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Atmospheric phenomena and their study

Earth's atmosphere is a shallow fluid held by gravity to the surface of a spinning sphere whose surface is heated by electromagnetic radiation from the sun. Roughly two thirds of the sphere is covered by water, which continuously undergoes evaporation, condensation, freezing, thawing and sublimation. There is continuous turbulent transport of water vapour and heat between atmosphere and surface. At global scale, the atmosphere is in continuous motion, driven by a relative excess of heating in equatorial regions relative to higher latitudes. The net effect of this motion is a latitudinal redistribution of heat, either directly or by a net transport of moist air from the tropics to higher latitudes where it condenses and falls as precipitation.

Atmospheric large scale motion results in a cascade of energy to smaller scales, producing a complex palimpsest of motion of various types, and at a wide range of scales from global (tens of thousands of kilometres) to a microscale on the order of millimetres. In spatial terms, these motions include some that are quasi two-dimensional, some that are fully three-dimensional, some that are strongly wave-like, and some that are appropriately described as chaotic. Temporally, the motions have time scales of variability that range from astronomically forced variations over tens of thousands of years to turbulent fluctuations of a few seconds in duration, and more recently, decade scale temporal trends driven by human industrial activities. In addition to the purely dynamical phenomena I have just described, the atmosphere includes phenomena whose dynamics are powerfully influenced by thermodynamic processes (such as cloud and precipitation processes) and a wide range of fascinating atmospheric optical phenomena (such as rainbows and circumsolar haloes). This book will primarily concentrate on atmospheric dynamical phenomena, whose time and space scales are graphically shown in Figure 1.1. These phenomena are conventionally grouped into *micro*, *meso* and *macro* scales, and most analyses of atmospheric phenomena focus only on one of the 'scales'. This narrowing of focus has become so sharp that most atmospheric scientists will label themselves according to the 'scale' they study. This three-way scale based classification is more than mere labelling, as it has a profound dynamical basis that will be covered in some detail through this book. At the most basic level, it is in fact a separation of scales, with the implicit assumption that phenomena at one 'scale' operate approximately independently of phenomena at larger and smaller scales. This separation is justified by an analysis of temporal variability in which it is shown that the larger scale phenomena change so slowly that they can be considered as 'frozen' boundary conditions to the phenomena of interest. Similarly, smaller scale phenomena are shown to be 'relaxed' in the sense that they execute many fluctuations during the lifetime of the phenomena of interest, and only their overall (averaged) effect need be considered. In the best of circumstances, it can be shown that a given 'scale' can be treated using an approximation to the full governing equations, and that details of the approximation implicitly or explicitly limit the scales of applicability of the approximate equations.

Space and time scales for atmospheric phenomena included in Figure 1.1 are derived from observations, and the figure is no more than a compact and effective representation of a scale based classification. As will be shown later, there exists an alternative approach to scale definition which employs observed time and space scales and the fundamental equations of atmospheric dynamics. This approach lifts the ideas of scale from a descriptive mode to an analytical mode. There exists yet a third approach to scale analysis that is based on the even more fundamental considerations that lie behind the ideas of dimensional analysis and the *Buckingham pi theorem*, which is presented in the Appendix.

Atmospheric modelling has two fundamental objectives: developing understanding and producing forecasts. The former objective is common to all sciences, and in our specific case involves an analysis of the workings of a model in order to understand the balance of impinging forces or forcings. Obviously, before the force balance analysis is performed, it is essential to establish that the model captures faithfully

