Index

C-adjoining, 242
C-factor, 211
maximal, 211

adapted basis, 153, 155, 177–179
approximating function, 9
Arithmetic Chaos Conjecture, 307
average width, 177, 178, 182

Bad, 7
badly approximable number, 126
badly approximable numbers, 7
badly approximable points, 31
Banach density, 190
Bernoulli shift, 193
best approximates, 5
best Sobolev constant, 164, 169, 171, 177
binary quadratic form, 296
Borel–Cantelli argument, 136, 142, 183
Borel–Cantelli Lemma, 11
Borel–Cantelli lemma for convergence, 12
Borel–Cantelli lemma for divergence, 12

Cantor-rich sets, 82
Cantor-winning sets, 85
Cassels’ problem, 73
characteristic factors, 216
circle rotation, 193
class number, 297
closed geodesic, 291
cohomological equation, 152, 155
conditional expectation, 200
connecting factor maps, 208
continued fraction, 294
continued fraction expansion, 5
continued fractions of badly approximable numbers, 7
continued fractions of quadratic irrationals, 8
convergence of non-conventional ergodic averages, 214, 247
of self-joinings under off-diagonal averaging, 214, 240
convergent, 5
coupling, 203
cutting sequence, 293

Dani correspondence, 279, 285
diagonal measure, 204
diagonal subgroup, 269
dimension function, 18
Diophantine approximation
inheritance, 282
simultaneous, 278
Diophantine approximation, 278
Dirichlet improvable, 278
\(\lambda\)-improvable, 278
principle, 278
Dirichlet theorem, 278
improvable, 283
Dirichlet’s Class Number Formula, 299
Dirichlet’s theorem, 2, 96
Index

Dirichlet’s theorem in $\mathbb{R}^n$, 27
discrepancy (modulo 1), 139
discriminant of a closed geodesic, 303
disintegration, 200
distal class, 234
distal system, 234
doubling measure, 13
Duffin–Schaeffer Conjecture, 17
Duffin–Schaeffer example, 15
Duke’s theorem, 289, 305
effective
  Gauss circle problem, 266
effective equidistribution, 137, 139
EKL theorem, 35
equidistribution, 258, 272
counting problems, 258
  Gauss circle problem, 258
large circles, 260, 272
ergodic, 281
  ergodic averages, 142, 157, 163, 167, 169, 172, 173, 177
  ergodic decomposition, 202, 218
  ergodic system, 199
Extremal manifolds, 60
  factor, 198
    generated by a factor map, 199
  Federer measure, 13
  filiform basis, 145–147, 153, 174
  filiform Lie group, 145
  filiform nilflow, 141, 145, 147
  filiform nilmanifold, 147
fractional linear action, 290
  fundamental discriminant, 300
  fundamental unit, 300, 301, 303
  Furstenberg correspondence principle, 195, 215
  Furstenberg–Katznelson theorem, 191, 197
G-space, 192
  G-system, 193
  Gallagher’s theorem, 35
  Gallagher’s zero-full law, 17
  Gauss circle problem, 258
  effective, 266
  Generalised Duffin–Schaeffer Conjecture, 26
  geodesic flow, 269
    path, 272
gerodesic flow, 290
  good points, 141, 142, 182
  Green operator, 156, 161
Hausdorff
distance, 280
  dimension, 19
  measure, 18
  height, 99
Heisenberg basis, 159, 161
  frame, 159, 161, 167, 169
  group, 144
  nilflows, 140, 145
  nilmanifold, 144
homogeneous dynamics, 270
  Dirichlet theorem, 280
  mixing, 270
Host–Kra–Ziegler Structure theorem, 237
Hurwitz’s theorem, 6
hyperbolic
green operator, 266
  geometry, 271
  plane, 267, 271
  boundary, 267
disc model, 271
  Riemannian metric, 267
idempotent class, 210
distal, 234
  join, 210
  Kronecker, 223
  partially invariant, 210
  pro-nil, 236
inheritance
  Diophantine approximation, 282
  injectivity radius, 161, 164, 177
  intertwining operator, 162, 175
  invariant distributions, 152, 161
  inverse limit, 209
  isometric extension, 233
    concretely, 233
  isometric factor, 223
  isometric system, 223
    concretely, 223
  structure of, 228
  isomorphism
    hyperbolic, 275
  Iwasawa decomposition, 275
Jarník’s theorem, 23
Jarník–Besicovitch theorem, 23
  joining, 203
Index

