Introduction to Environmental Modeling

The textbook presents an understanding of how basic physical descriptions can be translated into mathematical analogues that provide an opportunity to investigate environmental processes. Examples come from a range of hydrologic, atmospheric, and geophysical modeling problems. The emphasis is on simple examples and calculations that add to understanding. The book provides a sense for the meaning of mathematical expressions, a physical feel for their relations to the processes that they describe, and confidence in working with mathematical solutions.

Addressing environmental modeling problems from a scientific basis requires integration of physical understanding, mathematical skill, courage in making reasonable approximations, and transparency in identifying strengths, weaknesses, applicability, and limitations of the models formulated, employed, and solved. From this perspective, embarking on environmental modeling requires a student to depart from the familiar comforts of well-defined problems for which there is a well-defined solution. Students need to learn how to tackle often incompletely-defined environmental problems in an effort to understand the unknown elements of the past, present, or future and to learn where further studies are needed to increase understanding. The need to approximate, to span mathematics, science, and computational methods, to critique all elements of an analysis, and to admit limitations on what has been accomplished, all contribute to making environmental modeling an exciting, yet sometimes uncomfortable, activity. The goal of this book, in essence, is to present the timeless basic physical and mathematical principles and philosophy of environmental modeling, often to students who need to be taught how to think in a different way than they would for the average more narrowly-defined engineering or physics problem. Minimum prerequisites for the student reader (for any realistic modeling) include a knowledge of calculus through differential equations, but the book provides the mathematical and physical tools needed as the occasion arises.

Dr. William G. Gray holds a Ph.D. in chemical engineering from Princeton University. He has held full time faculty positions in civil engineering at Princeton and the University of Notre Dame and in environmental engineering and the Curriculum for the Environment and Ecology at the University of North Carolina (UNC), Chapel Hill. He has received teaching awards at Notre Dame and UNC. Dr. Gray is a Fellow of the American Geophysical Union, Distinguished Engineering Alumnus of the University of California at Davis, a Fulbright Scholar, and a Langbein Lecturer of the Hydrology section of the American Geophysical Union. He was editor of *Advances in Water Resources* for a dozen years and editor of *Water Resources Research* for four years. He is the author of approximately 140 articles in refereed journals and is a co-author of eight books.

Dr. Genetha A. Gray is a data scientist in the Talent Intelligence and Analytics organization at Intel where she analyzes talent supply chains and models career progression. Before joining Intel in 2014, Genetha spent 12 years at Sandia National Labs. There, she worked on problems related to the electrical and mechanical engineering of systems, the storage of nuclear waste, groundwater remediation, cyber security, and energy. She has a Ph.D. in Computational and Applied Mathematics from Rice University and specializes in analytics, optimization, data fusion, model validation, and uncertainty quantification. She has co-authored more than 25 research publications and given more than 60 presentations.

Cambridge University Press
978-1-107-57169-3 — Introduction to Environmental Modeling
William G. Gray , Genetha A. Gray
Frontmatter
More Information

Introduction to Environmental Modeling

WILLIAM G. GRAY
University of North Carolina at Chapel Hill, USA

GENETHA A. GRAY
Intel Corporation, USA

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107571693

First published 2017

Printed in the United States of America by Sheridan Books, Inc.

A catalog record for this publication is available from the British Library.

ISBN 978-1-107-57169-3

Additional resources for this publication at www.cambridge.org/9781107571693

**To the next generation of modelers,
Charlotte, Ben, Lucy, Gray, and Iris;
and to those who gave them life, love them,
and nurture their environment.**

Contents

List of Figures *page* xv
Preface xix

1 Philosophy and Approach 1
1.1 Introduction 1
1.2 Elements of environmental modeling 2
1.3 Education vs. training 4
1.4 Principles ensuring education 5
1.4.1 Education and expansion of thought processes 5
1.4.2 Challenge, persistence, critical analysis, and growth 6
1.4.3 Learning through reading, discussion, problems, and critical assessment 7
1.4.4 Increase fundamental and general understanding 7
1.5 Conclusion 7
1.6 Problems 8