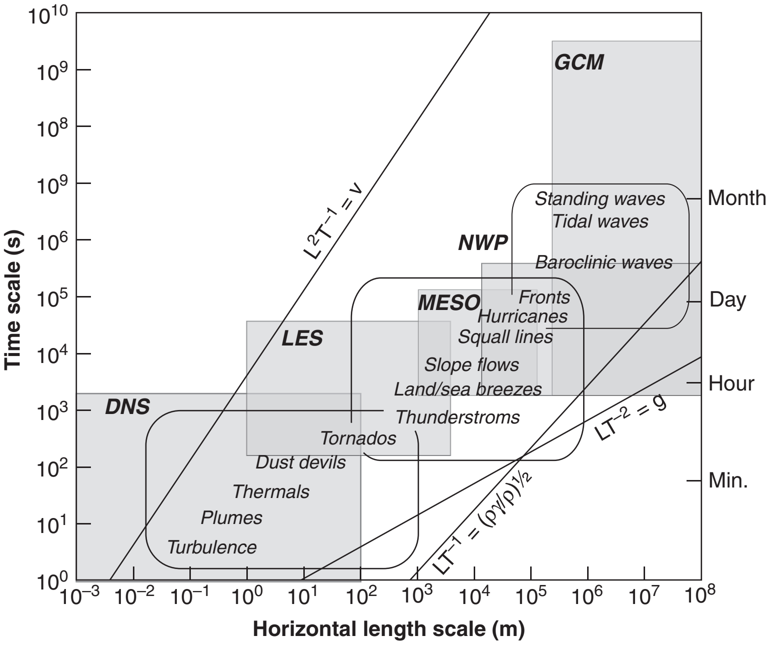

Figure 1.1 Time and length scales of atmospheric dynamical phenomena. The diagonal lines indicate excluded scale ranges which contain phenomena not conventionally treated as part of the atmosphere. In the upper left sector are phenomena in which viscosity dominates, in the lower right sector are phenomena whose characteristic speed is greater than that of sound, as well as phenomena whose acceleration is greater than that due to gravity. The hierarchy of atmospheric numerical models is superposed on the scale classification of phenomena. The models are: Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), Mesoscale (MESO), Numerical Weather Prediction (NWP) and General Circulation Model or Global Climate Model (GCM).

the observed behaviour of the phenomenon. This can only be achieved by comparison of model output with observations of the phenomenon being studied. There is thus no purely theoretical atmospheric modelling, as all modelling is informed by observations, and observations (at least in the last few decades) are based on a theoretical understanding of the phenomena being observed. Weather forecasting is the single most active and important application of atmospheric science, and is no more than a projection forward into time of the space-time variation of atmospheric variables. Because of the importance to society of weather forecasts, quality control consisting of an evaluation of forecast conditions against actually occurring conditions is continuously carried out. This evaluation is again based on observations of the atmosphere. It should thus be evident that atmospheric modelling and observation are intimately linked to each other.

1.1 Models as scientific tools

The term *model* has grown in scientific use in the last few decades. It is used to signify an *abstract analogue* in all sciences, though the levels of abstraction and type of analogue are often specific to particular sciences. It is worthwhile to examine the various uses of the term as it occurs in atmospheric science before setting out to understand atmospheric modelling. In very broad terms, atmospheric models can be Conceptual Models, Analogue Models, Physical Scale Models, Analytical (Mathematical) Models or Numerical Models. These five categories of model are simply different analogues of the actual atmospheric phenomena that are being modelled.

Conceptual models could be called *intuitive models* and are approaches to the understanding of a phenomenon based on the intuition of the scientist. This intuition is developed through education, training and practice of science, and is common to all scientific fields, but the model details will be field specific. The experimental particle physicist will have developed an intuition for the interaction of high energy sub-nuclear particles; an ecologist will have developed an intuitive understanding of the ways organisms or populations interact with each other and their environment; an organic chemist will have developed an intuitive model for chemical reaction mechanisms; an atmospheric scientist will have developed a conceptual model for the evolution of a mid-latitude cyclonic storm, and so on. In many ways, the conceptual models in all fields are developed through the use of the other types of models described here.

