1

MECHANICS

1.1 Introduction

Mechanics is the branch of science, which deals with forces. The entire classical mechanics is based on the Newton's laws of motion. Engineering mechanics is the application part of Newtonian mechanics. On one hand, it proposes suitable and simplified mathematical models for complicated situations, and, on the other hand, it develops the art and skill for efficient solutions.

Mechanics lies at the core of engineering analysis. Basic principles of mechanics are used in the study of subjects such as vibrations, strength of materials, stability of structures, robotics, rockets, space-craft, engine performance, fluid flow, electrical machines, etc.

1.2 Idealizations in mechanics

Some important idealizations in mechanics are defined and discussed as follows:

(a) Particle: A particle is considered as an object that has no size but mass. This is one of the widely used definitions in mechanics. In the true sense, nothing falls in this category. In practical sense, a body whose dimensions can be neglected in studying its motion or equilibrium is treated as a particle. For example, while studying the motion of moon around the earth, both the earth and the moon are treated as particles with mass concentrated at their center. Whenever the size of the body is unimportant in the mathematical formulation of the problem, the body is treated as a particle.

(b) Rigid body: A rigid body is the most commonly used concept in engineering mechanics. It is defined as the body, which does not deform under the application of forces. In the real sense, everybody undergoes deformation and the size of the body changes. This change, in most of the cases, is insignificant compared to the size of the body. Omission of this deformation does not affect the solution. In general, solid bodies are assumed to be rigid bodies, if the deformation is negligible compared to the size of the body.

(c) Space: The geometric region occupied by the body is termed as space. The position of the body is specified with reference to a co-ordinate system. The concepts of point, direction and length are required for measurement, and the location is space. A point is the indication of a location in space. A point has no size. Length is the concept for describing the size of the body by comparison with a second body of known size. For two-dimensional problems, two independent coordinates are needed.

(d) Time: Time is related to concepts like before and after and simultaneous occurrence of events. In other words, it is used for ordering or sequencing of events. It does not enter directly into the problems of statics, but it is one of the very important quantities for the problems of dynamics. Any particle or body cannot occupy two positions simultaneously or at the same point of time. Time will be involved during the change of position of the particle or the body.

(e) Matter: A substance from which bodies are made up of is called matter. It consists of atoms and molecules, and occupies space, volume, mass, etc.

(f) Mass: Mass may be defined as the quantity of matter in a body. Matter cannot exist without mass. It is the property of matter or body on which gravitational force acts and offers resistance to change of motion. Unit of mass in SI system is kilogram.

(g) Continuum: Continuum is defined as the continuous distribution of matter with no voids. Each body is made of atoms and molecules. At molecular level, there is free space between them. In spite of this fact, matter is assumed to be continuously distributed. The concept of continuum is used to study the measurable behaviours like mass, density, volume, etc.

(h) Force: Force is the physical action that tries to change the state of rest or motion of a body on which it is applied. It is a physical quantity which brings out the change in the velocity of moving bodies. It is a vector quantity and is completely described or characterized by its magnitude, direction and point of application.

The concept of force is related to the mass of the body in the Newton's second law of motion. It is independently related to the gravitational force of attraction between two bodies.

(i) Weight: The gravitational force of the earth on a body is referred to as weight. It is not a property of matter. It depends on the location of the body relative to the earth's center. The weight of the same body on different planets will be different. Unless otherwise stated, the weight refers to the gravitational force of the earth. The unit of weight in SI system is Newton (N).

(j) Scalar: A physical quantity, which is completely defined by its magnitude is called scalar. Scalar quantities can be added arithmetically. Mass, length, time, volume, energy, temperature, pressure, etc., are the few examples of scalar.

(k) Vector: A physical quantity which requires magnitude and direction for its description and follows the law of parallelogram of vector addition is called vector.

1.3 Essence of vector algebra

Introduction: All the physical quantities fall in one of the following two categories.
(a) Scalar and (b) Vector

1.3.1 Scalar quantity: Scalar quantities are fully described by their magnitude and unit.

1.3.2 Vector quantity: A physical quantity, which requires magnitude, unit and direction for its complete description, and follows the law of parallelogram of vector addition, is called vector quantity. Position, displacement, acceleration, force, impulse, momentum, etc., are some of the vector quantities.

Electric current requires both magnitude and direction for its description, but does not follow the law of vector addition. So, it is not a vector.

Symbolic representation of vector: Vector is symbolically represented by a bold face letter or an arrow head (→) over the letter representing the vector. For example, velocity vector is represented as **V** or $\vec{V}$. And the magnitude of the vector is represented by the letter representing the vector. Therefore, the magnitude of the velocity vector, **V** or $\vec{V}$ will be written as V or $|\vec{V}|$.

