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Excerpt
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Introduction: classical foundations

1.1 Preliminaries

1.1.1 Units

If I ask you, "How long is an Olympic swimming pool?", and you answer "164," you're correct, in a way, but your response is worthless to me, because I don't know whether you are talking about *feet*, or *meters*, or *yards*, or *light-years*. Most physical quantities carry **dimensions** (length, time, mass, etc.), and you must indicate the **units** you are using. In this book we will for the most part use the metric system: meters (m), seconds (s), kilograms (kg), and so on. This is arbitrary, of course, and you can use inches, hours, and pounds, if you prefer – as long as you are careful to include the units whenever you specify a physical quantity. (You *should* have said "164 feet.") In fact, it's a good practice to carry the units along at each step in a problem: if you're calculating a distance, and it comes out in seconds, you *know* you've made a mistake, and there is no point in continuing; go back and find the error before moving on.

Because not everybody uses the same units, it is important to be able to **convert** from one system to another. This is easy, if you systematically replace each unit with its equivalent in the new system. For example, 1 inch (in) is 2.54 centimeters (cm), 1 m = 100 cm, and there are 12 inches in a foot (ft), so the length of that pool is

$$\begin{aligned} 164\,\text{ft} &= (164) \times 12\,\text{in} = (164)(12) \times 2.54\,\text{cm} \\ &= (164)(12)(2.54) \times \frac{1}{100}\,\text{m} = 50\,\text{m}. \end{aligned}$$

Example 1. How many square feet are there in a square yard?

Solution: A yard (yd) is 3 feet, so

$$1\,\text{yd}^2 = (1) \times (3\,\text{ft})^2 = 9\,\text{ft}^2.$$

So there are 9 square feet in a square yard. (If this surprises you, draw a 3×3 checkerboard and count the squares.)

Problem 1. What is 60 mph (miles per hour) in meters per second? (A mile is 5280 ft.)

Problem 2. A milliliter (0.001 l) is a cubic centimeter. How many liters are there in a cubic meter?

1.1.2 Scientific notation

We're going to encounter some huge numbers, and some very tiny numbers (the age of the Universe is 432 000 000 000 000 000 s; the radius of a hydrogen atom is 0.000 000 000 0529 m). It's hard to make sense of numbers like that – you have to squint even to *count* all those zeros. Much better is the "power-of-ten" notation. Notice that

$$\begin{aligned} 10^1 &= 10, \\ 10^2 &= 10 \times 10 = 100, \\ 10^3 &= 10 \times 10 \times 10 = 1000; \end{aligned}$$

evidently the power of ten counts the number of zeros to the right of the number 1. As you increase the power of ten by one, you are *multiplying* by 10. Going in the other direction (reducing the power of ten) you are *dividing* by 10:

$$\begin{aligned} 10^0 &= 10/10 = 1, \\ 10^{-1} &= 1/10 = 0.1, \\ 10^{-2} &= 1/10^2 = 1/100 = 0.01, \end{aligned}$$

and so on; a *negative* power tells you how many places to the right of the decimal point the 1 lies. In this language I can write the age of the Universe as 4.32×10^{17} s (the decimal point is 17 places to the right of the 4) and the radius of hydrogen as 5.29×10^{-11} m (the decimal point is 11 places to the *left* of the 5).

To multiply numbers, in this notation, you multiply the numbers out front and add the exponents:

$$\begin{aligned} (2 \times 10^3) \times (3 \times 10^4) &= (2 \times 3) \times 10^{(3+4)} = 6 \times 10^7; \\ (3.14 \times 10^{-2}) \times (6.47 \times 10^{-3}) &= 20.3 \times 10^{-5} = 2.03 \times 10^{-4}. \end{aligned}$$

(It is customary to leave just one digit to the left of the decimal point, and let the power of 10 soak up the rest.) To divide, you divide the numbers out front, and subtract the exponents. If you wonder where these rules come from – or you forget how to do it – just make up a simple example for yourself, and you'll quickly figure it out:

$$400 \div 20 = \frac{400}{20} = \frac{4 \times 10^2}{2 \times 10^1} = \frac{4}{2} \times 10^{(2-1)} = 2 \times 10 = 20. \checkmark$$

By the way, *adding* (or subtracting) two numbers is awkward, in this notation. You first need to express both of them as the *same* power of 10:

$$\begin{aligned} 419 + 23 &= (4.19 \times 10^2) + (0.23 \times 10^2) = (4.19 + 0.23) \times 10^2 \\ &= 4.42 \times 10^2 = 442. \checkmark \end{aligned}$$

This all takes some getting used to, so if it is new to you, make sure you can solve the following problems. Once you are confident you understand how it works, get a calculator, and let it take care of the details!

