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Peter Lynch
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Weather Prediction by Numerical Process

> Perhaps some day in the dim future it will be possible to advance the computations faster than the weather advances and at a cost less than the saving to mankind due to the information gained. But that is a dream.
>
> (*WPNP,* p. vii; *Dover Edn.,* p. xi)

Lewis Fry Richardson's extraordinary book *Weather Prediction by Numerical Process*, published in 1922, is a strikingly original scientific work, one of the most remarkable books on meteorology ever written. In this book – which we will refer to as *WPNP* – Richardson constructed a systematic mathematical method for predicting the weather and demonstrated its application by carrying out a trial forecast. History has shown that his innovative ideas were fundamentally sound: the methodology proposed by him is essentially that used in practical weather forecasting today. However, the method devised by Richardson was utterly impractical at the time of its publication, and the results of his trial forecast appeared to be little short of outlandish. As a result, his ideas were eclipsed for decades and his wonderful opus gathered dust and was all but forgotten.

1.1 The problem

Imagine you are standing by the ocean shore, watching the sea rise and fall as wave upon wave breaks on the rocks. At a given moment the water is rising at a rate of one metre per second – soon it will fall again. Is there an ebb or a flood tide? Suppose you use the observed rate of change and extrapolate it over the six hours that elapse between tidal extremes; you will obtain an extraordinary prediction: the water level should rise by some 20 km, twice the height of Mt Everest. This forecast is meaningless! The water level is governed by physical processes with a wide range of timescales. The tidal variations, driven by lunar gravity, have a period of around 12 hours, linked to the Earth's rotation. But wind-driven waves and swell

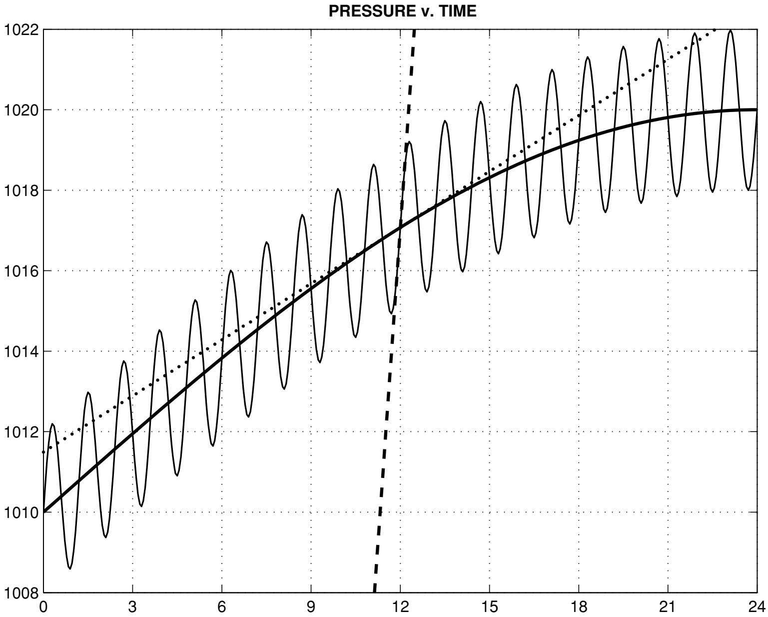

Figure 1.1 Schematic illustration of pressure variation over a 24 hour period. The thick line is the mean, long-term variation, the thin line is the actual pressure, with high frequency noise. The dotted line shows the rate of change, at 12 hours, of the mean pressure, and the dashed line shows the corresponding rate of change of the actual pressure. (After Phillips, 1973)

vary on a timescale of seconds. The instantaneous change in level due to a wave is no guide to the long-term tidal variations: if the observed rise is extrapolated over a period much longer than the timescale of the wave, the resulting forecast will be calamitous.