Geometrical representation of vector: A vector is geometrically represented by a straight line segment with an arrow head. Length of the line represents magnitude, direction of the line represents the direction of vector.

For example, a vector of 10 units in north east direction is shown in Figure 1.1. If a scale of 1 cm = 5 units is selected; the length of the line will be 2 cm.

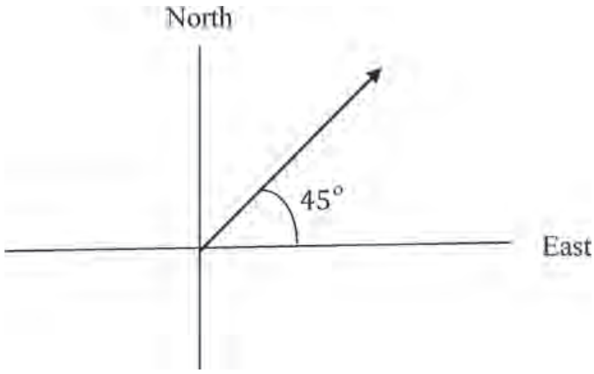

Figure 1.1

1.3.3 Unit Vector: Vector with magnitude of unit, 1, is called unit vector. To simplify the mathematical calculations on vectors, the system of unit vectors is selected according to well thought schemes. One such system of unit vectors is represented by *i, j* and *k*.

Unit vector, *i,* is parallel to x-axis and is in positive x direction.
Unit vector, *j*, is parallel to y-axis and is in positive y direction.
Unit vector, *k,* is parallel to z-axis and is in positive z direction.
On x-y plane, unit vectors *i* and *j* have been shown in Figure 1.2a.

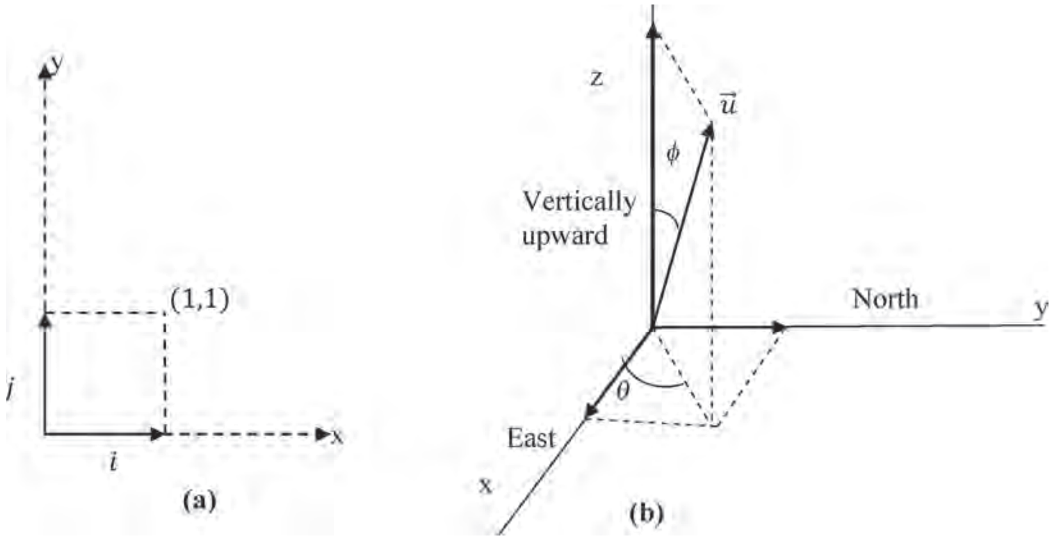

Figure 1.2: (a) Unit vector in x-y plane (b) unit vector in space

Any direction in space may be modelled in the following way:

Represent east direction by *i*, north direction by *j* and vertically upward direction by *k*. This selection is in agreement with right handed screw rule.

Suppose a unit vector makes an angle Ø from z-axis and its projection on x-y plane makes an angle of θ from x-axis as shown in Figure. 1.2. The unit vector is

$$\vec{u} = \cos Øk + \sin Ø(\cos\theta\, i + \sin\theta\, j)$$

$$\vec{u} = \cos Øk + \sin Ø \cos\theta\, i + \sin Ø \sin\theta\, j \tag{1.1}$$

It can be seen that $|\vec{u}| = 1$.