Problem 3. (a) How many seconds are there in a year? (b) What is the age of the Universe, in years?

Problem 4. A wheat field is 3 miles long and 2 miles wide. How many square inches is that?

Problem 5.

(a) $(3 \times 10^6) \times (12 \times 10^7) = ?$

(b) $\dfrac{(12 \times 10^{17})}{(4 \times 10^{13})} = ?$

(c) $(6.29 \times 10^4) + (7.1 \times 10^3) = ?$

1.1.3 Significant digits

Suppose I go to the blackboard, and, swinging my arm around, draw a big circle. *Question:* What's the circumference of that circle? Well, $c = 2\pi r$, where r is the radius, and I estimate my arm's length at about 70 cm (0.7 m). Punching this into my calculator (which has stored the value of π), I get

$$c = 4.398\,229\,715 \text{ m}.$$

What do you make of that result? Do you really believe the 5 down there in the ninth decimal place? After all, the value for r I used was only a rough estimate. It could easily be that my arm's length is actually 74 cm (in which case $c = 4.649\,557\,127$ m), or perhaps 67 cm ($c = 4.209\,734\,156$ m). Clearly the leading 4 is right, and the next digit seems to lie somewhere between 2 and 6, but all the rest is meaningless junk. I *should* have recorded the answer as

$$c = 4.4\,\text{m}.$$

Only those two digits bear any relation to the truth – we call them **significant digits**.

In an era when calculators and computers happily spit out 8 or 16 digits, it is tempting to list them all, even when a moment's reflection indicates that most of them are insignificant. It's not *wrong*, exactly, but it's grossly misleading, and it looks unprofessional. Don't do it. If you want to carry one or two extra digits, just to be on the safe side, I won't quarrel with you, but not more than that. The reader naturally assumes that whatever numbers you list are valid. More precisely, if you write 3.14, the reader will infer that the true value lies between 3.135 and 3.145. If you're pretty sure it's between 3.138 and 3.142, it's best to write 3.140 ± 0.002.

How can you tell how many digits in your answer are significant? The foolproof method is the one I used for the circumference of the circle: calculate the result using all the possible values of the input numbers. There's a whole mathematical apparatus for doing this more efficiently, but we don't need to get into that here. Very crudely, if you used n significant digits in your input, you deserve n significant digits in your output.

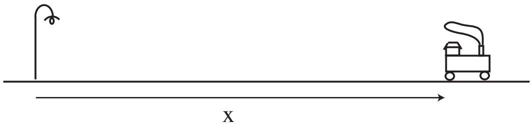
Problem 6. A notorious difficulty arises when you subtract two very nearly equal numbers. Here's an example. A machinist measures the lengths of two rods to fantastic precision: rod A is 4.793 02 m long, and rod B is 4.793 03 m long. (a) How many significant digits are there in each measurement? (b) What is the *difference* in their lengths? (c) How many significant digits are there in the difference?

1.2 Mechanics

Mechanics is **the study of motion**. It falls naturally into two parts: the *description* of motion (known technically as **kinematics**), and the *causes* of motion (**dynamics**).

1.2.1 Kinematics

Imagine a locomotive, constrained to move along a smooth, straight track.



We need some terminology to describe its motion.

