## Contents

**List of Figures**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>xv</td>
</tr>
</tbody>
</table>

**Preface**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>xix</td>
</tr>
</tbody>
</table>

**1 Philosophy and Approach**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Elements of environmental modeling</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Education vs. training</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Principles ensuring education</td>
<td>5</td>
</tr>
<tr>
<td>1.4.1 Education and expansion of thought processes</td>
<td>5</td>
</tr>
<tr>
<td>1.4.2 Challenge, persistence, critical analysis, and growth</td>
<td>6</td>
</tr>
<tr>
<td>1.4.3 Learning through reading, discussion, problems, and critical assessment</td>
<td>7</td>
</tr>
<tr>
<td>1.4.4 Increase fundamental and general understanding</td>
<td>7</td>
</tr>
<tr>
<td>1.5 Conclusion</td>
<td>7</td>
</tr>
<tr>
<td>1.6 Problems</td>
<td>8</td>
</tr>
</tbody>
</table>

**2 Thoughts on Use of Data**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Introduction</td>
<td>10</td>
</tr>
<tr>
<td>2.2 On numbers in elementary education</td>
<td>12</td>
</tr>
<tr>
<td>2.3 What’s the answer?</td>
<td>13</td>
</tr>
<tr>
<td>2.4 Given the process, what’s the answer?</td>
<td>16</td>
</tr>
<tr>
<td>2.5 Given the answer, what’s the process?</td>
<td>19</td>
</tr>
<tr>
<td>2.6 Given an answer, is it useful?</td>
<td>21</td>
</tr>
<tr>
<td>2.6.1 Given the process, is the proposed answer useful?</td>
<td>22</td>
</tr>
<tr>
<td>2.6.2 Given the data, is the answer useful?</td>
<td>23</td>
</tr>
<tr>
<td>2.7 Conclusion</td>
<td>24</td>
</tr>
<tr>
<td>2.8 Problems</td>
<td>25</td>
</tr>
</tbody>
</table>

**3 Models as a Framework for Study of Data**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Introduction</td>
<td>26</td>
</tr>
<tr>
<td>3.2 Models</td>
<td>29</td>
</tr>
<tr>
<td>3.2.1 Physical models</td>
<td>30</td>
</tr>
<tr>
<td>3.2.2 Conceptual models</td>
<td>32</td>
</tr>
<tr>
<td>3.2.3 Mathematical models</td>
<td>33</td>
</tr>
<tr>
<td>3.3 Definitions</td>
<td>37</td>
</tr>
</tbody>
</table>
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 Elements of a deterministic model</td>
<td>41</td>
</tr>
<tr>
<td>3.4.1 Conceptual model of the mathematical model</td>
<td>44</td>
</tr>
<tr>
<td>3.5 Modeling principles</td>
<td>47</td>
</tr>
<tr>
<td>3.6 Approaches to modeling</td>
<td>50</td>
</tr>
<tr>
<td>3.7 Summary of the modeling procedure</td>
<td>51</td>
</tr>
<tr>
<td>3.8 Quantifying the utility of a model</td>
<td>54</td>
</tr>
<tr>
<td>3.8.1 How uncertainty gets introduced</td>
<td>54</td>
</tr>
<tr>
<td>3.8.2 Reducing uncertainty</td>
<td>55</td>
</tr>
<tr>
<td>3.8.3 Verification and validation development</td>
<td>58</td>
</tr>
<tr>
<td>3.8.4 Verification</td>
<td>58</td>
</tr>
<tr>
<td>3.8.5 Validation</td>
<td>59</td>
</tr>
<tr>
<td>3.8.6 Uncertainty quantification</td>
<td>60</td>
</tr>
<tr>
<td>3.8.7 Calibration</td>
<td>61</td>
</tr>
<tr>
<td>3.8.8 Final thoughts on validation</td>
<td>63</td>
</tr>
<tr>
<td>3.9 Conclusion</td>
<td>64</td>
</tr>
<tr>
<td>3.10 Problems</td>
<td>64</td>
</tr>
<tr>
<td>4 Length and Time Scales</td>
<td>66</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>66</td>
</tr>
<tr>
<td>4.2 Scales of observation</td>
<td>68</td>
</tr>
<tr>
<td>4.3 Continuum scale, averaging, and the rev concept</td>
<td>69</td>
</tr>
<tr>
<td>4.4 Reflections on measurement of properties of a continuum</td>
<td>73</td>
</tr>
<tr>
<td>4.5 Reflections on a general governing equation</td>
<td>75</td>
</tr>
<tr>
<td>4.6 Length scales</td>
<td>77</td>
</tr>
<tr>
<td>4.7 Time scales</td>
<td>80</td>
</tr>
<tr>
<td>4.8 Problems</td>
<td>82</td>
</tr>
<tr>
<td>5 Mechanisms of Change</td>
<td>85</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>85</td>
</tr>
<tr>
<td>5.2 Elements of body sources</td>
<td>86</td>
</tr>
<tr>
<td>5.2.1 Sources of total mass</td>
<td>87</td>
</tr>
<tr>
<td>5.2.2 Sources of chemical species</td>
<td>87</td>
</tr>
<tr>
<td>5.2.3 Sources of momentum</td>
<td>87</td>
</tr>
<tr>
<td>5.2.4 Sources of energy</td>
<td>88</td>
</tr>
<tr>
<td>5.3 Elements of surface sources and transport</td>
<td>89</td>
</tr>
<tr>
<td>5.3.1 Velocity</td>
<td>89</td>
</tr>
<tr>
<td>5.3.2 Diffusion and dispersion processes</td>
<td>92</td>
</tr>
<tr>
<td>5.3.3 Mass, momentum, and energy diffusion</td>
<td>92</td>
</tr>
<tr>
<td>5.3.4 Mass, momentum, and energy dispersion</td>
<td>94</td>
</tr>
<tr>
<td>5.3.5 Convection, advection, and flux</td>
<td>97</td>
</tr>
<tr>
<td>5.3.6 Waves</td>
<td>98</td>
</tr>
<tr>
<td>5.4 Conclusion</td>
<td>99</td>
</tr>
<tr>
<td>5.5 Problems</td>
<td>100</td>
</tr>
</tbody>
</table>
## Table of Contents