If Ø = 90°, the unit vector lies in x-y plane. The unit vector makes an angle of θ from x-axis. The unit vector in x-y plane is

$$\vec{u} = \cos\theta i + \sin\theta\, j \tag{1.2}$$

Example 1.1: A force of 70 N is inclined at an angle of 20° from vertical and its projection on horizontal plane makes an angle of 30° from x-axis. Express the force vector in terms of *i*, *j* and *k*.

Solution:

$$\vec{F} = \left(70\cos 20k + 70\sin 20\left(\cos 30i + \sin 30\, j\right)\right)\ \text{N}$$

$$\vec{F} = \left(20.73i + 11.97\, j + 65.78k\right)\ \text{N}$$

1.3.4 Simultaneous action of vectors or law of vector addition:

Description of the following situations will be helpful in understanding the importance of vector addition.

Situation 1: Suppose a swimmer can swim with a speed of 5 m/s in any direction in still water. A river water flows with a speed of 2 m/s. If the swimmer swims in a direction perpendicular to the velocity of the flow of river, he will not be able to reach the other bank of the river, just in front of him. Where will the swimmer reach? Simple arithmetic is unable to answer this question. This can be answered with the help of vector algebra, which is based upon the law of parallelogram of vector addition.

Situation 2: Suppose a single force of 10 N is applied on a mass of 1 kg in east direction, then according to the Newton's second law of motion, the body will accelerate with an acceleration of 10 m/s^2 in the east direction. Similarly, if a single force of 5 N is applied on the same body of 1 kg in north direction, the body will accelerate with an acceleration of 5 m/s^2 in the north direction. Now suppose two forces, one of 10 N in east direction and the other of 5 N in north direction, are simultaneously applied, then what will be the magnitude and the direction of the acceleration? Simple arithmetic is unable to answer this question. This can be answered with the help of vector algebra, which is based upon the law of parallelogram of vector addition. Fortunately, single rule is applicable to all the physical quantities, which are vectors. Vector Algebra consists of a rule of addition and two multiplications.

1.3.5 Law of parallelogram of vector addition

If two vectors $\vec{P}$ and $\vec{Q}$ are represented by the two sides of a parallelogram, the diagonal of the parallelogram represents the resultant of the two vectors.

The magnitude and direction of the resultant can be found in different ways, as follows:

(a) Graphical method: By drawing the parallelogram and measuring the diagonal.
(b) Analytical method:

- **(i) Method of Projection**
- **(ii) Vector approach**

In Figure 1.3, vectors $\vec{P}$ and $\vec{Q}$ have been represented geometrically by lines OA and OB. Parallelogram OACB has been completed by drawing a line from point A parallel to OB and a line from point B parallel to OA. They intersect at point C. Diagonal OC represents the magnitude of the resultant of vectors. Direction of the resultant is from O to C.

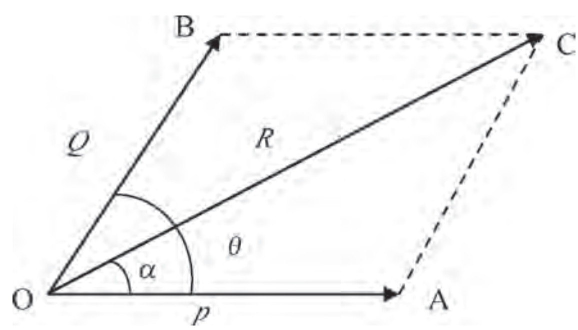

Figure 1.3

Construction of the parallelogram and measurement of α, and OC will give the answer to the problem.

Mathematically equivalent statement of the law of parallelogram of vector addition is given by the following equation:

$$\vec{R} = \vec{P} + \vec{Q}$$

The length of the diagonal of the parallelogram and the angle between the side and diagonal can also be obtained analytically.

Method of projection: Before presenting the theorem on projection, we first define the projection of a point on a line and a line segment on another line.

Projection of a point on a line: The foot of perpendicular drawn from a point on a line is called projection of the point on the line. In Figure 1.4 (a) P' is the projection of point P on line AB.

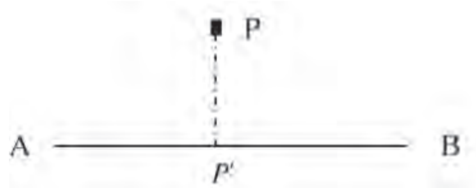

Figure 1.4a

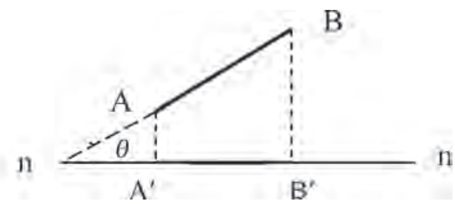

Figure 1.4b

Projection of a line segment on another line: If we drop perpendiculars from the ends A and B of line segment AB on another line n-n (Figure 1.4(b)), the line segment between the feet of perpendiculars A′B′ is called projection of line segment AB on *n-n*.