- **Position.** First of all, how might I communicate to someone the *location* of the locomotive? One thing I could do is specify its distance from the lamppost, x. If I tell you that $x = 12$ m, you will know that the locomotive is 12 meters to the right of the lamppost. What would it mean if I reported $x = -3$ m? I suppose that would indicate that it is 3 meters to the *left* of the lamppost. Good: x is the **position** of the object.
- **Velocity.** When the locomotive moves, its position changes, and I might like to know how *fast* it changes. **Velocity** is the *rate of change* of position – the distance traveled, divided by the time it took to get there. For example, suppose we use a stopwatch to measure the time, t (in seconds), and we observe that the locomotive goes from $x = 10$ m, at $t = 3$ s, to $x = 25$ m, at $t = 8$ s. The distance traveled is 15 m, and the time it took was 5 s, so the velocity is

$$v = \frac{15\,\mathrm{m}}{5\,\mathrm{s}} = 3\,\mathrm{m/s}.$$

Notice how we arrived at those numbers. To get the distance traveled, we subtracted the initial position from the final position: $x_{\text{final}} - x_{\text{initial}}$; to get the elapsed time, we subtracted the initial time from the final time: $t_{\text{final}} - t_{\text{initial}}$. Evidently

$$v = \frac{x_{\text{final}} - x_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}}.$$

But this looks awfully cumbersome; there is a nice shorthand, using the Greek letter Δ (delta), which means “the change in” whatever comes next:

$$\Delta z = z_{\text{final}} - z_{\text{initial}},$$

(whatever z may be). Then

$$v = \frac{\Delta x}{\Delta t}. \tag{1.1}$$

This doesn’t tell us anything we didn’t already know – it just says it in a tidier way.

What would you conclude if I told you the velocity of the locomotive is *negative* (say, -3 m/s)? Evidently in that case it is moving to the *left*. Physicists use the word **speed** to denote the *magnitude* of the velocity, regardless of its direction. In this example we would say the speed is 3 m/s (no minus sign – speed is always positive).

Motion at constant velocity. Suppose the locomotive started from the lamppost at $t = 0$, and traveled to the right at a constant velocity of 3 m/s. What is its position after 1 s? $x = 3$ m, obviously. After 2 s? $x = 6$ m. After

3 s? $x = 9$ m; etc. What's the general rule here? Evidently you multiply the velocity (v) by the elapsed time (t). We can express this in a simple formula:

$$x = vt \quad \text{(for motion at constant velocity)}. \tag{1.2}$$

That's assuming it started at the lamppost ($x = 0$); if it started out at x_0, then after a time t it would be at

$$x = x_0 + vt. \tag{1.3}$$

- **Acceleration.** What if the locomotive speeds up or slows down, so its velocity is *not* constant? The rate of change of velocity is what we call **acceleration**. Acceleration is to velocity as velocity is to position:

$$a = \frac{\Delta v}{\Delta t}. \tag{1.4}$$

Say (for instance) the train was going 3 m/s at $t = 7$ s, and it's going 9 m/s at $t = 10$ s. Then $\Delta v = 9\,\text{m/s} - 3\,\text{m/s} = 6\,\text{m/s}$, $\Delta t = 10\,\text{s} - 7\,\text{s} = 3\,\text{s}$, so

$$a = \frac{6\,\text{m/s}}{3\,\text{s}} = 2\,\text{m/s}^2.$$

(Notice that the units of acceleration are m/s^2.)

Motion at constant acceleration. Suppose the locomotive starts from rest (velocity zero) at the lamppost (position zero), at $t = 0$ (noon, say), and undergoes constant acceleration, a. After a time t its velocity will be

$$v = at \quad \text{(for motion at constant acceleration)} \tag{1.5}$$

(just like the position, under motion at constant velocity).

How about its *position*? This is tricky. You might be inclined to say $x = vt = (at)t = at^2$, but this is incorrect, because *most* of the time the velocity was *less* than the final value at. (After all, it started out with velocity *zero*; Eq. (1.2) applies only to motion at *constant* velocity.) What *should* the formula be? I think it is plausible (and if this were a regular physics textbook I would prove it) that we want the *average* velocity: not the initial value (0) and not the final value (at), but halfway between ($v_{\text{average}} = at/2$):

$$x = v_{\text{ave}}t = \tfrac{1}{2}at^2 \quad \text{(for motion at constant acceleration)}. \tag{1.6}$$

That's assuming it started from rest, at the lamppost; if it was already going at velocity v_0, then after a time t its velocity would be

$$v = v_0 + at, \tag{1.7}$$

and if it started from x_0 it would now be at

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 \tag{1.8}$$

(x_0 is where it began, $v_0 t$ is the distance it would have gone if it had maintained its initial velocity, and $(1/2)at^2$ is the extra distance it went because of the acceleration).