In 1922, Richardson presented such a forecast to the world. He calculated a change of atmospheric pressure, for a particular place and time, of 145 hPa in 6 hours. This was a totally unrealistic value, too large by two orders of magnitude. The prediction failed for reasons similar to those that destroy the hypothetical tidal forecast. The spectrum of motions in the atmosphere is analogous to that of the ocean: there are long-period variations dominated by the effects of the Earth's rotation – these are the meteorologically significant rotational modes – and short-period oscillations called gravity waves, having speeds comparable to that of sound. The interaction between the two types of variation is weak, just as is the interaction between wind-waves and tidal motions in the ocean; and, for many purposes, the gravity waves, which are normally of small amplitude, may be treated as irrelevant noise.

Although they have little effect on the long-term evolution of the flow, gravity waves may profoundly influence the way it changes on shorter timescales. Figure 1.1

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(after Phillips, 1973) schematically depicts the pressure variation over a period of one day. The smooth curve represents the variation due to meteorological effects; its gentle slope (dotted line) indicates the long-term change. The rapidly varying curve represents the actual pressure changes when gravity waves are superimposed on the meteorological flow: the slope of the oscillating curve (dashed line) is precipitous and, if used to determine long-range variations, yields totally misleading results. What Richardson calculated was the instantaneous rate of change in pressure for an atmospheric state having gravity-wave components of large amplitude. This tendency, $\partial p/\partial t \approx 0.7\,\mathrm{Pa\,s^{-1}}$, was a sizeable but not impossible value. Such variations are observed over short periods in intense, localised weather systems.[1] The problem arose when Richardson used the computed value in an attempt to deduce the long-term change. Multiplying the calculated tendency by a time step of six hours, he obtained the unacceptable value quoted above. The cause of the failure is this: *the instantaneous pressure tendency does not reflect the long-term change.*

This situation looks hopeless: how are we to make a forecast if the tendencies calculated using the basic equations of motion do not guide us? There are several possible ways out of the dilemma; their success depends crucially on the decoupling between the gravity waves and the motions of meteorological significance – we can distort the former without seriously corrupting the latter.

The most obvious approach is to construct a forecast by combining many time steps which are short enough to enable accurate simulation of the detailed high-frequency variations depicted schematically in Fig. 1.1. The existence of these high-frequency solutions leads to a stringent limitation on the size of the time step for accurate results; this limitation or *stability criterion* was discovered in a different context by Hans Lewy in Göttingen in the 1920s (see Reid, 1976), and was first published in Courant *et al.* (1928). Thus, although these oscillations are not of meteorological interest, their presence severely limits the range of applicability of the tendency calculated at the initial time. Small time steps are required to represent the rapid variations and ensure accuracy of the long-term solution. If such small steps are taken, the solution will contain gravity-wave oscillations about an essentially correct meteorological flow. One implication of this is that, if Richardson could have extended his calculations, taking a large number of small steps, his results would have been noisy but the mean values would have been meteorologically reasonable (Phillips, 1973). Of course, the attendant computational burden made this impossible for Richardson.

The second approach is to modify the governing equations in such a way that the gravity waves no longer occur as solutions. This process is known as filtering

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[1] For example, Loehrer and Johnson (1995) reported a surface pressure drop of 4 hPa in five minutes in a mesoscale convective system, or $\partial p/\partial t \approx -1.3\,\mathrm{Pa\,s^{-1}}$.

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the equations. The approach is of great historical importance. The first successful computer forecasts (Charney *et al.*, 1950) were made with the barotropic vorticity equation (see Chapter 10), which has low-frequency but no high-frequency solutions. Later, the quasi-geostrophic equations were used to construct more realistic filtered models and were used operationally for many years. An interesting account of the development of this system appeared in Phillips (1990). The quasi-geostrophic equations are still of great theoretical interest (Holton, 2004) but are no longer considered to be sufficiently accurate for numerical prediction.

The third approach is to adjust the initial data so as to reduce or eliminate the gravity-wave components. The adjustments can be small in amplitude but large in effect. This process is called *initialisation*, and it may be regarded as a form of smoothing. Richardson realised the requirement for smoothing the initial data and devoted a chapter of *WPNP* to this topic. We will examine several methods of initialisation in this work, in particular in Chapter 8, and will show that the digital-filtering initialisation method yields realistic tendencies when applied to Richardson's data.

