Worked example

Set C contains the odd numbers from 1 to 10 inclusive. Find n(C).

$C = \{1, 3, 5, 7, 9\}$. There are 5 elements in the set, so:
$n(C) = 5$

Inclusion in a set

The symbols $\in$ and $\notin$ indicate whether or not an item is an element of a set.

1.07 Worked example

Set $A = \{2, 5, 6, 9\}$. Describe which of the numbers 2, 3 or 4 are elements and which are not elements of set A.

Set A contains the element 2, therefore $2 \in A$.
Set A does not contain the elements 3 or 4, therefore $3, 4 \notin A$.

The universal set and the complement of a set

The universal set, ξ, for any problem is the set which contains all the available elements for that problem.

The complement of a set A, A', is the set of elements of ξ which do not belong to A.

1.08 Worked example

The universal set is all of the odd numbers up to and including 11.

$\xi = \{1, 3, 5, 7, 9, 11\}$

1.09 Worked example

If $A = \{3, 5\}$ and the universal set is the same as the previous example, write down the complement of A.

$A' = \{1, 7, 9, 11\}$

The empty set

This is a set that contains no elements, and is denoted $\varnothing$ or $\{\ \}$. For example, for some readers, the set of people who wear glasses in their family will have no members. The empty set is sometimes referred to as the null set.

Subsets

If all the elements of a set A are also elements of a set B then A is said to be a **subset** of B, $A \subseteq B$.

Every set has at least two subsets, itself and the null set.

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TIP

The number of subsets can be found by using the formula 2^n, where n = number of elements in the set.

1.10 Worked example

List all the subsets of {a, b, c}.

The subsets are ∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c} and {a, b, c}, because all of these elements can occur in their own right inside the main set.

Subsets A ⊆ B and proper subsets

If all the elements of a set A are also elements of a set B then A is said to be a **subset** of B.

Every set has at least two subsets, itself and the null set.

Proper subsets will contain all elements except the whole set and the null set.

1.11 Worked example

Set A = {2, 3, 5}, find:

a the subsets **b** the proper subsets

a { }{2}, {3}, {5}, {2, 3}, {2, 5}, {3, 5}, {2, 3, 5}, note { } is the empty set and can also be written as
b {2}, {3}, {5}, {2, 3}, {2, 5}, {3, 5}

Intersection and union

The **intersection** of two sets A and B is the set of elements which are common to both A and B, and is denoted by $A \cap B$.

The **union** the sets A and B is the set of all the elements contained in A and B, and is denoted by $A \cup B$.

1.12 Worked example

If $A = \{2, 3, 5, 8, 9\}$ and $B = \{1, 3, 4, 8\}$, find:

a $A \cap B$ **b** $A \cup B$

a $A \cap B = \{3, 8\}$, because these elements are common to both sets.

b $A \cup B = \{1, 2, 3, 4, 5, 8, 9\}$, because these are all the elements contained in both A and B.

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Progress check

1.07 Describe the difference between the union and intersection of two or more sets.

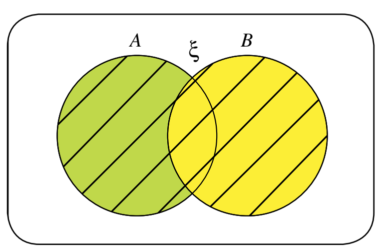
1.08 What does the complement of a set mean?

1.09 If set A is $\{1, 2, 3, 5\}$ and set B is $\{3, 4, 6, 9\}$, write down:
  a $n(A)$ b $A \cap B$ c $A \cup B$

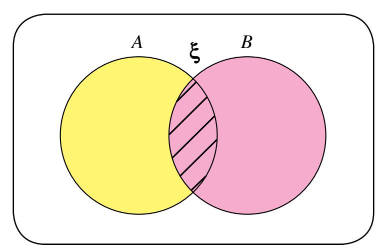
Venn diagrams

Set problems may be solved by using Venn diagrams. The universal set is represented by a rectangle and subsets of this set are represented by circles or loops. Some of the definitions explained earlier can be shown using these diagrams.

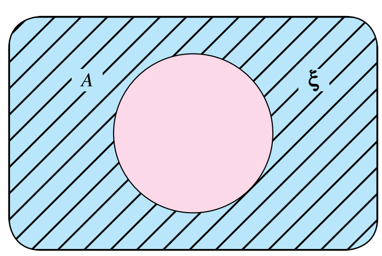
$A \cup B$



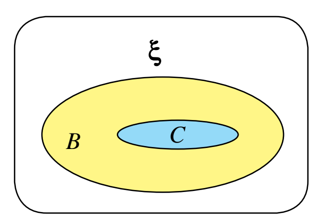
$A \cap B$



A'

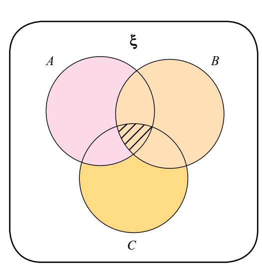


$C \subset B$

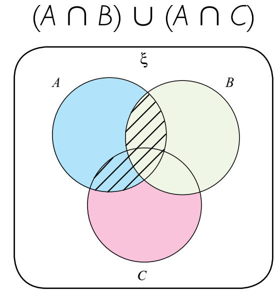


Some more complex Venn diagrams:

$A \cap B \cap C$



$(A \cap B) \cup (A \cap C)$



Obviously there are many different arrangements possible with these diagrams but now let us try some more difficult problems.