A familiar example of motion at constant acceleration is **free fall**. When you drop an object, it accelerates (downward, of course) at $a = 9.81\ \mathrm{m/s^2}$ (the so-called **acceleration of gravity**, for which we use the letter g). Remarkably, all objects fall with the *same* acceleration, if we neglect the effect of air resistance.

Example 2. (a) Drop a rock off a high tower. After 3 s, how fast is it going, and how far has it fallen?

Solution:

$$v = at = (10\,\mathrm{m/s^2})(3\,\mathrm{s}) = 30\,\mathrm{m/s};$$
$$x = \frac{1}{2}at^2 = \frac{1}{2}(10\,\mathrm{m/s^2})(9\,\mathrm{s^2}) = 45\,\mathrm{m}.$$

(I used $g = 10\,\mathrm{m/s^2}$, instead of $9.8\,\mathrm{m/s^2}$, because I'm lazy.)

(b) The Golden Gate Bridge is about 80 m above the ocean (actually, it's more like 67 m, but the numbers work out nicer this way). If you drop a quarter, how long will it take to hit the water?

Solution:

$$80 = \frac{1}{2}(10)t^2 \Rightarrow t^2 = \frac{2 \times 80}{10} = 16, \text{ so } t = 4\,\mathrm{s}.$$

More precisely (but this still ignores air resistance)

$$67 = \frac{1}{2}(9.81)t^2 \Rightarrow t^2 = \frac{2 \times 67}{9.81} = 13.7, \text{ so } t = \sqrt{13.7} = 3.7\,\mathrm{s}.$$

(If you are a fastidious person you will want to attach the appropriate units to every quantity, as you go along. Sometimes this is distracting, however, and I'm not always conscientious about it. But the final answer definitely needs its units.)

Circular motion. Acceleration involves *change* in velocity. Ordinarily, this means speeding up or slowing down. But a change in *direction* also constitutes acceleration. The classic example is circular motion – a ball tied to a string whirled around your head, a planet orbiting around the Sun, or an electron circling the nucleus of an atom. If it's going in a circle, it is accelerating, toward the center, in the amount

$$a_c = \frac{v^2}{r}, \tag{1.9}$$

where v is the speed and r is the radius of the circle. We call it **centripetal** ("center-seeking") acceleration.[1]

> **Example 3.** You're on a merry-go-round, sitting on a horse 5 m from the center. It takes 10 s to complete a revolution. What is your speed? What is your centripetal acceleration?
>
> *Solution:* For one revolution, $vt = 2\pi r$, so $v = 2\pi r/t = 2\pi(5)/10 = 3.14$ m/s. Your acceleration, then, is
>
> $$a_c = \frac{3.14^2}{5} = 2.0\,\text{m/s}^2.$$
>
> Just for comparison, that's about 1/5 the acceleration of gravity.
>
> Of course, you *feel* thrown outward, but your acceleration is actually inward. That's like taking off in an airplane: you're accelerating forward, so you *feel* pushed backward, into the seat.

Problem 7. A car driving north at constant velocity on SE 39th passes Tolman (600 meters south of Woodstock) at 12:07 p.m. and reaches Holgate (2400 meters north of Woodstock) at 12:12 p.m. Find: (a) Δx (in meters), (b) Δt (in seconds), and (c) the velocity of the car (in meters per second).

Problem 8. An ocean liner makes a 3600-mile voyage in 8 days, 8 hours. What was its (average) velocity?

Problem 9. An object moving initially with a velocity of 12 m/s is uniformly accelerated at a rate of 3 m/s^2. What is its velocity after 8 seconds of acceleration? If it started at position 8 m, what is its position after 8 s?

Problem 10. An automobile starts from rest and after 3 seconds is moving with a speed of 21 m/s. If the acceleration was constant, how far did the automobile move in the first 2 seconds? How far did it move during the third second?