2 Thoughts on Use of Data 10
2.1 Introduction 10
2.2 On numbers in elementary education 12
2.3 What's the answer? 13
2.4 Given the process, what's the answer? 16
2.5 Given the answer, what's the process? 19
2.6 Given an answer, is it useful? 21
2.6.1 Given the process, is the proposed answer useful? 22
2.6.2 Given the data, is the answer useful? 23
2.7 Conclusion 24
2.8 Problems 25

3 Models as a Framework for Study of Data 26
3.1 Introduction 26
3.2 Models 29
3.2.1 Physical models 30
3.2.2 Conceptual models 32
3.2.3 Mathematical models 33
3.3 Definitions 37

3.4 Elements of a deterministic model 41
3.4.1 Conceptual model of the mathematical model 44
3.5 Modeling principles 47
3.6 Approaches to modeling 50
3.7 Summary of the modeling procedure 51
3.8 Quantifying the utility of a model 54
3.8.1 How uncertainty gets introduced 54
3.8.2 Reducing uncertainty 55
3.8.3 Verification and validation development 58
3.8.4 Verification 58
3.8.5 Validation 59
3.8.6 Uncertainty quantification 60
3.8.7 Calibration 61
3.8.8 Final thoughts on validation 63
3.9 Conclusion 64
3.10 Problems 64

4 Length and Time Scales 66
4.1 Introduction 66
4.2 Scales of observation 68
4.3 Continuum scale, averaging, and the rev concept 69
4.4 Reflections on measurement of properties of a continuum 73
4.5 Reflections on a general governing equation 75
4.6 Length scales 77
4.7 Time scales 80
4.8 Problems 82

5 Mechanisms of Change 85
5.1 Introduction 85
5.2 Elements of body sources 86
5.2.1 Sources of total mass 87
5.2.2 Sources of chemical species 87
5.2.3 Sources of momentum 87
5.2.4 Sources of energy 88
5.3 Elements of surface sources and transport 89
5.3.1 Velocity 89
5.3.2 Diffusion and dispersion processes 92
5.3.3 Mass, momentum, and energy diffusion 92
5.3.4 Mass, momentum, and energy dispersion 94
5.3.5 Convection, advection, and flux 97
5.3.6 Waves 98
5.4 Conclusion 99
5.5 Problems 100

6 Dimensional Analysis 102
6.1 Introduction 102
6.2 Parameter selection 102
6.2.1 Problem identification 103
6.2.2 Modeling objectives 104
6.3 Dimensional analysis in general 105
6.4 Dimensional analysis of solid–liquid problem 109
6.4.1 Well-mixed solid–liquid problem 109
6.4.2 Solid–liquid problem without mixing 111
6.5 Dimensional analysis of sedimentation 115
6.6 Dimensional analysis of pipe flow 119
6.7 Dimensional analysis of porous media flow 123
6.8 Some important dimensionless numbers 126
6.8.1 Reynolds number 127
6.8.2 Prandtl number 127
6.8.3 Schmidt number 128
6.8.4 Peclet number 128
6.8.5 Biot number 128
6.8.6 Froude number 128
6.8.7 Rossby number 129
6.8.8 Bond and capillary numbers 129
6.9 Conclusion 130
6.10 Problems 131

7 Mathematical Instruments of Change 133
7.1 Introduction 133
7.2 First order discrete time models 134
7.2.1 Malthusian model 135
7.2.2 Logistic model 138
7.2.3 Gompertz model 142
7.2.4 Harvest models 144
7.3 Stability for discrete models 147
7.3.1 Stability theorem 147
7.3.2 Stability for Malthusian models 149
7.3.3 Stability for logistic models 151
7.4 Other contexts 152
7.4.1 Finance 153
7.4.2 Roots of equations 155
7.5 Conclusion 159
7.6 Problems 159

8 Derivatives and Scales 164
8.1 Continuous change in time and space 164
8.2 Preliminary concepts for continuous modeling 164

8.3 Scales in model formulation 165
8.4 Time derivatives 166
8.4.1 General time derivative 167
8.4.2 Eulerian approach 171
8.4.3 Lagrangian approach 172
8.5 Comments on time derivatives 175
8.6 Dot product or inner product 176
8.7 Cross product 177
8.8 Gradient 178
8.9 Comment on coordinate systems 181
8.10 Normal direction to a surface 182
8.11 Normal velocity of a surface 183
8.11.1 Case I: fixed CV with $\mathbf{w} = 0$ at every point on the boundary 184
8.11.2 Case II: the CV is moving with $\mathbf{w} \neq 0$ at some boundary locations 184
8.12 Divergence 185
8.12.1 Divergence in cylindrical coordinates 187
8.13 Conclusion 189
8.14 Problems 190

