DATA ENVELOPMENT ANALYSIS

Using the neoclassical theory of production economics as the analytical framework, this book provides a unified and easily comprehensible, yet fairly rigorous, exposition of the core literature on data envelopment analysis (DEA) for readers based in different disciplines. The various DEA models are developed as nonparametric alternatives to the econometric models. Apart from the standard fare consisting of the basic input- and output-oriented DEA models formulated by Charnes, Cooper, and Rhodes and Banker, Charnes, and Cooper, the book covers more recent developments, such as the directional distance function, free disposal hull (FDH) analysis, nonradial measures of efficiency, multiplier bounds, mergers and breakup of firms, and measurement of productivity change through the Malmquist total factor productivity index. The chapter on efficiency measurement using market prices provides the critical link between DEA and the neoclassical theory of a competitive firm. The book also covers several forms of stochastic DEA in detail.

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Professor Ray's research has been published in major professional journals including *American Economic Review; Management Science; The Economic Journal; European Journal of Operational Research; Journal of Productivity Analysis; American Journal of Agricultural Economics; Journal of Money, Credit, and Banking; Journal of Banking and Finance; International Journal of Systems Science; Journal of Forecasting; International Journal of Forecasting*; and *Journal of Quantitative Economics.* He served as one of the guest editors of a special issue of *Journal of Productivity Analysis* honoring William Cooper. Professor Ray coauthored *Applied Econometrics: Problems with Data Sets* in 1992 with William F. Lott.

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Theory and Techniques for Economics and Operations Research

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For Shipra, who has cheerfully given up many precious evenings and weekends that rightfully belonged to her in order to make it possible for me to complete this book.

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Preface

Researchers from diverse fields ranging from economics to accounting, information management, and operational research use Data Envelopment Analysis (DEA) to measure technical efficiency of firms (often called Decision-Making Units [DMUs]). Scholars from the different disciplines, in general, approach the question of measuring efficiency from different perspectives. Often, an operations research analyst is primarily interested in the solution algorithm of an inequality-constrained optimization problem but is less careful in defining the inputs and outputs. At times, the input variables may include both the number of workers and wage expenses even though, under the implicit assumption of competitive wages, they are broadly proportional to one another. Similarly, sometimes both sales revenue and profits earned are defined as outputs, even though profit maximization is the implicit objective of the firm. Clearly, the efficiency measure derived from an optimization model becomes more meaningful when the choice variables and the constraints correspond to an explicitly conceptualized theory of firm behavior. At the other end of the spectrum, there are numerous empirical applications in economics where some DEA model is employed to evaluate efficiency without careful attention to the appropriateness of the specific version of DEA for the production technology and the implicit objective of the firm. For the applied researcher, a clear understanding of the differences between the various DEA models is absolutely necessary for a proper interpretation of the results.

My principal research interest in production economics has convinced me over the years that one must treat the production technology and the objectives of firm behavior under the constraints specified as fundamental to any analysis of efficiency and, just as in econometric modeling one estimates a frontier production, cost, or profit function for measuring efficiency, in much the same way one has to specify the appropriate DEA model in order to obtain a proper measure of the efficiency of a firm. Thus, the neoclassical model of production

economics, in its primal–dual forms, is the basic analytical framework of this book as it provides the economic rationale of the various DEA models.

The principal objective of this book is to provide a unified and easily comprehensible yet fairly rigorous exposition of the essential features of the core literature on DEA for the interested readers coming from different disciplines. The standard concepts of technical, scale, and cost efficiency are first explained using simple parametric functional forms. Subsequently, the various DEA models are developed as nonparametric alternatives to the parametric models. This should be particularly helpful for the average economist more familiar with parametrically specified production, cost, or profit functions. At the same time, various numerical examples of the parametric models have been included for the benefit of the reader whose principal background is in operations research or management science, even though such examples may appear superfluous to readers familiar with neoclassical production economics.