### 6 Dimensional Analysis

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 Introduction</td>
<td>102</td>
</tr>
<tr>
<td>6.2 Parameter selection</td>
<td>102</td>
</tr>
<tr>
<td>6.2.1 Problem identification</td>
<td>103</td>
</tr>
<tr>
<td>6.2.2 Modeling objectives</td>
<td>104</td>
</tr>
<tr>
<td>6.3 Dimensional analysis in general</td>
<td>105</td>
</tr>
<tr>
<td>6.4 Dimensional analysis of solid–liquid problem</td>
<td>109</td>
</tr>
<tr>
<td>6.4.1 Well-mixed solid–liquid problem</td>
<td>109</td>
</tr>
<tr>
<td>6.4.2 Solid–liquid problem without mixing</td>
<td>111</td>
</tr>
<tr>
<td>6.5 Dimensional analysis of sedimentation</td>
<td>115</td>
</tr>
<tr>
<td>6.6 Dimensional analysis of pipe flow</td>
<td>119</td>
</tr>
<tr>
<td>6.7 Dimensional analysis of porous media flow</td>
<td>123</td>
</tr>
<tr>
<td>6.8 Some important dimensionless numbers</td>
<td>126</td>
</tr>
<tr>
<td>6.8.1 Reynolds number</td>
<td>127</td>
</tr>
<tr>
<td>6.8.2 Prandtl number</td>
<td>127</td>
</tr>
<tr>
<td>6.8.3 Schmidt number</td>
<td>128</td>
</tr>
<tr>
<td>6.8.4 Peclet number</td>
<td>128</td>
</tr>
<tr>
<td>6.8.5 Biot number</td>
<td>128</td>
</tr>
<tr>
<td>6.8.6 Froude number</td>
<td>128</td>
</tr>
<tr>
<td>6.8.7 Rossby number</td>
<td>129</td>
</tr>
<tr>
<td>6.8.8 Bond and capillary numbers</td>
<td>129</td>
</tr>
<tr>
<td>6.9 Conclusion</td>
<td>130</td>
</tr>
<tr>
<td>6.10 Problems</td>
<td>131</td>
</tr>
</tbody>
</table>