The absence of gravity waves from the initial data results in reasonable initial rates of change, but it does not automatically allow the use of large time steps. The existence of high-frequency solutions of the governing equations imposes a severe restriction on the size of the time step allowable if reasonable results are to be obtained. The restriction can be circumvented by treating those terms of the equations that govern gravity waves in a numerically implicit manner; this distorts the structure of the gravity waves but not of the low-frequency modes. In effect, implicit schemes slow down the faster waves thus removing the cause of numerical instability (see §5.2 below). Most modern forecasting models avoid the pitfall that trapped Richardson by means of initialisation followed by semi-implicit integration.

1.2 Vilhelm Bjerknes and scientific forecasting

At the time of the First World War, weather forecasting was very imprecise and unreliable. Observations were scarce and irregular, especially for the upper air and over the oceans. The principles of theoretical physics played a relatively minor role in practical forecasting: the forecaster used crude techniques of extrapolation, knowledge of climatology and guesswork based on intuition; forecasting was more an art than a science. The observations of pressure and other variables were plotted in symbolic form on a weather map and lines were drawn through points with equal pressure to reveal the pattern of weather systems – depressions, anticyclones, troughs and ridges. The concept of *fronts,* surfaces of discontinuity between warm and cold airmasses, had yet to emerge. The forecaster used his experience, memory

Figure 1.2 A recent painting (from photographs) of the Norwegian scientist Vilhelm Bjerknes (1862–1951) standing on the quay in Bergen. (© *Geophysical Institute, Bergen*. Artist: Rolf Groven)

of similar patterns in the past and a menagerie of empirical rules to produce a forecast map. Particular attention was paid to the reported pressure changes or tendencies; to a great extent it was assumed that what had been happening up to now would continue for some time. The primary physical process attended to by the forecaster was *advection*, the transport of fluid characteristics and properties by the movement of the fluid itself.

The first explicit analysis of the weather-prediction problem from a scientific viewpoint was undertaken at the beginning of the twentieth century when the Norwegian scientist Vilhelm Bjerknes set down a two-step plan for rational forecasting (Bjerknes, 1904):

> If it is true, as every scientist believes, that subsequent atmospheric states develop from the preceding ones according to physical law, then it is apparent that the necessary and sufficient conditions for the rational solution of forecasting problems are the following:
> 1. A sufficiently accurate knowledge of the state of the atmosphere at the initial time.
> 2. A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.

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Bjerknes used the medical terms *diagnostic* and *prognostic* for these two steps (Friedman, 1989). The diagnostic step requires adequate observational data to define the three-dimensional structure of the atmosphere at a particular time. There was a severe shortage of observations, particularly over the seas and for the upper air, but Bjerknes was optimistic:

> We can hope . . . that the time will soon come when either as a daily routine, or for certain designated days, a complete diagnosis of the state of the atmosphere will be available. The first condition for putting forecasting on a rational basis will then be satisfied.

In fact, such designated days, on which upper air observations were made throughout Europe, were organised around that time by the International Commission for Scientific Aeronautics.

The second, or prognostic, step was to be taken by assembling a set of equations, one for each dependent variable describing the atmosphere. Bjerknes listed seven basic variables: pressure, temperature, density, humidity and three components of velocity. He then identified seven independent equations: the three hydrodynamic equations of motion, the continuity equation, the equation of state and the equations expressing the two laws of thermodynamics. (As pointed out by Eliassen (1999), Bjerknes was in error in listing the second law of thermodynamics; he should instead have specified a continuity equation for water substance.) Bjerknes knew that an exact analytical integration was beyond our ability. His idea was to represent the initial state of the atmosphere by a number of charts giving the distribution of the variables at different levels. Graphical or mixed graphical and numerical methods, based on the fundamental equations, could then be applied to construct a new set of charts describing the state of the atmosphere, say, three hours later. This process could be repeated until the desired forecast length was reached. Bjerknes realised that the prognostic procedure could be conveniently separated into two stages, a purely hydrodynamic part and a purely thermodynamic part; the hydrodynamics would determine the movement of an airmass over the time interval and thermodynamic considerations could then be used to deduce changes in its state. He concluded:

> It may be possible some day, perhaps, to utilise a method of this kind as the basis for a daily practical weather service. But however that may be, the fundamental scientific study of atmospheric processes sooner or later has to follow a method based upon the laws of mechanics and physics.