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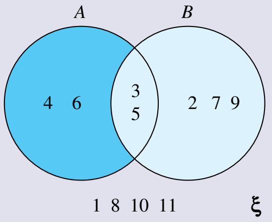
1.13 Worked example

If $A = \{3, 4, 5, 6\}$, $B = \{2, 3, 5, 7, 9\}$ and $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, draw a Venn diagram to represent this information. Hence write down the elements of:

a A' **b** $A \cap B$ **c** $A \cup B$

We only have 2 sets (A, B), so there are two circles inside the universal set.



From the Venn diagram it can be seen that:

a $A' = \{1, 2, 7, 8, 9, 10, 11\}$
b $A \cap B = \{3, 5\}$
c $A \cup B = \{2, 3, 4, 5, 6, 7, 9\}$

Problems with the number of elements of a set

For two intersecting sets A and B we can use the rule:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

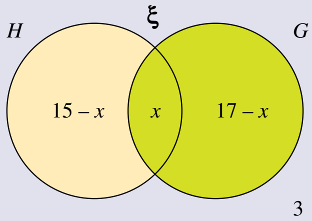
This formula can be used for any problem involving two sets; here is an example.

1.14 Worked example

In a class of 25 members, 15 take history, 17 take geography and 3 take neither subject. How many class members take both subjects?

Let H = set of history students, so $n(H) = 15$.
Let G = set of geography students, so $n(G) = 17$.
Also, $n(H \cup G) = 22$ (since 3 students take neither subject).
Let x represent the number taking both subjects.

Now we can draw the Venn diagram.



Using the formula:

$$n(H \cup G) = n(H) + n(G) - n(H \cap G)$$
$$22 = 15 + 17 - x$$

So $x = 10$.

Hence the number taking both subjects is 10.
The history-only region is labelled $15 - x$.
This is because a number of these students also take geography.

Progress check

1.10 In a class of 35 students, 19 take French, 18 take German and 3 take neither. Calculate how many take:
- a both French and German
- b just French
- c just German.

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1.07 Exponential sequences

These take the form a^n, where a is a positive integer. For example, 2^n, 2^{n+1}, 3^n.

Here is a table showing some typical exponential sequences:

n	1	2	3	4	5	6
2^n	$2^1 = 2$	$2^2 = 4$	8	16	32	64
3^n	$3^1 = 3$	$3^2 = 9$	27	81	243	729

Notice that the differences between successive terms are steadily increasing, but they represent powers of 2, 3, etc.

1.15 Worked example

Write down the first five terms of the sequence $2^{n+1} + n^2$.

$$n = 1, 2^{(1)+1} + (1)^2 = 5$$
$$n = 2, 2^{(2)+1} + (2)^2 = 13$$
$$n = 3, 2^{(3)+1} + (3)^2 = 25$$
$$n = 4, 2^{(4)+1} + (4)^2 = 48$$
$$n = 5, 2^{(5)+1} + (5)^2 = 89$$

So the first five terms are 5, 13, 25, 48, 89.

1.16 Worked example

Find the nth term and hence the 50th term for the sequence 0, 3, 8, 15, 24, 35, …

Construct a table:

n	1	2	3	4	5	6
Sequence	0	3	8	15	24	35

1st difference 3 5 7 9 11

2nd difference 2 2 2 2

Now we notice that the differences are equal in the *second* row, so the formula involves n^2. If we square the first few terms of n we get 1, 4, 9, 16, etc. We can see that we have to subtract 1 from these numbers to get the terms in the sequence. So:

$$n\text{th term} = n^2 - 1$$

Now we have the nth term, to find the 50th term we use simple substitution:

$$50\text{th term} = (50)^2 - 1 = 2499$$

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TERMS

- $n(A)$
  The number of elements in set *A*.
- $\in, \notin$
  An element is or is not an element of a set.
- ξ
  The universal set, which contains all possible elements for the problem.
- A'
  The complement of a set: the elements in the universal set but not in set *A*.
- $\cap$
  The intersection of two sets: the common elements of both sets.
- $\cup$
  The union of two sets: those elements in either set *A* or set *B*.
- $\subseteq$
  The subset of a set: another set made from some or all of the elements in the main set.

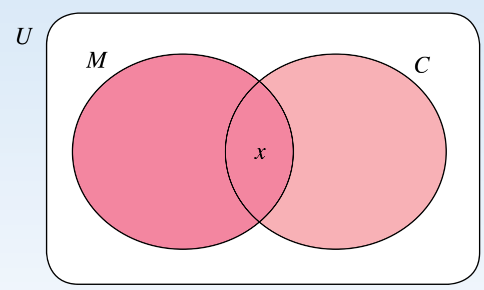
Exam-style questions

1.04 All the students in a class of 20 took tests in mathematics and chemistry. The following table shows the results of these two tests.

	Pass	Fail
Mathematics	12	8
Chemistry	11	9

M is the set of students who passed the mathematics test.
C is the set of students who passed the chemistry test.
x is the number of students who passed both tests.

a Write 3 expressions **in terms of x** to complete the Venn diagram.



b Two pupils failed both mathematics and chemistry. Find the value of x, the number of students who passed both tests.

Source: *Cambridge IGCSE Mathematics 0580 Paper 21 Q10a, b June 2011.*