### 7 Mathematical Instruments of Change

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 Introduction</td>
<td>133</td>
</tr>
<tr>
<td>7.2 First order discrete time models</td>
<td>134</td>
</tr>
<tr>
<td>7.2.1 Malthusian model</td>
<td>135</td>
</tr>
<tr>
<td>7.2.2 Logistic model</td>
<td>138</td>
</tr>
<tr>
<td>7.2.3 Gompertz model</td>
<td>142</td>
</tr>
<tr>
<td>7.2.4 Harvest models</td>
<td>144</td>
</tr>
<tr>
<td>7.3 Stability for discrete models</td>
<td>147</td>
</tr>
<tr>
<td>7.3.1 Stability theorem</td>
<td>147</td>
</tr>
<tr>
<td>7.3.2 Stability for Malthusian models</td>
<td>149</td>
</tr>
<tr>
<td>7.3.3 Stability for logistic models</td>
<td>151</td>
</tr>
<tr>
<td>7.4 Other contexts</td>
<td>152</td>
</tr>
<tr>
<td>7.4.1 Finance</td>
<td>153</td>
</tr>
<tr>
<td>7.4.2 Roots of equations</td>
<td>155</td>
</tr>
<tr>
<td>7.5 Conclusion</td>
<td>159</td>
</tr>
<tr>
<td>7.6 Problems</td>
<td>159</td>
</tr>
</tbody>
</table>

### 8 Derivatives and Scales

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1 Continuous change in time and space</td>
<td>164</td>
</tr>
<tr>
<td>8.2 Preliminary concepts for continuous modeling</td>
<td>164</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8.3</td>
<td>Scales in model formulation</td>
</tr>
<tr>
<td>8.4</td>
<td>Time derivatives</td>
</tr>
<tr>
<td>8.4.1</td>
<td>General time derivative</td>
</tr>
<tr>
<td>8.4.2</td>
<td>Eulerian approach</td>
</tr>
<tr>
<td>8.4.3</td>
<td>Lagrangian approach</td>
</tr>
<tr>
<td>8.5</td>
<td>Comments on time derivatives</td>
</tr>
<tr>
<td>8.6</td>
<td>Dot product or inner product</td>
</tr>
<tr>
<td>8.7</td>
<td>Cross product</td>
</tr>
<tr>
<td>8.8</td>
<td>Gradient</td>
</tr>
<tr>
<td>8.9</td>
<td>Comment on coordinate systems</td>
</tr>
<tr>
<td>8.10</td>
<td>Normal direction to a surface</td>
</tr>
<tr>
<td>8.11</td>
<td>Normal velocity of a surface</td>
</tr>
<tr>
<td>8.11.1</td>
<td>Case I: fixed CV with ( w = 0 ) at every point on the boundary</td>
</tr>
<tr>
<td>8.11.2</td>
<td>Case II: the CV is moving with ( w \neq 0 ) at some boundary locations</td>
</tr>
<tr>
<td>8.12</td>
<td>Divergence</td>
</tr>
<tr>
<td>8.12.1</td>
<td>Divergence in cylindrical coordinates</td>
</tr>
<tr>
<td>8.13</td>
<td>Conclusion</td>
</tr>
<tr>
<td>8.14</td>
<td>Problems</td>
</tr>
</tbody>
</table>

### 9 Integral Theorems and Volume Kinematics

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td>Introduction</td>
<td>194</td>
</tr>
<tr>
<td>9.2</td>
<td>Divergence theorem</td>
<td>195</td>
</tr>
<tr>
<td>9.3</td>
<td>Gradient form of the divergence theorem</td>
<td>199</td>
</tr>
<tr>
<td>9.4</td>
<td>Digression</td>
<td>201</td>
</tr>
<tr>
<td>9.5</td>
<td>General transport theorem</td>
<td>202</td>
</tr>
<tr>
<td>9.5.1</td>
<td>Kinematics of a general control volume</td>
<td>205</td>
</tr>
<tr>
<td>9.6</td>
<td>Reynolds transport theorem</td>
<td>205</td>
</tr>
<tr>
<td>9.6.1</td>
<td>Kinematics of a material volume</td>
<td>206</td>
</tr>
<tr>
<td>9.7</td>
<td>Transport theorem for a fixed volume</td>
<td>207</td>
</tr>
<tr>
<td>9.7.1</td>
<td>Kinematics of a fixed volume</td>
<td>207</td>
</tr>
<tr>
<td>9.8</td>
<td>Conclusion</td>
<td>208</td>
</tr>
<tr>
<td>9.9</td>
<td>Problems</td>
<td>208</td>
</tr>
</tbody>
</table>