From Figure 1.4(b), we get

$$A'B' = AB\cos\theta$$

Rule: The projection of resultant of any number of vectors on any line is equal to the algebraic sum of projection of its components.

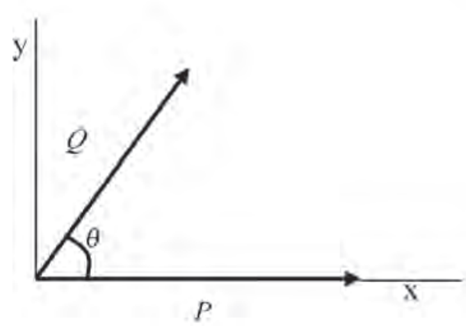

Figure 1.5a

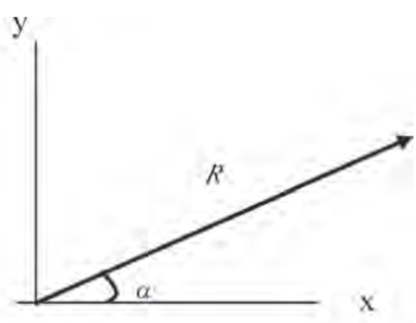

Figure 1.5b

In Figure1.5a, vectors P and Q are shown. Their resultant R is shown in Figure 1.5b.

Projection of P on x-axis is P
Projection of Q on x-axis is $Q\cos\theta$
The sum of projection of P and Q on x-axis is $(P + Q\cos\theta)$
Projection of P on y-axis is $(P\sin\theta = 0)$
Projection of Q on y-axis is $Q\sin\theta$
The sum of projections of P and Q on y-axis $Q\sin\theta$
Projection of R on x-axis is $R\cos\alpha$
Projection of R on y-axis is $R\sin\alpha$
Using the rule given we get:

$$R\cos\alpha = (P + Q\cos\theta) \tag{i}$$

$$R\sin\alpha = Q\sin\theta \tag{ii}$$

Eliminating α from equations (i) and (ii), we get

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta \tag{1.3}$$

And dividing the corresponding sides of equations (ii) and (ii), we get

$$\tan\alpha = \frac{Q\sin\theta}{(P + Q\cos\theta)} \tag{1.4}$$

Note: The method of projection is very convenient for numerical analysis in case of more than two vectors.

1.3.6 Analytical Methods:

Method 1: Determination of angle α and length of diagonal OC by analyzing the geometry of the parallelogram as shown in Figure 1.3

Magnitude of the resultant R and the angle of inclination α from the direction of P
From Δ OCA,
Applying the sine rule, we get

$$\frac{OA}{\sin(\theta - \alpha)} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin(180^o - \theta)}$$

Since OA, AC and DC are proportional to P, Q and R, the above equation becomes

$$\frac{P}{\sin(\theta - \alpha)} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\theta}$$

Above equality is equal to the following two equations

$$\frac{P}{\sin(\theta-\alpha)} = \frac{Q}{\sin\alpha} \qquad \text{(i)}$$

$$\frac{Q}{\sin\alpha} = \frac{R}{\sin\theta} \qquad \text{(ii)}$$

From equation (i),

$$Q[\sin\theta\cos\alpha - \cos\theta\sin\alpha] = P\sin\alpha$$

Collecting the coefficients of $\sin\alpha$ and $\cos\alpha$, we get

$$(Q\sin\theta)\cos\alpha = (P + Q\cos\theta)\sin\alpha$$

or, $\tan\alpha = \dfrac{Q\sin\theta}{(P+Q\cos\theta)}$ Same as the result obtained in equation 1.4

From equation (ii),

$$R = \frac{Q\sin\theta}{\sin\alpha} = Q\sin\theta\operatorname{cosec}\alpha \qquad \text{(iii)}$$

Now, $\operatorname{cosec}\alpha = \sqrt{1+\cot^2\alpha}$

Substituting the value of cot α from the equation (iii), we get

$$\operatorname{cosec}\alpha = \frac{\sqrt{P^2+Q^2+2PQ\cos\theta}}{Q\sin\theta} \qquad \text{(iv)}$$

Substituting the value of cosec α from the equation (iv) in the equation (iii), we get

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$ Same as the result obtained in equation 1.3.