9 Integral Theorems and Volume Kinematics 194
9.1 Introduction 194
9.2 Divergence theorem 195
9.3 Gradient form of the divergence theorem 199
9.4 Digression 201
9.5 General transport theorem 202
9.5.1 Kinematics of a general control volume 205
9.6 Reynolds transport theorem 205
9.6.1 Kinematics of a material volume 206
9.7 Transport theorem for a fixed volume 207
9.7.1 Kinematics of a fixed volume 207
9.8 Conclusion 208
9.9 Problems 208

10 Mass Conservation 210
10.1 Introduction 210
10.2 Mathematical tools 211
10.3 Integral forms of conservation of mass 213
10.3.1 Conservation of mass for a general volume 213
10.3.2 Conservation of mass for a material volume 216
10.3.3 Conservation of mass for a fixed volume 216
10.4 Point form of conservation of mass 217
10.4.1 Point conservation of mass from a general volume 219

10.4.2 Point conservation of mass from a material volume 221
10.4.3 Point conservation of mass from a fixed volume 224
10.5 Total mass conservation in a stirred tank 225
10.5.1 Differential mass conservation in a CSTR 226
10.5.2 Discrete mass conservation in a CSTR 227
10.6 Hydrologic routing 228
10.6.1 Hydrologic routing for a reservoir 230
10.6.2 Hydrologic routing for a channel 231
10.7 Conclusion 232
10.8 Problems 233

11 Species Mass Conservation 237
11.1 Introduction 237
11.2 General species mass conservation principle 237
11.2.1 Conservation of species mass for a general volume 238
11.2.2 Conservation of species mass for a material volume 241
11.2.3 Conservation of species mass for a fixed volume 242
11.3 Point form of mass conservation for a chemical species 242
11.4 Conservation of moles 244
11.5 On the velocity of a multispecies solution 245
11.5.1 Barycentric or mass average velocity 245
11.5.2 Molar average velocity 247
11.6 Introduction to the diffusion/dispersion vector 249
11.7 Approximate form of the diffusion/dispersion vector 250
11.7.1 Mass dispersion vector 250
11.7.2 Molar dispersion vector 255
11.8 Species mass conservation in a stirred tank 256
11.8.1 General solution for species mass fraction 257
11.8.2 Constant inflow and outflow 258
11.8.3 Discrete solution for species mass fraction 261
11.9 Advection–Dispersion equation 262
11.10 Solution of the advection–dispersion equation 265
11.10.1 One-dimensional analytic solution 266
11.10.2 Superposition for linear ADE solution 267
11.11 Conclusion 268
11.12 Problems 269

12 Statement of Conservation of Momentum 275
12.1 Introduction 275
12.2 Elements of the momentum conservation equation 275
12.3 Second order tensor 277
12.3.1 Mathematical relations 277
12.3.2 Physical meaning of two example tensors 280
12.4 Integral momentum conservation equations 282

12.4.1 Momentum conservation for a general volume 282
12.4.2 Momentum conservation for a material volume 283
12.4.3 Momentum conservation for a fixed volume 283
12.5 Point form of the momentum conservation equation 284
12.6 Viscous stress tensor 286
12.7 Navier–Stokes equation 289
12.8 Bernoulli equation 291
12.9 Microscale mechanical energy balance 294
12.10 System-scale mechanical energy balance 296
12.11 Conclusion 297
12.12 Problems 298
12.13 Appendix: angular momentum equation 301

13 Conservation of Total Energy 303
13.1 Introduction 303
13.2 Statement of the total energy equation 304
13.3 Point forms of total energy conservation 306
13.4 Thermal energy balance 308
13.5 Internal energy equation in terms of temperature 309
13.5.1 Introduction of heat capacity 309
13.5.2 Introduction of Fourier's law 310
13.6 Conclusion 311
13.7 Problems 311