Analogue models are employed when one has developed an understanding of a phenomenon in a particular context, and uses that understanding to describe the behaviour of an analogous or similar phenomenon in a different context. The contextual difference may be simply a matter of location or time, or could be rather more profound. An atmospheric example of this is the development of an understanding of the great red spot on Jupiter by analogy with mid-latitude anticyclones in Earth's atmosphere.

Physical scale models are not the primary focus of this book, but a brief explanation of what they involve is needed for completeness. In physical scale models, some or all of the important variables are scaled, either upward or (as is generally the case) downward, in order to overcome difficulties in making observations on the full-scale prototype, or to allow manipulation of a variable that is generally not controllable in the full-scale prototype. There are three major questions in physical scale modelling: technical details needed to construct the scale model; scaling arguments which ensure that the scale model is operating in the same dynamical regime as the full-scale prototype; evaluation of scale model results against the full-scale prototype. Common examples of physical scale models of the atmosphere are wind tunnels (often used to study wind loading on structures, or the dispersion of pollutants around buildings and topographical features) or flow tanks (in which flowing water is used to study atmospheric flow over terrain). A major technical difficulty is incorporating the effects of Earth's rotation in atmospheric scale models. In physical scale models, the scaled version of the prototype is the analogue, and the abstraction is assumptions and approximations that justify the correctness of the scaling.

Analytical (mathematical) models are the primary focus of this book, and their nature, use and meaning will be discussed at some length. In these models, the analogue employed is that measurable physical quantities are represented by variables in a set of mathematical equations, coupled with the assumption that the way the variables in the equation behave, individually and collectively, exactly matches the way the corresponding physical quantities behave. That this is possible can be viewed as either an unfathomable mystery (and therefore akin to an axiom), or simply as the nature of applied mathematics (and therefore no mystery at all). The level of abstraction in an analytical model can generally be identified as the set of assumptions and/or approximations needed to render the complete equations soluble, or at least tractable. This matter is of central importance in analytical modelling of atmospheric phenomena, and will be shown to be closely linked to the definition of scales of phenomena illustrated in Figure 1.1.

Numerical models are numerical implementations of mathematical models. In principle they are no different than analytical mathematical models, and in some cases are simply numerical extensions to analytical models. In practice, numerical models are very different from analytical models because of the nature of the 'solution' they constitute. Rather than the solution being a closed form function, the 'solution' is a large volume of numerical values for all variables over the solution domain. Numerical models are common research and forecasting tools in atmospheric science because the governing equations contain very strong non-linearities that make analytical advances difficult, if not impossible. It should not be a surprise that grid definition, discretization and approximation techniques and control of roundoff errors are enormously complicated and technical questions in atmospheric numerical modelling. In parallel with the classification of atmospheric phenomena into the three scales in Figure 1.1, atmospheric numerical models exist for application at defined scales. Direct Numerical Simulation (DNS) models are used to study fine scale atmospheric motion at the lower left corner of Figure 1.1. Because of computational demands, these models are only employed for study domains up to a few hundred metres in vertical and horizontal extent. Turbulent processes in the lower atmosphere are studied using Large Eddy Simulation (LES) models. Both DNS and LES models are volume averaged implementations of the full equations, and so produce time varying fields. LES models are averaged so as to just resolve the largest energy-carrying turbulent structures (eddies). Mesoscale atmospheric numerical models are based on ensemble averaged equations which have been simplified using the Boussinesq approximation to the full equations. Their output is generally as hourly averages over grid resolutions that span the mesoscale, whose phenomena they are used to study and forecast. All weather forecasts, worldwide, are based on atmospheric numerical models designed to capture atmospheric phenomena at the mesoscale and macroscale. These models are run continuously by national weather forecast agencies, with model output being electronically distributed to regional weather offices for interpretation and the preparation of local forecasts. While the models run continuously, meteorological observations are available on a standard observational cycle. These observations (including satellite derived data) are then ingested into the model run-stream in a process known as 4-D data assimilation. The most important examples of large scale atmospheric numerical models are the General Circulation Models (GCMs) used to study global climates over time scales of decades to centuries. Apart from operational weather forecast models, most atmospheric numerical models exist in the research realm and are often available for free download.