Particular Cases:

Case 1: When P and Q are collinear to each other;

Angle between P and Q is zero. So, $\theta = 0$ and cos 0 = 1

From the equation 1.4, $\tan\alpha = 0$ or $\alpha = 0$. The resultant is along vector P.

From the equation 1.3, $R = \sqrt{P^2+Q^2+2PQ}$

or, $R = P + Q$

Case 2: When P and Q are perpendicular to each other (Figure 1.6a);

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Excerpt
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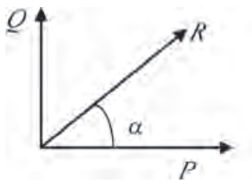

Figure 1.6a

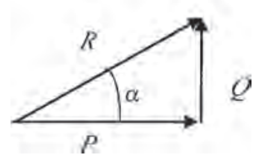

Figure 1.6b

Angle between P and Q is 90^o. So, $\theta = 90^o$ and $\cos\theta = 0$.

The equation (1.3) gives $$R = \sqrt{P^2 + Q^2} \quad (1.5a)$$

The equation (1.4) gives $$\tan\alpha = \frac{Q}{P} \quad (1.5b)$$

The angle between P and R is α. The above results of the equation 1.5a and the equation 1.5b are represented in the Figure 1.6b. The resultant of vectors of magnitudes P and Q is R. The angle between P and R is α. By inversing the above results, we get

$$P = R\cos\alpha \quad (1.6a)$$

$$Q = R\sin\alpha \quad (1.6b)$$

Putting into words, two rectangular components of R are $R\cos\alpha$ and $R\sin\alpha$.

Resolved Components: The resultant of two vectors is unique. However, a vector can be resolved in two components in many ways.

If a vector is resolved in two parts, which are perpendicular to each other, the two components are called resolved parts or components. The resolved components of $\vec{R}$ are $R\cos\alpha$ and $R\sin\alpha$ (Figure. 1.6b).

Method 2: Determination of R and α with the use of unit vectors i and j

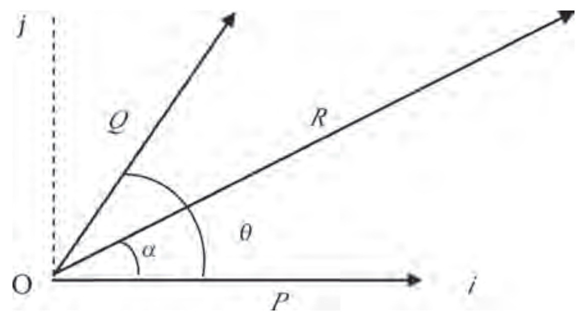

Figure 1.7

Imagine a unit vector i along P and unit vector j perpendicular to P (Figure 1.7);

$$\vec{P} = Pi$$

$$\vec{Q} = Q(\cos\theta i + \sin\theta j)$$

$$\vec{R} = R(\cos\alpha i + \sin\alpha j)$$

$$\vec{R} = \vec{P} + \vec{Q}$$

or, $$R(\cos\alpha\, i + \sin\alpha\, j) = (P + Q\cos\theta)i + Q\sin\theta\, j$$

Comparing the coefficients of *i* we get

$$R\cos\alpha = P + Q\cos\theta \quad \text{(i)}$$

Similarly,

$$R\sin\alpha = Q\sin\theta \quad \text{(ii)}$$

From equation (i) and equation (ii), we get

or, $$\tan\alpha = \frac{Q\sin\theta}{(P + Q\cos\theta)}$$

and, $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Same as the results obtained in the equations 1.3 and 1.4.

Use of unit vectors simplifies the calculations, when the number of vectors in the problem is three or more.

Example 1.2: Forces 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2 kN, respectively act at one of the angular points of a regular hexagon towards other angular points as shown in Figure 1.8. Determine the magnitude and the direction of the resultant force.

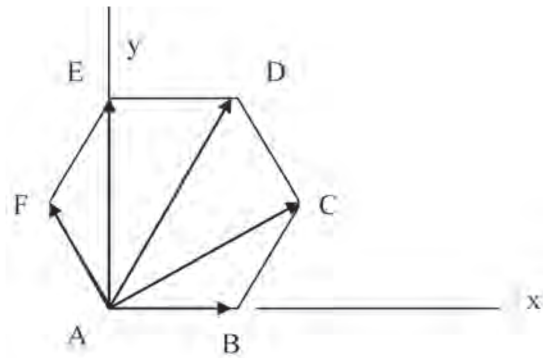

Figure 1.8