14 Mixed-scale Modeling 313
14.1 Introduction 313
14.2 Starting point 314
14.3 Equations for hydraulic routing 316
14.3.1 Mass conservation for hydraulic routing 317
14.3.2 Momentum conservation for hydraulic routing 319
14.3.3 Special form of the momentum equation 324
14.4 Friction slope parameterization 325
14.5 Implementation of hydraulic routing techniques 328
14.5.1 Unsteady, non-uniform flow (full dynamic equation) with $\beta = 1$ 330
14.5.2 Steady, non-uniform flow (quasi-steady dynamic wave approximation) 330
14.5.3 Steady, non-uniform flow (diffusion wave approximation) 331
14.5.4 Steady, uniform flow rate Q (normal flow) 332
14.5.5 Kinematic wave approximation 332
14.6 Data support 333
14.7 Mechanical energy equation for hydraulic routing 335
14.8 Shallow-water equations 338
14.8.1 Mass conservation for shallow-water flow 338

14.8.2 Momentum conservation for shallow-water flow 340
14.9 Conclusion 346
14.10 Problems 347

15 Porous Media and Groundwater Systems 349
15.1 Introduction 349
15.2 Considerations for porous media flow and transport 351
15.3 Fluid mass conservation in a porous medium 353
15.4 Macroscale fluid mass conservation equation 356
15.5 Matrix deformation and material compressibility 358
15.6 Darcy's law 362
15.7 Flow equation in terms of pressure 366
15.8 Flow equation in terms of head 367
15.9 Constant density flow in a deformable, homogeneous porous medium 367
15.10 Steady-state flow with constant ρ^w and K 368
15.11 Summary of main points for flow equations 369
15.12 Porous medium-scale convective–dispersive equation 370
15.13 Conclusion 373
15.14 Problems 374
15.15 Appendix: anisotropic hydraulic conductivity 377

16 Advection–Dispersion Equation Solution 379
16.1 Introduction 379
16.2 Simplified advection–dispersion equation 380
16.3 One-dimensional formulation 380
16.4 Discrete form of the advection–dispersion equation 381
16.4.1 Dimensionless form of the advection–dispersion equation 381
16.4.2 Discrete approximations of derivatives 382
16.4.3 One-dimensional discrete advection–dispersion equation 384
16.5 Two-dimensional formulation 388
16.5.1 Construction of a difference equation 389
16.5.2 Steady-state case 391
16.6 Conclusion 394
16.7 Problems 395

17 Stability Revisited 398
17.1 Introduction 398
17.2 Lake/CSTR simulation 399
17.2.1 Discrete form and stability 400
17.3 First example of lake operation 401
17.4 Second example of lake operation 403
17.5 Third example of lake operation 405

17.6 Conclusion 406
17.7 Problems 407

References 408
Index 420

Figures

2.1 Plot of available data for January rainfall in Grangeville, Idaho, from 1950 to 2000. 19
3.1 Descent from perfect determinism through statistical uncertainty, scenario uncertainty, and recognized ignorance to total ignorance (after [217]). 27
3.2 Schematic of model types and subtypes (after [105]). The shaded box is the primary focus of this book. 30
3.3 Conceptual model of the hydrologic cycle (after [32]). Water stored in each of the four elements has units of 10^{15} kg. Flows between elements have units of 10^{15} kg/yr. 33
3.4 Excessively simplified, though commonly employed, modeling strategy. 41
3.5 Improved strategy for development of a useful deterministic model. 43
3.6 Conceptual model of Mathematical Model box of Figure 3.5 with subelements indicated. 45
3.7 Simplified conceptual model of a modeling system (after [53]). 50
3.8 Conceptual model of key elements and actions of the modeling process based on the list in Section 3.5. Unnumbered boxes are all part of item 9, the critical review. 52
4.1 Normalized mass density as a function of the averaging volume size. 70
5.1 Lake Michigan considered as a single system in (a) and subdivided into two systems in (b). 86
5.2 Spreading of contaminant by movement of lower plate produces apparent dispersion over the system cross section: (a) contaminated fluid on left, pure fluid on right; (b) distribution of contaminated and uncontaminated fluid after translation of lower plate; (c) average concentration of contaminant over cross section after translation of lower plate. 95
5.3 Contamination flows in from the left. At point A, the flow splits into two parts that come together at point B. The contamination concentrations at point B are determined in the problems. 100
6.1 Moody diagram plot of Eq. (6.80) (after Glasgow College of Nautical Studies, www.asabe.org/media/147982/pipefric.pdf). 122
7.1 Solutions to the discrete logistic equation (dots), Eq. (7.20), and to the differential logistic equation given as Eq. (7.24) (curve) with $p_0 = 0.1$, $r\Delta t = 0.5$. For these parameter values, both solutions are monotonic. 141
7.2 Solutions to the discrete logistic equation (dots), Eq. (7.20), and to the differential logistic equation given as Eq. (7.24) (curve) with $p_0 = 0.1$,

