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## RHEOPHYSICS

### The Deformation and Flow of Matter

Why is it necessary to strike while the iron is hot? What makes a good liquid crystal display? Why is rubber so elastic? These questions can be answered through rheophysics, using the mechanics of continuous media at the macroscopic scale, and statistical mechanics and the physics of defects at the microscopic level.

This book addresses problems involving the flow of matter, covering the main aspects of the mechanical response of fluids and solids to applied stress or strain. It includes the hydrodynamics of ordinary liquids, the elasticity and plasticity of solids, and the rheology of complex fluids such as suspensions, polymers and liquid crystals. Dislocations are described thoroughly, and special attention is given to instabilities.

Concepts and physical properties are illustrated by numerous experiments, historical anecdotes, and applications to aeronautics, metallurgy and geophysics, making this a valuable reference for researchers and graduate students in physics, engineering and materials science.

PATRICK OSWALD is Director of Research with CNRS at the Physics Laboratory at the École Normale Supérieure, University of Lyon, France. His research spans many areas in soft condensed matter, including thermotropic and lyotropic liquid crystals, crystal growth, defects, plasticity and rheology.

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R H E O P H Y S I C S  
The Deformation and Flow of Matter

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Translated by Doru Constantin



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To my wife Jocelyne,  
and to my daughters Séverine and Magalie

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## Foreword

Should anyone still believe in a clear demarcation between fundamental science and applied science – that one is noble and the other laborious, that one moves in the higher spheres of universalism and the other makes do with down-to-earth utilitarianism – then reading this book will make them think again.

We are invited to travel through the border country where the world of ideas, principles and theories is not separated from but meets and mixes with the world of phenomena, empirical laws and concrete objects. Few topics lend themselves quite so readily to such cross-fertilisation as does rheophysics in the way Patrick Oswald presents it for us here.

Already in his famous book on *dislocations*, Jacques Friedel had accustomed us to bringing together the most classic and often the most ancient observations and knowledge – those of metallurgists (blacksmiths, founders, jewellers, wiredrawers and the like) or of potters, or even of flint cutters – with the science of continuous matter, then the science of crystal states and finally with the physics of solids. This gave rise to a whole new outlook for practitioners and engineers; they began to understand, at least in outline, the deformational behaviour of the materials they had erstwhile worked, processed and deformed quite empirically. The extreme complexity of these behaviours was reduced, using plain geometrical concepts (first at macroscopic and then at atomic scale), to insightful models that made interpretation possible. And so, as Jean Perrin put it, something complicated and visible was replaced by something simple and invisible, before the invisible appeared before our very eyes with the advent of the electron microscope in the late 1950s and its stupendous further development.

Here, in the same spirit, Patrick Oswald addresses the vast and important topic of *rheophysics* – a neologism created from *rheology*, the science of deformation and material flow. The topic is vast as it covers virtually all condensed matter. It is important because all matter is subjected to some force, be it gravity only, and so tends to deform, whether we like it (when we want to shape it) or not (when we want it to remain stable).

This treatise begins with an overview of materials in their extreme diversity and then with refreshers on continuum mechanics and hydrodynamics of simple liquids. The stage being set, a detailed analysis is presented of the various behaviours – elastic, elastodynamic, plastic, viscoelastic, hydrodynamic, etc. – that may occur when matter is subjected to forces. Wherever possible the major laws of macroscopic behaviour are associated with

atomic or molecular models, the profusion of which is tempered by a constant endeavour to synthesise and to generalise.

Amid this wealth of data, hypotheses, laws, equations, experiments, formulas, criteria and materials (from metal alloys to rocks, from colloidal crystals to polymers, from suspensions to emulsions, from liquid crystals to rubbers, from micelles to glass, etc.) a firm line had to be kept. This was made possible by the global vision Patrick Oswald has acquired of a domain in which he is one of foremost experimental and theoretical specialists. He provides us here with a dense and thorough treatise, where experimental illustrations and practical examples justify and corroborate mathematical developments.

This book, which is to be lauded for the effort put into unifying the notation in a handy table and for the sheer quality of its presentation, will prove very useful in academic and industrial laboratories alike. May it also help students who, even if they will probably not read it from cover to cover, will discover a thousand and one topics and exercises, and will come to realise that between the mathematician's desk, the physicist's laboratory and the engineer's workshop there should be no barriers but, on the contrary, constant, supportive and stimulating feedback.

Yves Quéré  
Emeritus Professor at the École Polytechnique

## Preface

*Rheophysics* is the study of the deformation and flow of matter in all its states (gaseous, liquid, solid, and even glassy or mesomorphic<sup>1</sup>). Thus defined, the topic is extremely vast, comprising the hydrodynamics of simple liquids, the elasticity and plasticity of solids and the rheology of complex fluids exhibiting a behaviour intermediate between those of liquids and of solids.

There are two different (and yet complementary) approaches to rheophysics.

The *macroscopic* approach attempts to describe the behaviour of a material over *scales much larger than that of its microstructure*. One then employs the concepts and techniques of the *mechanics of continuous media*. This theory, which stems directly from the fundamental laws of macroscopic physics (Newton's laws, the first and second principles of thermodynamics), can describe and quantify the average structure of flows<sup>2</sup> provided the *constitutive laws* of the material are known. Most important among them is that relating *stress* and *strain*. We recall that this law is often established in a *phenomenological* manner, relying on symmetry arguments and using the results of rheological measurements performed on model systems.

The second approach is *microscopic*. Its goal is to *identify the mechanisms of material deformation at the scale of its microstructure*. Though it depends strongly on the type of material under consideration, it remains nevertheless fundamental, as it brings into play extremely general mechanisms and concepts (such as *defects*). It also has great practical importance, since understanding the relation between the macroscopic constitutive law of a material and its microstructure enables us to act more effectively on its rheological behaviour. Note that microscopic theories require elementary notions of statistical mechanics (a brief reminder of which is given in the chapter on polymers) and of the physics of defects, to be considered in detail in the chapters on solids and on liquid crystals.