[1] If you're interested in seeing a *derivation* of the centripetal acceleration formula, look in any introductory physics textbook.

Problem 11. Suppose you drop a rock down a well, and 3 seconds later you hear the splash.

(a) How deep is the well? (That is: how far down is the surface of the water? Neglect air resistance and the time it took the sound to reach you.)
(b) How fast was the brick going when it hit the water?
(c) Sound travels at 340 m/s. If we *do* take into account the time it took the sound to reach you, what is the corrected depth of the well?

Problem 12. You're on an island at the equator when for some reason the Earth's rotation starts to speed up, until eventually (when $a_c = g$) gravity can no longer hold things down, and you have to tie yourself to a palm tree to keep from flying off. What is the length of a day (the time it takes the Earth to complete a full rotation) when this happens?

1.2.2 Dynamics

It's harder to move a heavy object than a light one. That commonplace observation is the basis of dynamics. I need to explain, precisely and quantitatively, what it means.

Let's begin with the concept of **mass**. Mass is a measure of the "amount of stuff" in an object. It is directly related to the more familiar notion of **weight**. We measure it in **kilograms** (kg) – one kilogram corresponds to 2.2 lbs, so if you weigh 132 lbs, your mass is 60 kg. But technically "weight" is the force exerted by gravity – in outer space (where there is no gravity) everything is "weightless." By contrast, your mass is the same wherever you go.

Aristotle taught that objects in motion naturally come to rest, unless somebody is there to keep pushing on them. That certainly sounds right. If I shove a book across the table, it does indeed come to a stop. What Aristotle didn't realize is that an unseen force is acting on the book: the force of **friction**. Galileo was apparently the first to understand that if you could get rid of the friction, the book would keep on moving. But because friction is ubiquitous,[2] Aristotle's claim seems plausible, and it took 1500 years to fix his error. Serious physics began with **Newton's first law** of motion.

Objects keep moving in a straight line, with constant velocity, unless acted upon by some force.
(Of course, if they start out at rest – velocity zero – they remain at rest.)

[2] Historically, planets, freely falling objects, and pendulums were about as close as you could get to frictionless motion, and that's one reason why they played such an important role in the early development of physics.

If a force *does* act, the velocity will change – the object will *accelerate*. The amount of acceleration is given by **Newton's second law** of motion:

$$F = ma. \tag{1.10}$$

Here F is the applied force, m is the mass, and a is the resulting acceleration. Newton's second law is the foundation for all of mechanics. In a sense, it subsumes the first law as a special case: if the force is zero, then the acceleration is zero, so the velocity is constant. Notice that mass is a measure of **inertia** – the object's *resistance to acceleration*. The greater the mass, the larger the force that will be required to achieve a given acceleration.

But what exactly is "force"? Well, it's a "push" or "pull." Ordinarily, the agency responsible is pretty obvious: a rope pulling on a wagon, the chain lifting the carcass of a wrecked car, my hand pushing a book across the table. But sometimes the force is not so visible – the gravitational force on a rock in free fall; the frictional force on a sliding book; the magnetic force that sticks things to the refrigerator. . . . Indeed, much of physics involves discovering the mechanism by which one object exerts forces on another.

Force is measured in newtons (N):

$$1\,\mathrm{N} = 1\,\mathrm{kg\,m/s^2}. \tag{1.11}$$

How big is a newton? Well, you can easily exert a force of 10 N with your little finger.

Example 4. A 60 kg skater is pulled across a frozen lake[3] with a constant force of 120 N. What is her acceleration?

Solution:

$$120 = 60\,a \quad \Rightarrow a = \frac{120}{60} = 2\,\mathrm{m/s^2}.$$

Knowing the acceleration, we can go back to kinematics (Eqs. (1.5) and (1.6)) to figure out (for example) how fast she is going after 3 s ($v = at = 2 \times 3 = 6$ m/s), or how far she goes in 4 s ($x = (1/2)at^2 = (1/2)(2)(4)^2 = 16$ m).

Example 5. What is the force of gravity on a rock (mass m) in free fall?

Solution: Its acceleration is g, so

$$F = ma = mg.$$

[3] Physicists love icy surfaces; it's a secret code, meaning "frictionless."