$r\Delta t = 2.0$. For these parameter values, the discrete solution is oscillatory while the continuous solution is monotonic. 141
7.3 Plot of the function f in Eq. (7.29) vs. p for various values of the parameter ϵ. When $\epsilon = 1$, the logistic model results. When $\epsilon \to \infty$, the Gompertz model is obtained. 143
7.4 Equilibrium dimensionless yield vs. equilibrium dimensionless populations for Gompertz and logistic models with harvesting. 146
8.1 The volume in (a) is composed of two juxtaposed volumes with typical unit normal vectors to each volume indicated along with the fact that the outward unit normals are collinear but in opposite directions at a common surface. The volume in (b) depicts outwardly directed normal vectors at two points on the surface. 183
8.2 Cuboidal volume with side lengths Δx, Δy, and Δz. 186
8.3 Annular volume with side lengths Δr, Δz, $r\Delta\theta$, and $(r + \Delta r)\,\Delta\theta$. 188
9.1 Juxtaposition of small domains. (a) Six boxes with fluxes at boundary of each box indicated. (b) Composite box such that only external boundary fluxes are considered because flux dotted with outward normal at common surfaces cancel. 196
10.1 (a) Global volume; (b) Subdivided volume for description of some variability within the volume and flow between subregions; (c) Further subdivided volume. In the limit of subdivision into very small study volumes, a continuous description of the variability of a quantity of interest is obtained. 217
10.2 Discretization of Gulf of Mexico into 33 272 triangles with 17 306 nodes. Range of node spacing is from approximately 1 to 125 km [93]. 218
10.3 Well-mixed CSTR with a single inflow and a single outflow. 225
11.1 Solution given by Eq. (11.72) for various values of $\kappa = kV/Q$. 260
12.1 Elemental volume with side lengths Δx, Δy, and Δz with elements of the stress tensor depicted. 281
12.2 Dam at one end of a reservoir. 299
12.3 Treatment tank of cross-sectional area A with outflow orifice of cross-section a. The volumetric inflow is Q, volumetric outflow is q, and the height of fluid in the tank is H. The fluid density, ρ, is constant. 299
14.1 Diagram of a one-dimensional river with flow parallel to the river bed: (a) side view with x direction parallel to river bed, control volume indicated in grey; (b) front view; (c) top view. 316
14.2 Diagram of a section of estuary for derivation of a vertically averaged model. Depth of flow is $H = \zeta + h$. Averaging domain is of height H and cross-section $\Delta x\Delta y$. 339
15.1 Sketch of a spherical averaging region within an aquifer. Below the land surface is the unsaturated zone with water and air fluid phases. Below the water table is a saturated region where water is the only fluid present. Then, below the unconfined aquifer is the aquifer confined above and below. The REV is the spherical region Ω_{REV} with volume v_{REV}. The

region occupied by the fluid is $\Omega_{\text{REV}w}$ with volume $v_{\text{REV}w}$. The region occupied by the solids is indicated as the dark grains and has volume $v_{\text{REV}s}$ such that $v_{\text{REV}} = v_{\text{REV}w} + v_{\text{REV}s}$. The part of the external boundary of the REV that intersects fluid is designated $\Gamma_{\text{REV}w}$, and the internal boundary between the fluid and solid is designated Γ_{REVi}. 353

15.2 Experimental setup of Henry Darcy. 363

15.3 Linear relation between q and $\Delta h/L$ based on Darcy's experiment (data from [24]). 363

16.1 Nodes involved in the approximation of Eq. (16.8) in the vicinity of the unknown solution $C(\tau_{k+1/2}, \xi_j)$. 385