Bjerknes' speculations are reminiscent of Richardson's 'dream' of practical scientific weather forecasting.[2]

A tentative first attempt at mathematically forecasting synoptic changes by the application of physical principles was made by Felix Exner, working in Vienna. His account (Exner, 1908) appeared only four years after Bjerknes' seminal paper. Exner

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[2] Bjerknes' ideas on rational forecasting were adumbrated by Cleveland Abbe. See note added in proof, p. 27.

makes no reference to Bjerknes' work, which was also published in *Meteorologische Zeitschrift*. Though he may be presumed to have known about Bjerknes' ideas, Exner followed a radically different line: whereas Bjerknes proposed that the full system of hydrodynamic and thermodynamic equations be used, Exner's method was based on a system reduced to the essentials. He assumed that the atmospheric flow is geostrophically balanced and that the thermal forcing is constant in time. Using observed temperature values, he deduced a mean zonal wind. He then derived a prediction equation representing advection of the pressure pattern with constant westerly speed, modified by the effects of diabatic heating. It yielded a realistic forecast in the case illustrated in Exner's paper. Figure 1.3 shows his calculated pressure change (top) and the observed change (bottom) over the four-hour period between 8 p.m. and midnight on 3 January 1895; there is reasonable agreement between the predicted and observed changes. However, the method could hardly be expected to be of general utility. Exner took pains to stress the limitations of his method, making no extravagant claims for it. But despite the very restricted applicability of the technique devised by him, the work is deserving of attention as a first attempt at systematic, scientific weather forecasting. Exner's numerical method was summarised in his textbook (Exner, 1917, §70). The only reference by Richardson to the method was a single sentence (*WPNP*, p. 43) 'F. M. Exner has published a prognostic method based on the source of air supply.' It would appear from this that Richardson was not particularly impressed by it!

In 1912, Bjerknes became the first Director of the new Geophysical Institute in Leipzig. In his inaugural lecture he returned to the theme of scientific forecasting. He observed that 'physics ranks among the so-called exact sciences, while one may be tempted to cite meteorology as an example of a radically inexact science'. He contrasted the methods of meteorology with those of astronomy, for which predictions of great accuracy are possible, and described the programme of work upon which he had already embarked: *to make meteorology into an exact physics of the atmosphere.* Considerable advances had been made in observational meteorology during the previous decade, so that now the diagnostic component of his two-step programme had become feasible.

> ... now that complete observations from an extensive portion of the free air are being published in a regular series, a mighty problem looms before us and we can no longer disregard it. We must apply the equations of theoretical physics not to ideal cases only, but to the actual existing atmospheric conditions as they are revealed by modern observations. These equations contain the laws according to which subsequent atmospheric conditions develop from those that precede them. It is for us to discover a method of practically utilising the knowledge contained in the equations. From the conditions revealed by the observations we must learn to compute those that will follow. The problem of accurate pre-calculation that was solved for astronomy centuries ago must now be attacked in all earnest for meteorology. *(Bjerknes, 1914a)*