### 10 Mass Conservation

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>Introduction</td>
<td>210</td>
</tr>
<tr>
<td>10.2</td>
<td>Mathematical tools</td>
<td>211</td>
</tr>
<tr>
<td>10.3</td>
<td>Integral forms of conservation of mass</td>
<td>213</td>
</tr>
<tr>
<td>10.3.1</td>
<td>Conservation of mass for a general volume</td>
<td>213</td>
</tr>
<tr>
<td>10.3.2</td>
<td>Conservation of mass for a material volume</td>
<td>216</td>
</tr>
<tr>
<td>10.3.3</td>
<td>Conservation of mass for a fixed volume</td>
<td>216</td>
</tr>
<tr>
<td>10.4</td>
<td>Point form of conservation of mass</td>
<td>217</td>
</tr>
<tr>
<td>10.4.1</td>
<td>Point conservation of mass from a general volume</td>
<td>219</td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>10.4.2</td>
<td>Point conservation of mass from a material volume</td>
<td>221</td>
</tr>
<tr>
<td>10.4.3</td>
<td>Point conservation of mass from a fixed volume</td>
<td>224</td>
</tr>
<tr>
<td>10.5</td>
<td>Total mass conservation in a stirred tank</td>
<td>225</td>
</tr>
<tr>
<td>10.5.1</td>
<td>Differential mass conservation in a CSTR</td>
<td>226</td>
</tr>
<tr>
<td>10.5.2</td>
<td>Discrete mass conservation in a CSTR</td>
<td>227</td>
</tr>
<tr>
<td>10.6</td>
<td>Hydrologic routing</td>
<td>228</td>
</tr>
<tr>
<td>10.6.1</td>
<td>Hydrologic routing for a reservoir</td>
<td>230</td>
</tr>
<tr>
<td>10.6.2</td>
<td>Hydrologic routing for a channel</td>
<td>231</td>
</tr>
<tr>
<td>10.7</td>
<td>Conclusion</td>
<td>232</td>
</tr>
<tr>
<td>10.8</td>
<td>Problems</td>
<td>233</td>
</tr>
<tr>
<td>11</td>
<td>Species Mass Conservation</td>
<td>237</td>
</tr>
<tr>
<td>11.1</td>
<td>Introduction</td>
<td>237</td>
</tr>
<tr>
<td>11.2</td>
<td>General species mass conservation principle</td>
<td>237</td>
</tr>
<tr>
<td>11.2.1</td>
<td>Conservation of species mass for a general volume</td>
<td>238</td>
</tr>
<tr>
<td>11.2.2</td>
<td>Conservation of species mass for a material volume</td>
<td>241</td>
</tr>
<tr>
<td>11.2.3</td>
<td>Conservation of species mass for a fixed volume</td>
<td>242</td>
</tr>
<tr>
<td>11.3</td>
<td>Point form of mass conservation for a chemical species</td>
<td>242</td>
</tr>
<tr>
<td>11.4</td>
<td>Conservation of moles</td>
<td>244</td>
</tr>
<tr>
<td>11.5</td>
<td>On the velocity of a multispecies solution</td>
<td>245</td>
</tr>
<tr>
<td>11.5.1</td>
<td>Barycentric or mass average velocity</td>
<td>245</td>
</tr>
<tr>
<td>11.5.2</td>
<td>Molar average velocity</td>
<td>247</td>
</tr>
<tr>
<td>11.6</td>
<td>Introduction to the diffusion/dispersion vector</td>
<td>249</td>
</tr>
<tr>
<td>11.7</td>
<td>Approximate form of the diffusion/dispersion vector</td>
<td>250</td>
</tr>
<tr>
<td>11.7.1</td>
<td>Mass dispersion vector</td>
<td>250</td>
</tr>
<tr>
<td>11.7.2</td>
<td>Molar dispersion vector</td>
<td>255</td>
</tr>
<tr>
<td>11.8</td>
<td>Species mass conservation in a stirred tank</td>
<td>256</td>
</tr>
<tr>
<td>11.8.1</td>
<td>General solution for species mass fraction</td>
<td>257</td>
</tr>
<tr>
<td>11.8.2</td>
<td>Constant inflow and outflow</td>
<td>258</td>
</tr>
<tr>
<td>11.8.3</td>
<td>Discrete solution for species mass fraction</td>
<td>261</td>
</tr>
<tr>
<td>11.9</td>
<td>Advection–Dispersion equation</td>
<td>262</td>
</tr>
<tr>
<td>11.10</td>
<td>Solution of the advection–dispersion equation</td>
<td>265</td>
</tr>
<tr>
<td>11.10.1</td>
<td>One-dimensional analytic solution</td>
<td>266</td>
</tr>
<tr>
<td>11.10.2</td>
<td>Superposition for linear ADE solution</td>
<td>267</td>
</tr>
<tr>
<td>11.11</td>
<td>Conclusion</td>
<td>268</td>
</tr>
<tr>
<td>11.12</td>
<td>Problems</td>
<td>269</td>
</tr>
<tr>
<td>12</td>
<td>Statement of Conservation of Momentum</td>
<td>275</td>
</tr>
<tr>
<td>12.1</td>
<td>Introduction</td>
<td>275</td>
</tr>
<tr>
<td>12.2</td>
<td>Elements of the momentum conservation equation</td>
<td>275</td>
</tr>
<tr>
<td>12.3</td>
<td>Second order tensor</td>
<td>277</td>
</tr>
<tr>
<td>12.3.1</td>
<td>Mathematical relations</td>
<td>277</td>
</tr>
<tr>
<td>12.3.2</td>
<td>Physical meaning of two example tensors</td>
<td>280</td>
</tr>
<tr>
<td>12.4</td>
<td>Integral momentum conservation equations</td>
<td>282</td>
</tr>
</tbody>
</table>
## Contents