16.2 Computational molecule in two spatial dimensions and time centered around $t_{k+1/2}, x_i, y_j$. 389

16.3 Computational molecule in two spatial dimensions for use in a steady-state simulation centered around x_i, y_j. 392

16.4 Discretized region for example problem $\nabla^2\omega = 0$. Domain is the rectangle $0 \le x \le L$, $0 \le y \le B$. Specified function for ω at $x = 0$ and $x = L$; zero normal gradient at $y = 0$ and $y = B$. 393

16.5 Computational molecule at boundary of simulation region where normal gradient is specified. 393

17.1 Well-mixed lake to be modeled as a stirred tank. 399

Preface

The possibility of using a computer to predict the physical, chemical, and biological responses of environmental systems is an exciting notion that attracts student interest. The elements and processes of environmental modeling, however, are in many ways foreign to the kind of thinking that young investigators exercise during their educational experience. Modeling is inexact, requires physical insight, makes use of mathematics that expresses physical concepts, and is subject to revision and continuing study in light of data and observed system behavior. In developing their scientific background, students often see solving problems of physics as grabbing the right equation for the job. Solution of mathematical equations likewise involves identifying and applying the right techniques. Mathematics and physics books for undergraduates only present problems that are fully specified, can be solved, and that have a single answer. Many of those answers are given in the back of the book. Thus work on a problem is reduced to shooting for a known target solution or at least finding the direct path to the only possible answer. Environmental questions, on the other hand, involve problems that an investigator must define in terms of their component elements to find answers that were previously unknown. Formulation of equations requires information about time and spatial scales. Information that would make even simple models solvable is often lacking. The absence of an a priori answer requires that the modeler not only propose a model and its results but also be the harshest critic of the inadequacies of the model. This dual role requires that the modeler be well versed in fundamental processes and in how descriptions of those fundamental processes can be applied to particular environmental problems of interest. Learning to do this requires much more than a semester or a single book of exposure; it requires experience, success, failure, time, insight, and patience.

This book is intended to provide an introduction to environmental modeling; it is not a book that transforms an individual into an environmental modeler. Those who work through this material, appreciate the thought processes, gain some grasp of the principles involved, and develop fundamental understanding will have been introduced to environmental modeling. Hopefully, they will appreciate the hard work that lies ahead to be able to model environmental systems. The references provided are typically more advanced than the material in this book, with the thought being that the introduction here is adequate to provide an entrée into the more specialized and advanced information. This book, although a modeling book, does not make the leap to implementing either new or existing computer codes. Using codes to obtain collections of numbers and attractive graphics can be hazardous without understanding the principles incorporated into the code, the limitations of the code, the meaning and limitations of the numbers calculated, and ways to make a code particularly adapted to a setting of interest. Such activity should only be undertaken after forewarning and gaining an appreciation for the fact that the ability to hit the enter key such that a code executes to expected completion is a minute part of successful modeling. Exercising available codes to good effect is not an introductory element of environmental modeling, and writing new codes is also a process that often requires advanced computer science skills.

This book covers the thought processes for modeling, as well as some of the fundamental principles that enable one to model physical processes from a mechanistic perspective. Chapter 1 presents the perspective that modeling requires thinking rather than rote repetition of information. Chapter 2 supports the idea that data alone does not describe a system, but data in light of the context in which the data is collected can describe a system. Chapter 3 encourages examination of data to answer a question in light of a framework, but cautions that the examination is almost never exact so that sources of error must be appreciated and controlled. Chapter 4 stresses the importance of length and time scales in formulating a description of a problem of interest. An overview of the mechanisms that must be operative for a system to undergo change is presented. The mechanisms in this chapter are subsequently incorporated into statements of how systems change. Chapter 6 on dimensional analysis points to the importance of dimensional homogeneity in equation formulation and also illustrates the use of the Buckingham Pi theorem to aid in efficient description of system behavior.