This book will focus primarily on analytical models of atmospheric phenomena, but will wherever possible refer to advances that are possible using numerical models. The technical details of atmospheric numerical models will not be covered. There will be no further mention made of atmospheric scale modelling.

The existence of often dominant non-linear processes in the atmosphere means that predictability of atmospheric phenomena must be a vexing question. Over very short times, the dominant processes are part of atmospheric turbulence, and so are essentially unpredictable in detail. Fortunately for weather forecasting professionals, the atmosphere has reasonable predictability over times of three to five days, but very little predictability much beyond that up to seasonal scales when predictability increases. GCMs are able to predict the consequences of various CO_2 emission scenarios with accuracy greater than is possible for the associated human influences and responses to global warming. This varying predictability is simply another aspect of the scale dependence of all atmospheric phenomena.

1.2 Forces in a rotating frame of reference

The atmosphere is bound to, and rotates with, Earth. We observe and analyse the atmosphere relative to this rotating system. It is therefore natural and convenient that the equations of atmospheric motion are stated in this non-inertial frame of reference.

Confining ourselves for the moment to two dimensions, and considering absolute (inertial) and relative (rotating) frames of reference, an absolute velocity vector **U** is given by:

$$\mathbf{U} = \frac{dX}{dt}\mathbf{I} + \frac{dY}{dt}\mathbf{J},$$

where X and Y are coordinates of a point in a fixed frame of reference, and **I** and **J** are unit vectors in that frame. The velocity **U** will have components

$$U = u - \Omega y, \quad V = v + \Omega x$$

in the rotating frame, where Ω is the angular rotation rate of the relative frame of reference[1] and u and v are components of the same velocity

[1] $\Omega = 2\pi/86\,400 = 7.272 \times 10^{-5}\ \mathrm{s}^{-1}$.

in the relative frame. These two equations define absolute and relative velocities. Taking a second time derivative of the position vector yields an acceleration vector **A**, which has components:

$$A = a - 2\Omega v - \Omega^2, \quad B = b + 2\Omega u + \Omega^2$$

where a and b are components of the same acceleration in the relative frame. These two equations define absolute and relative accelerations. These equations show that relative acceleration consists of absolute acceleration and two new terms due to the rotating coordinates. The first new term (proportional to Ω) is called the Coriolis acceleration, while the second (proportional to Ω^2) is called the centripetal acceleration. Extension to three dimensions is straightforward since the third axis (that of the rotation) is common to both systems and, in vector form,

$$\mathbf{U} = \mathbf{u} + \mathbf{\Omega} \times \mathbf{r}$$
$$\mathbf{A} = \mathbf{a} + 2\mathbf{\Omega} \times \mathbf{u} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}),$$

where $\mathbf{r}$ is the radius vector from Earth's centre, and $\mathbf{\Omega} = \Omega\mathbf{r}$ is the Earth rotation rate vector. When the additional two accelerations appear in Newton's second law, they are placed on the right side of the equation and interpreted as virtual forces. As must be clear from the fact that loose matter at Earth's surface does not go flying out into space, gravity overwhelms the centrifugal force (identified with the centripetal acceleration). The centrifugal force does result in Earth having an oblate shape,[2] with the consequence that centrifugal and gravitational forces can be combined into a predominantly gravity component, leaving only the Coriolis force to be accounted for.

Since our analyses are going to be conducted in a local Cartesian framework (in which we replace angular coordinates of longitude/latitude with x/y linear coordinates), we will have to express the Coriolis force in that framework, as depicted in Figure 1.2.