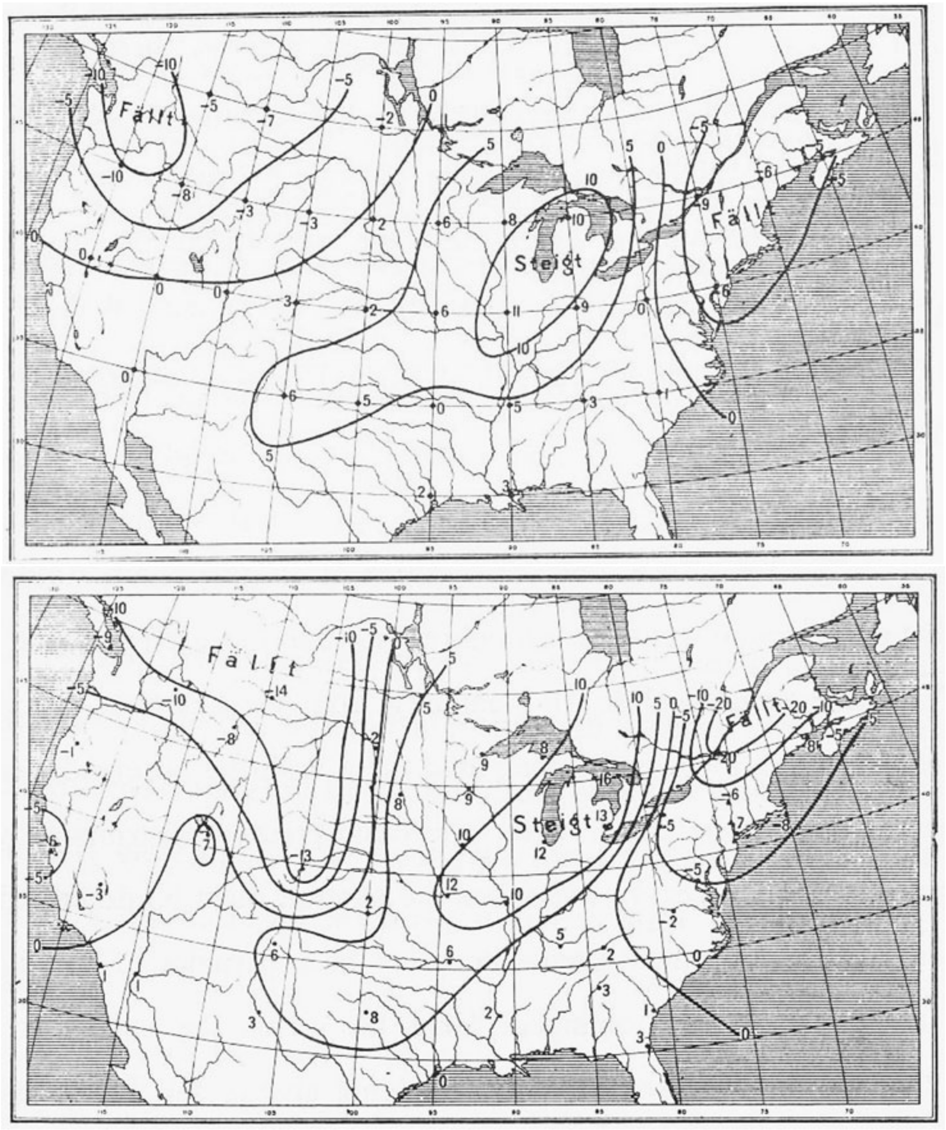

Figure 1.3 Top: Exner's calculated pressure change between 8 p.m. and midnight, 3 January 1895. Bottom: observed pressure change for the same period [Units: hundredths of an inch of mercury. *Steigt* = rises; *Fällt* = falls]. (Exner, 1908)

Bjerknes expressed his conviction that the acid test of a science is its utility in forecasting: 'There is after all but one problem worth attacking, *viz.*, the precalculation of future conditions.' He recognised the complexity of the problem and realised that a rational forecasting procedure might require more time than the atmosphere itself takes to evolve, but concluded:

> If only the calculation shall agree with the facts, the scientific victory will be won. Meteorology would then have become an exact science, a true physics of the atmosphere. When that point is reached, *then* the practical results will soon develop.
>
> It may require many years to bore a tunnel through a mountain. Many a labourer may not live to see the cut finished. Nevertheless this will not prevent later comers from riding through the tunnel at express-train speed.

At Leipzig, Bjerknes instigated the publication of a series of weather charts based on the data that were collected during the internationally-agreed intensive observation days and compiled and published by Hugo Hergesell in Strasbourg (these charts are discussed in detail in Chapter 6). One such publication (Bjerknes, 1914b), together with the 'raw data' in Hergesell (1913), was to provide Richardson with the initial conditions for his forecast.