Because algebraic, discrete differences or changes are intuitively easier to understand than differential changes, Chapter 7 formulates some discrete models and discusses their solution. In addition, the effects of numerical instability in solving the equations are demonstrated. Mathematically, Chapter 8 is a review of elements of calculus that a student should have encountered previously. However, the material is presented from the perspective of the relations being physical as well as mathematical statements. This approach is continued in Chapter 9 where integral theorems are introduced that enable equation forms to be rearranged and, in particular, facilitate the transition of the statement of an equation for a finite-sized volume to one that is infinitesimal. Chapters 10 through 13 develop the conservation equations for mass, chemical species, momentum, and energy as the fundamental principles that describe how systems must behave. Both point and integral forms of these equations are encountered. In Chapter 14, approaches are employed for modeling a system in a reduced number of spatial coordinates. As introduced in calculus courses, not considering variability in a coordinate direction usually is achieved by dropping the derivatives in that direction. Here, we emphasize that to ensure conservation properties are honored, variation in a coordinate direction is accomplished by integration over that coordinate. Included in this chapter are the shallow-water equations and the St. Venant equations for channel flow. In Chapter 15, we develop equations that are employed in modeling of porous media. The prime intent is to show that the relevant equations are based on the fundamental conservation principles formulated at a different scale. At this point, equations have been developed for a wide array of systems. Chapter 16 introduces the idea of solving these equations using a numerical method. The presentation is for a simple contaminant transport problem with the objective being to illustrate the thought processes behind numerical algorithms that can serve as a springboard to the study of more complex algorithms and processes. Chapter 17 revisits stability considerations for numerical solution of discrete equations. The objective is to illustrate that instabilities, which sometimes seem to be surprising mathematical artifacts, can be thought of as manifestations of improperly operating a system.

In all chapters, questions are inserted within the text and problems are provided at the end of the chapters. The questions are relatively simple and are inserted strategically at places in the text as aids in cementing down processes or concepts just introduced. In a sense, they are speed bumps that encourage a reader to make sure he or she understands what has been written before going on to the next concept. These questions also provide a basis for productive discussions. The problems at the end of the chapter are more comprehensive. They make use of material in the preceding chapter and also integrate material presented previously in the book. The problems do not seek regurgitation of material presented but ask the student to be creative in bringing the material to bear on a specific scenario. Answers to the problems are not necessarily unique; problems may require that some additional information be added so that the problem is completely defined. Work on the problems by groups rather than individuals can be beneficial in some cases, as different perspectives will enrich formulation of problem solutions.

This book is based in part on a course on environmental modeling offered through the Curriculum for the Environment and Ecology (CEE) at the University of North Carolina in Chapel Hill. The authors are grateful to those discerning people who have provided them with insights over the years on modeling and/or on education in general, especially George F. Pinder (University of Vermont), Cass T. Miller (University of North Carolina at Chapel Hill), Randall L. Kolar (University of Oklahoma), Stephen Whitaker (University of California at Davis), Mohamed S. B. H. Ghidaoui (Hong Kong University of Science and Technology), Monica Martinez-Canales (Intel), Herbie Lee (University of California, Santa Cruz), and Robert B. Gramacy (Virginia Tech). The outstanding students who have encountered this material in course offerings have added many insights and much enthusiasm. Their contributions in providing perspective on this material and in confirming its importance to their education has inspired some of the approaches employed here. We are grateful to Gregory Gangi, who encouraged such excellent students to engage with environmental modeling, and to Jaye E. Cable whose educational leadership fostered this course under the CEE umbrella. Two of the students who took the course, Elizabeth C. Christenson and Jillian L. Simmerman, deserve particular mention because their involvement, as well as subsequent contributions as teaching assistants, added greatly to the collaborative atmosphere of the course and to insights into the way material is structured and presented. We thank Matt Lloyd of Cambridge University Press for his unswerving support for this manuscript from the first time it was informally proposed through its completion. We greatly appreciate the efforts of Judith Shaw, who copy-edited the manuscript, and of Caroline Mowatt, who ushered this effort through to publication.

Over the years, many individuals have provided perspectives on the topics of this book through interactions with the authors in professional settings, books and manuscripts, informal conversations, and anonymous writings. Every effort has been made to acknowledge all sources, but the collection of perspectives through the years has resulted in the identity

of some sources fading from memory while their insights remain. We are grateful indeed to all parties who have informed us on elements of environmental modeling and regret any oversights in explicit recognition. We welcome comments and suggestions involving the use of this book and regret the shortcomings of our presentation.

The first author is ever grateful to and for Genny Gray who has provided generous, unflagging confidence in him throughout the years. Without her participation, the life of the first author would have been incomplete, and the life of the second author would have been impossible. The second author is grateful to Rob McGormley for his encouragement and support during the creation of this work. This collaboration has been a professional and personal activity that both authors recognize as a special privilege.