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- 2.53 means the reference is to be found in question 53 of chapter 2.
 2.53⁻ means that the reference is to be found before that question.
 2.53⁺ means that the reference is to be found after that question.
 2.53–58 refers to questions 53 to 58 of chapter 2.
 2.53,64 refers to questions 53 and 64 of chapter 2.
 2H means that the reference is to be found in the historical note to chapter 2.
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