The goal of this book is thus to present rheophysics as generally as possible, striving to maintain the appropriate balance between these two approaches, at the experimental level as well as at the theoretical one. This is obviously a very difficult endeavour, considering the wide variety of materials and their rheological behaviour, impossible to exhaust in a

<sup>1</sup> Liquid crystals, whose structures are intermediate between liquids and solids, represent the mesomorphic states of matter. Note that the term mesomorphic stems from the Greek words 'meso' signifying intermediate and 'morphé', meaning form (of intermediate form).

<sup>2</sup> Namely, 'smoothed' over a scale larger than that of the microstructure, but still smaller than the sample size.

work of this size. For that reason, we had to make (sometimes difficult) choices between several possible topics, to the point of omitting certain important phenomena,<sup>3</sup> such as the turbulence of fluids, the rheology of glasses or the flow of dry granular media.

As to the matter of the book, it is presented in detail in the table of contents. Let us only point out that it is structured in eight chapters, grouped in four distinct parts.

Thus, the first two chapters, making up the first part, contain a general discussion of materials and their rheological behaviour, as well as a brief overview of the mechanics of continuous media. They are not indispensable at a first reading, but it is advisable to have a good grasp of the material here before tackling the remainder of the book.

Next comes the chapter on the hydrodynamics of simple liquids at small and large Reynolds numbers. Alongside the usual fundamental topics (lift force on wings, vortices, viscous drag on a moving body, etc.), this chapter deals with subjects seldom treated elsewhere, such as the spreading of a liquid on a rotating plate ('spin-coating'), microorganism propulsion or the Saffman–Taylor instability in porous media. This chapter, aimed primarily at hydrodynamicists, also contains an instrumental section on rheometers and their use, a paragraph on the microscopic theories of the viscosity of gases and liquids, as well as an appendix on shock waves, and more generally on discontinuity surfaces in fluid mechanics. It makes up the second part of the book.

One finds then two chapters on solids. The first one deals with linear elasticity in the static and dynamic regimes (waves, in particular). It discusses several classical topics of elasticity (the flexion of a plate, the torsion of a beam and its buckling instability under compression) alongside less common applications, such as seismic waves, aeroelastic instabilities, or the peculiar elasticity of colloidal crystals. The following chapter treats the problems of plasticity and brittle fracture, as well as the physics of defects, a subject both modern and fundamental, which only came into its own right after the Second World War, although it encompasses much more than the mere area of rheophysics. These two chapters, representing the third part of this book, should be of interest not only for physicists and metallurgists and but also for geologists.

We are left with the last three chapters on complex materials, two of them being dedicated to isotropic viscoelastic fluids and solids (polymers and dispersions, essentially), while the last one deals with liquid crystals, whose behaviour is clearly at the crossroads between hydrodynamics and metallurgy. These three chapters make up the fourth and final part of the book. It is addressed to readers with an interest in soft matter, whether physicists, mechanicians or engineers.

In conclusion, let us point out that some important references to fundamental papers or to recent work are cited at the end of each chapter, complementing the (in no way exhaustive) list of reference books given at the end of the book. Note that research papers are indicated in the following way: (Frank, 1954) (first author followed by the publication year of the paper), while the books are referenced by a number: [33], for instance. A list of the notation employed is also given at the end of the book. Finally, the equations of hydrodynamics and elasticity are written in cylindrical and spherical coordinates at the end of Chapters 3 and 4.

<sup>3</sup> Not completely understood to date.

## Acknowledgments

This work appeared as a result of several lectures given between 1983 and 1993 at the Écoles Normales Supérieures de Fontenay-aux-Roses and Saint-Cloud, then, when they merged in 1987, at the École Normale Supérieure de Lyon. At the time, it was a revision course of the mechanics curriculum for physics students, including some elements of fluid hydrodynamics and solid elasticity. It was only in 2002 that I volunteered to present a lecture course in rheophysics to the department of material sciences of the École Normale Supérieure de Lyon, a proposal that was immediately supported by the directors of studies at that time, Professor E. Charlaix and Professor B. Berge, whom I thank once again for their confidence. It was while teaching this course that I started writing the book, taking advantage of the numerous questions and suggestions of the students to improve it along the way. I also benefited from the precious help of J.-F. Palierne, who advised me in my reading choices and who explained several of the more subtle points of polymer viscoelasticity. I should also acknowledge the outstanding part played by Doru Constantin who, while doing his Ph.D. under my supervision, took the initiative of studying the rheology of surfactant solutions, giving me no option but to get up to date on this topic. I would like to thank my early and assiduous readers, Doru Constantin (once again), Patrick Cordier, Éric Freyssingeas and Michel Saint-Jean, for their very constructive remarks and suggestions. Special thanks go to Professor Yves Quéré, member of the French Academy of Sciences, for whom I hold the greatest admiration and who gave me the honour of reading the manuscript and of writing the Foreword. It is a pleasure to thank my friend Pawel Pieranski, who never stopped encouraging me during the three years of work on the present book. I am equally grateful to my editors, C. Counillon at Belin and S. Capelin at Cambridge University Press, who made the English edition of this work possible. I would also like to express all my gratitude to Doru Constantin, who skilfully translated this book into English.

I will end with the most important: my family. Writing a book requires a lot of time, mainly taken on that which I should have dedicated to family (on weekends, in particular). That is why I would like to thank my wife Jocelyne, and my two daughters Séverine and Magalie, for their unfailing support; without them, this book would have never been written.