12.4.1 Momentum conservation for a general volume 282
12.4.2 Momentum conservation for a material volume 283
12.4.3 Momentum conservation for a fixed volume 283
12.5 Point form of the momentum conservation equation 284
12.6 Viscous stress tensor 286
12.7 Navier–Stokes equation 289
12.8 Bernoulli equation 291
12.9 Microscale mechanical energy balance 294
12.10 System-scale mechanical energy balance 296
12.11 Conclusion 297
12.12 Problems 298
12.13 Appendix: angular momentum equation 301

13 Conservation of Total Energy

13.1 Introduction 303
13.2 Statement of the total energy equation 304
13.3 Point forms of total energy conservation 306
13.4 Thermal energy balance 308
13.5 Internal energy equation in terms of temperature 309
13.5.1 Introduction of heat capacity 309
13.5.2 Introduction of Fourier’s law 310
13.6 Conclusion 311
13.7 Problems 311

14 Mixed-scale Modeling

14.1 Introduction 313
14.2 Starting point 314
14.3 Equations for hydraulic routing 316
14.3.1 Mass conservation for hydraulic routing 317
14.3.2 Momentum conservation for hydraulic routing 319
14.3.3 Special form of the momentum equation 324
14.4 Friction slope parameterization 325
14.5 Implementation of hydraulic routing techniques 328
14.5.1 Unsteady, non-uniform flow (full dynamic equation) with $\beta = 1$ 330
14.5.2 Steady, non-uniform flow (quasi-steady dynamic wave approximation) 330
14.5.3 Steady, non-uniform flow (diffusion wave approximation) 331
14.5.4 Steady, uniform flow rate $Q$ (normal flow) 332
14.5.5 Kinematic wave approximation 332
14.6 Data support 333
14.7 Mechanical energy equation for hydraulic routing 335
14.8 Shallow-water equations 338
14.8.1 Mass conservation for shallow-water flow 338
# Table of Contents