In these coordinates, Earth's rotation vector is:

$$\mathbf{\Omega} = \Omega \cos\phi \,\mathbf{j} + \Omega \sin\phi \,\mathbf{k}.$$

The absolute acceleration

$$\frac{d\mathbf{u}}{dt} + 2\mathbf{\Omega} \times \mathbf{u},$$

[2] Earth's polar radius is about 6357 km, compared with the equatorial radius of 6378 km.

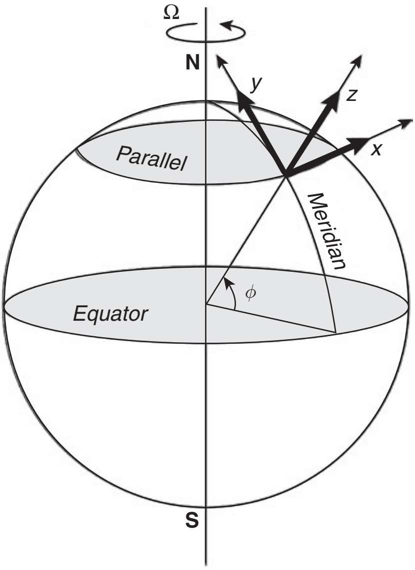

Figure 1.2 A local, Cartesian coordinate system on a spherical Earth: x is East–West, y is North–South, z is vertical and ϕ is the latitude in degrees.

has components:

$$x: \qquad \frac{du}{dt} + f_R w - f v$$

$$y: \qquad \frac{dv}{dt} + f u$$

$$z: \qquad \frac{dw}{dt} - f_R u.$$

We call $f = 2\Omega \sin\phi$ and $f_R = 2\Omega \cos\phi$ the Coriolis and reciprocal Coriolis parameters, respectively. The parameter f is positive in the Northern Hemisphere, zero at the equator and negative in the Southern Hemisphere, whereas f_R is positive everywhere.

1.3 Governing equations

It is worthwhile briefly to consider forces that cause acceleration in fluids since we will be developing an equation for the conservation of momentum of a fluid parcel by a generalization of Newton's second law.

Pressure gradient force

We are all aware that the force we exert on a bicycle pump results in an acceleration of air out of the pump. The force driving this acceleration is due to the gradient of pressure between the body of the pump and the air outside. The pressure gradient force is one of the forces that must be considered when analysing atmospheric motion. This force acts from high pressure to low pressure – down the pressure gradient – and is linear in the gradient itself. The pressure gradient force per unit mass $\mathbf{f}_{PG}$ expressed in kinematic terms is thus:

$$\mathbf{f}_{PG} = -\frac{1}{\rho}\nabla p,$$

where p is the pressure field and ρ is the density of air.[3]

Force of gravitation

The force of gravity accelerates air parcels towards the centre of Earth. Atmospheric density decreases geometrically with altitude, and at 100 km above the Earth surface is one millionth of its surface value. Since this height is much smaller than the Earth radius, we can consider the acceleration due to gravity to be constant throughout the atmosphere. The magnitude of this force is given by Newton's law of gravitation, and the force per unit mass $\mathbf{f}_g$ is:

$$\mathbf{f}_g = -g\frac{\mathbf{r}}{r},$$

where $\mathbf{r}$ and r are the Earth radius in vector and magnitude, respectively. As explained in Section 1.2, gravity and centrifugal forces are combined and are represented by a single g. The 'standard' value of g is $g = 9.8066\ \mathrm{m\,s^{-2}}$, while the actual acceleration due to gravity and centrifugal force is latitude dependent, and varies by about $0.03\ \mathrm{m\,s^{-2}}$.

Force of friction

Air, like all fluids, has a viscosity, which expresses itself as a force on fluid elements due to distortion of the fluid body. While viscosity is

[3] Air is a mixture of nitrogen, oxygen and small amounts of other gases. Its density depends on temperature and pressure, and is $\rho = 1.275\ \mathrm{kg\,m^{-3}}$ at $0\,^\circ\mathrm{C}$ and 1000 mbar.