joining topology, 204
joint distribution of factors, 204

Khintchine’s theorem, 14
Khintchine’s theorem in higher dimensions, 32
Khintchine’s Transference Principle, 45
Khintchine–Groshev theorem, 43
Khintchine–Jarník theorem, 24
Khintchine–Sullivan theorem, 171
Kirillov’s theory, 152, 153
Kronecker class, 223
Kronecker factor, 223
Kronecker’s theorem, 98

lattice, 269
lim sup set, 10
Liouville number, 116
Liouville’s inequality, 105
Littlewood’s conjecture, 34
local quasi-independence on average, 13
Local–Global Conjecture, 312

Mahler compactness, 282
Mass Transference Principle, 25
metric
Riemannian, 271
Middle third Cantor set, 20
Minkowski theorem, 280
Minkowski’s theorem for convex bodies, 28
Minkowski’s theorem for systems of linear forms, 29
mixing, 270, 277
Möbius transformation, 268
modular surface, 290
moduli space, 159, 161
Multidimensional Multiple Recurrence Theorem, 194, 197, 212
Multiple Recurrence Theorem, 194

nilflows, 139
nilsystem, 236, 239
non-degenerate manifolds, 55
non-divergence quantitative, 282

off-diagonal invariance of limit joinings, 217
off-diagonal transformation, 213

pairwise independence, 12
Parametric geometry of numbers, 124
partially invariant class, 210

partially invariant factor, 199
path, 267
Pell equation, 300
Pigeonhole principle, 2, 278
Pleasant Extensions theorem, 240, 247
Poincaré Recurrence theorem, 194
pro-nil class, 236
pro-nil system, 236
psi-approximable number, 9
quantitative non-divergence, 282
quasi-independence on average, 13

relative product, 207
relatively independent joining, 207
renormalization flow, 159, 160, 169
renormalization group, 153, 174, 179
resonant points, 47
rho-cover, 18
Riemann zeta function, 266
Riemannian metric, 161, 164, 271
Roth’s theorem, 191, 230

s-dimensional Hausdorff measure, 19
Sated Extensions theorem, 245
Schmidt’s conjecture, 81
Schmidt’s games, 79
Schubfachprinzip, 278
self-joining, 203
Simplex lemma, 79
simply transitive, 268
simultaneously psi-well approximable points, 32
skew-shifts, 139, 143, 149
Sobolev norm, 266
Sobolev embedding theorem, 152, 163, 171, 178
Sobolev space, 140, 156, 164, 168, 172
Sobolev trace theorem, 164, 177–179
speed of a path, 267
symbolic dynamics, 296
system, 193
c-sated, 243
pleasant, 239
Szemerédi’s theorem, 191

tangent bundle, 267
unit, 269
tau-well approximable numbers, 18
Index

Taylor expansion, 262
Theorem of Cassels and Swinnerton-Dyer, 34
thin integer set, 312
Transference theorem, 106
transitive, 268
transverse Laplacian, 155
transverse Sobolev norms, 155
transverse Sobolev space, 178
Ubiquitous system, 48
Ubiquity statements, 49
unfolding, 275
uniform bounds, 136, 142
uniform distribution (modulo 1), 139
unipotent dynamics, 285
unitary representations, 153
upper half-plane, 267
Van der Waerden’s theorem, 190
Vinogradov symbols, 15
volume form, 260
weak mixing, 222, 224
Weyl Equidistribution theorem, 229
Weyl exponent, 138
Weyl sums, 137–141, 143, 145, 148, 150, 159, 182
winning sets, 79
Wirsing’s conjecture, 109
Zaremba’s conjecture, 309
zero-full law, 14
zero-one law, 14