14.8.2 Momentum conservation for shallow-water flow 340
14.9 Conclusion 346
14.10 Problems 347

15 Porous Media and Groundwater Systems 349
   15.1 Introduction 349
   15.2 Considerations for porous media flow and transport 351
   15.3 Fluid mass conservation in a porous medium 353
   15.4 Macroscale fluid mass conservation equation 356
   15.5 Matrix deformation and material compressibility 358
   15.6 Darcy’s law 362
   15.7 Flow equation in terms of pressure 366
   15.8 Flow equation in terms of head 367
   15.9 Constant density flow in a deformable, homogeneous porous medium 367
   15.10 Steady-state flow with constant $\rho w$ and $K$ 368
   15.11 Summary of main points for flow equations 369
   15.12 Porous medium-scale convective–dispersive equation 370
   15.13 Conclusion 373
   15.14 Problems 374
   15.15 Appendix: anisotropic hydraulic conductivity 377

16 Advection–Dispersion Equation Solution 379
   16.1 Introduction 379
   16.2 Simplified advection–dispersion equation 380
   16.3 One-dimensional formulation 380
   16.4 Discrete form of the advection–dispersion equation 381
      16.4.1 Dimensionless form of the advection–dispersion equation 381
      16.4.2 Discrete approximations of derivatives 382
      16.4.3 One-dimensional discrete advection–dispersion equation 384
   16.5 Two-dimensional formulation 388
      16.5.1 Construction of a difference equation 389
      16.5.2 Steady-state case 391
   16.6 Conclusion 394
   16.7 Problems 395

17 Stability Revisited 398
   17.1 Introduction 398
   17.2 Lake/CSTR simulation 399
      17.2.1 Discrete form and stability 400
   17.3 First example of lake operation 401
   17.4 Second example of lake operation 403
   17.5 Third example of lake operation 405
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.6 Conclusion</td>
<td>406</td>
</tr>
<tr>
<td>17.7 Problems</td>
<td>407</td>
</tr>
</tbody>
</table>

*References*  
*Index*
Figures

2.1 Plot of available data for January rainfall in Grangeville, Idaho, from 1950 to 2000. 19
3.1 Descent from perfect determinism through statistical uncertainty, scenario uncertainty, and recognized ignorance to total ignorance (after [217]). 27
3.2 Schematic of model types and subtypes (after [105]). The shaded box is the primary focus of this book. 30
3.3 Conceptual model of the hydrologic cycle (after [32]). Water stored in each of the four elements has units of $10^{15}$ kg. Flows between elements have units of $10^{15}$ kg/yr. 33
3.4 Excessively simplified, though commonly employed, modeling strategy. 41
3.5 Improved strategy for development of a useful deterministic model. 43
3.6 Conceptual model of Mathematical Model box of Figure 3.5 with subelements indicated. 45
3.7 Simplified conceptual model of a modeling system (after [53]). 50
3.8 Conceptual model of key elements and actions of the modeling process based on the list in Section 3.5. Unnumbered boxes are all part of item 9, the critical review. 52
4.1 Normalized mass density as a function of the averaging volume size. 70
5.1 Lake Michigan considered as a single system in (a) and subdivided into two systems in (b). 86
5.2 Spreading of contaminant by movement of lower plate produces apparent dispersion over the system cross section: (a) contaminated fluid on left, pure fluid on right; (b) distribution of contaminated and uncontaminated fluid after translation of lower plate; (c) average concentration of contaminant over cross section after translation of lower plate. 95
5.3 Contamination flows in from the left. At point A, the flow splits into two parts that come together at point B. The contamination concentrations at point B are determined in the problems. 100
7.1 Solutions to the discrete logistic equation (dots), Eq. (7.20), and to the differential logistic equation given as Eq. (7.24) (curve) with $p_0 = 0.1$, $r\Delta t = 0.5$. For these parameter values, both solutions are monotonic. 141
7.2 Solutions to the discrete logistic equation (dots), Eq. (7.20), and to the differential logistic equation given as Eq. (7.24) (curve) with $p_0 = 0.1$, 

© in this web service Cambridge University Press

www.cambridge.org
7.3 Plot of the function $f$ in Eq. (7.29) vs. $p$ for various values of the parameter $\epsilon$. When $\epsilon = 1$, the logistic model results. When $\epsilon \to \infty$, the Gompertz model is obtained.