Richardson first heard of Bjerknes' plan for rational forecasting in 1913, when he took up employment with the Meteorological Office. In the preface to *WPNP* he writes:

> The extensive researches of V. Bjerknes and his School are pervaded by the idea of using the differential equations for all that they are worth. I read his volumes on *Statics* and *Kinematics* soon after beginning the present study, and they have exercised a considerable influence throughout it.

Richardson's book opens with a discussion of then-current practice in the Met Office. He describes the use of an Index of Weather Maps, constructed by classifying old synoptic charts into categories. The Index (Gold, 1920) assisted the forecaster to find previous maps resembling the current one and therewith to deduce the likely development by studying the evolution of these earlier cases:

> The forecast is based on the supposition that what the atmosphere did then, it will do again now. There is no troublesome calculation, with its possibilities of theoretical or arithmetical error. The past history of the atmosphere is used, so to speak, as a full-scale working model of its present self. (*WPNP,* p. vii; *Dover Edn.,* p. xi)

Bjerknes had contrasted the precision of astronomical prediction with the 'radically inexact' methods of weather forecasting. Richardson returned to this theme in his preface:

> – the *Nautical Almanac,* that marvel of accurate forecasting, is not based on the principle that astronomical history repeats itself in the aggregate. It would be safe to say that a particular disposition of stars, planets and satellites never occurs twice. Why then should we expect a present weather map to be exactly represented in a catalogue of past weather?... This alone is sufficient reason for presenting, in this book, a scheme of weather prediction which resembles the process by which the *Nautical Almanac* is produced, in so far as it is founded upon the differential equations and not upon the partial recurrence of phenomena in their ensemble.

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Richardson's forecasting scheme amounts to a precise and detailed implementation of the prognostic component of Bjerknes' programme. It is a highly intricate procedure: as Richardson observed, 'the scheme is complicated because the atmosphere is complicated'. It also involved an enormous volume of numerical computation and was quite impractical in the pre-computer era. But Richardson was undaunted, expressing his dream that 'some day in the dim future it will be possible to advance the computations faster than the weather advances'. Today, forecasts are prepared routinely on powerful computers running algorithms that are remarkably similar to Richardson's scheme – his dream has indeed come true.

Before discussing Richardson's forecast in more detail, we will digress briefly to consider his life and work from a more general viewpoint.

1.3 Outline of Richardson's life and work

Richardson's life and work are discussed in a comprehensive and readable biography (Ashford, 1985). The Royal Society Memoir of Gold (1954) provides a more succinct description and the *Collected Papers* of Richardson, edited by Drazin (LFR I) and Sutherland (LFR II), include a biographical essay by Hunt (1993); see also Hunt (1998). Brief introductions to Richardson's work in meteorology (by Henry Charnock), in numerical analysis (by Leslie Fox) and on fractals (by Philip Drazin) are also included in Volume 1 of the *Collected Papers*. The article by Chapman (1965) is worthy of attention and some fascinating historical background material may be found in the review by Platzman (1967). In a recent popular book on mathematics, Körner (1996) devotes two chapters (69 pages) to various aspects of Richardson's mathematical work. The National Cataloguing Unit for the Archives of Contemporary Scientists has produced a comprehensive catalogue of the papers and correspondence of Richardson, which were deposited by Oliver Ashford in Cambridge University Library (NCUACS, 1993). The following sketch of Richardson's life is based primarily on Ashford's book.

Lewis Fry Richardson was born in 1881, the youngest of seven children of David Richardson and Catherine Fry, both of whose families had been members of the Society of Friends for generations. He was educated at Bootham, the Quaker school in York, where he showed an early aptitude for mathematics, and at Durham College of Science in Newcastle. He entered King's College, Cambridge in 1900 and graduated with a First Class Honours in the Natural Science Tripos in 1903. In 1909, he married Dorothy Garnett. They had no offspring but adopted two sons and a daughter, Olaf (1916–83), Stephen (1920–) and Elaine (1927–).

Over the ten years following his graduation, Richardson held several short research posts (Appendix 2 contains a chronology of the milestones of his life and career). As a scientist with National Peat Industries, he investigated the optimum