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978-1-107-53894-8 - Introduction to Dynamical Systems
Michael Brin and Garrett Stuck
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Introduction to Dynamical Systems

This book provides a broad introduction to the subject of dynamical systems, suitable for a one- or two-semester graduate course. In the first chapter, the authors introduce over a dozen examples, and then use these examples throughout the book to motivate and clarify the development of the theory. Topics include topological dynamics, symbolic dynamics, ergodic theory, hyperbolic dynamics, one-dimensional dynamics, complex dynamics, and measure-theoretic entropy. The authors top off the presentation with some beautiful and remarkable applications of dynamical systems to such areas as number theory, data storage, and Internet search engines.

This book grew out of lecture notes from the graduate dynamical systems course at the University of Maryland, College Park, and reflects not only the tastes of the authors, but also to some extent the collective opinion of the Dynamics Group at the University of Maryland, which includes experts in virtually every major area of dynamical systems.

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GARRETT STUCK



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To Eugenia, Pamela, Sergey, Sam, Jonathan, and Catherine for
their patience and support. And in memory of our dear
colleague Dan Rudolph.

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Introduction

The purpose of this book is to provide a broad and general introduction to the subject of dynamical systems, suitable for a one- or two-semester graduate course. We introduce the principal themes of dynamical systems both through examples and by explaining and proving fundamental and accessible results. We make no attempt to be exhaustive in our treatment of any particular area.

This book grew out of lecture notes from the graduate dynamical systems course at the University of Maryland, College Park. The choice of topics reflects not only the tastes of the authors, but also to a large extent the collective opinion of the Dynamics Group at the University of Maryland, which includes experts in virtually every major area of dynamical systems.

Early versions of this book have been used by several instructors at Maryland, the University of Bonn, and Pennsylvania State University. Experience has shown that, with minor omissions, the first five chapters of the book can be covered in a one-semester course. Instructors who wish to cover a different set of topics may safely omit some of the sections at the end of Chapter 1, §§2.7–2.8, §§3.5–3.8, and §§4.8–4.12, and then choose from topics in later chapters. Examples from Chapter 1 are used throughout the book. Chapter 6 depends on Chapter 5, but the other chapters are essentially independent. Every section ends with exercises (starred exercises are the most difficult).

The exposition of most of the concepts and results in this book have been refined over the years by various authors. Since most of these ideas have appeared so often and in so many variants in the literature, we have not attempted to identify the original sources. In many cases, we followed the written exposition from specific sources listed in the bibliography. These sources cover particular topics in greater depth than we do here, and we recommend them for further reading. We also benefited from the advice and

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