7.4 Equilibrium dimensionless yield vs. equilibrium dimensionless populations for Gompertz and logistic models with harvesting.

8.1 The volume in (a) is composed of two juxtaposed volumes with typical unit normal vectors to each volume indicated along with the fact that the outward unit normals are collinear but in opposite directions at a common surface. The volume in (b) depicts outwardly directed normal vectors at two points on the surface.

8.2 Cuboidal volume with side lengths $\Delta x$, $\Delta y$, and $\Delta z$.

8.3 Annular volume with side lengths $\Delta r$, $\Delta \theta$, and $(r + \Delta r) \Delta \theta$.

9.1 Juxtaposition of small domains. (a) Six boxes with fluxes at boundary of each box indicated. (b) Composite box such that only external boundary fluxes are considered because flux dotted with outward normal at common surfaces cancel.

10.1 (a) Global volume; (b) Subdivided volume for description of some variability within the volume and flow between subregions; (c) Further subdivided volume. In the limit of subdivision into very small study volumes, a continuous description of the variability of a quantity of interest is obtained.

10.2 Discretization of Gulf of Mexico into 33,272 triangles with 17,306 nodes. Range of node spacing is from approximately 1 to 125 km [93].

10.3 Well-mixed CSTR with a single inflow and a single outflow.

11.1 Solution given by Eq. (11.72) for various values of $\kappa = kV/Q$.

12.1 Elemental volume with side lengths $\Delta x$, $\Delta y$, and $\Delta z$ with elements of the stress tensor depicted.

12.2 Dam at one end of a reservoir.

12.3 Treatment tank of cross-sectional area $A$ with outflow orifice of cross-section $a$. The volumetric inflow is $Q$, volumetric outflow is $q$, and the height of fluid in the tank is $H$. The fluid density, $\rho$, is constant.

14.1 Diagram of a one-dimensional river with flow parallel to the river bed: (a) side view with $x$ direction parallel to river bed, control volume indicated in grey; (b) front view; (c) top view.

14.2 Diagram of a section of estuary for derivation of a vertically averaged model. Depth of flow is $H = \zeta + h$. Averaging domain is of height $H$ and cross-section $\Delta x \Delta y$.

15.1 Sketch of a spherical averaging region within an aquifer. Below the land surface is the unsaturated zone with water and air fluid phases. Below the water table is a saturated region where water is the only fluid present. Then, below the unconfined aquifer is the aquifer confined above and below. The REV is the spherical region $\Omega_{REV}$ with volume $v_{REV}$. The
region occupied by the fluid is $\Omega_{\text{REV}_w}$ with volume $v_{\text{REV}_w}$. The region occupied by the solids is indicated as the dark grains and has volume $v_{\text{REV}_s}$ such that $v_{\text{REV}} = v_{\text{REV}_w} + v_{\text{REV}_s}$. The part of the external boundary of the REV that intersects fluid is designated $\Gamma_{\text{REV}_w}$, and the internal boundary between the fluid and solid is designated $\Gamma_{\text{REV}_i}$.353

15.2 Experimental setup of Henry Darcy. 363
15.3 Linear relation between $q$ and $\Delta h/L$ based on Darcy’s experiment (data from [24]). 363
16.1 Nodes involved in the approximation of Eq. (16.8) in the vicinity of the unknown solution $C(t_{k+1/2}, \xi_j)$. 385
16.2 Computational molecule in two spatial dimensions and time centered around $t_{k+1/2}, x_i, y_j$. 389
16.3 Computational molecule in two spatial dimensions for use in a steady-state simulation centered around $x_i, y_j$. 392
16.4 Discretized region for example problem $\nabla^2 \omega = 0$. Domain is the rectangle $0 \leq x \leq L, 0 \leq y \leq B$. Specified function for $\omega$ at $x = 0$ and $x = L$; zero normal gradient at $y = 0$ and $y = B$. 393
16.5 Computational molecule at boundary of simulation region where normal gradient is specified. 393
17.1 Well-mixed lake to be modeled as a stirred tank. 399