Apart from the standard fare consisting of the basic input- and output-oriented DEA models formulated by Charnes, Cooper, and Rhodes (CCR) and Banker, Charnes, and Cooper (BCC), the book includes detailed coverage of more recent developments like the directional distance function, free disposal hull (FDH) analysis, nonradial measures of efficiency, multiplier bounds, merger and breakup of firms, and measurement of productivity change through the Malmquist total factor productivity index. The chapter on efficiency measurement using market prices provides the critical link between DEA and the neoclassical theory of a competitive firm. In the chapter on nonparametric approaches to production analysis, a number of models that complement DEA are presented to establish the common intellectual lineage of these two approaches – one coming from economics and the other from operations research. Similarly, for the interested reader, a detailed discussion of Shephard's distance function is provided in an appendix to Chapter 2. Finally, several forms of stochastic DEA are discussed in detail.

This book is designed to provide the theoretical and methodological background that would enable interested readers to formulate the relevant DEA model for the specific problem under investigation. The emphasis is on setting up the appropriate linear programming models in the primal–dual forms. Although, for most types of models, sample computer programs in SAS are provided as examples, it is expected that readers will either write their own programs for any software that serves their purpose or get a skilled programmer to translate the DEA optimization problems that they formulate into a set of computer commands.

I have personally been interested in DEA as an analytical tool in production economics right from its inception into the literature. In 1978, while I was a graduate student at the University of California, Santa Barbara, Llad Phillips, who was teaching a course in Labor Economics, introduced me to the neoclassical theory of duality in production. Shortly thereafter, Jati Sengupta brought to my attention a paper by CCR published in the *European Journal of Operational Research* on measurement of technical efficiency using a new method called Data Envelopment Analysis (DEA). Later, in 1979, I joined Phillips and one of his past Ph.D. students, Manuel Olave, from INCAE, Managua, Nicaragua, as a research assistant for their project on measuring the productive efficiency of primary health care and family planning centers in Costa Rica and Guatemala. My own contribution to the study was to complement their Translog cost function analysis with the new approach of DEA. The data set included various manpower hours (physicians, nursing, and other personnel) for inputs and different types of cases treated (like maternity, family planning, and others) for outputs. The units observed were health care facilities from different regions categorized as urban, rural, or tribal (Indian), and observations were recorded for different semesters over years. In our first application, based on our intuition from production economics, we used the regional characteristics as ordered categorical variables, thereby anticipating a subsequent development in the literature. Similarly, we conceptualized nonregressive technical change and constructed a series of sequential frontiers for the chronologically ordered time periods. Looking back, ours must have been one of the earlier applications of DEA, which has remained unrecognized in the chronology of the literature. This is explained largely by the fact that during the subsequent political turmoil in Nicaragua, I lost contact with Manuel Olave and, over the years, the project report slipped into oblivion. Over the decade that followed, my interest in productivity analysis deepened and I continued to work on DEA just by myself with little intellectual discourse with anyone else. This led to two papers that appeared in *Socio-Economic Planning Sciences* and *Management Science*, respectively. Finally, in 1991, I presented a paper in the DEA stream of the EURO-TIMS Meetings held in Aachen, Germany. My first exposure to the community of researchers working on productivity and efficiency analysis was a most exciting and intellectually rewarding experience. It was at this meeting that I first met some of the leading scholars in the field such as Bill Cooper, Knox Lovell, and Rolf Färe. Thereafter, I became a regular participant in the Productivity Workshops held in the United States and in Europe in alternate years. Interaction with fellow researchers at these meetings has greatly contributed to the development of this book. I am particularly grateful to Knox Lovell,

who at various times has been a very constructive critic of my work. At a different level, Bill Cooper has always been a source of inspiration and encouragement for me. Subal Kumbhakar, a long-time friend and a leading exponent of the stochastic frontier analysis, has always been an open-minded listener to my ideas and has judged the essence of any research idea from the broad perspective of neoclassical production economics rather than through the narrow lenses of a methodologist of a particular conviction. Steve Miller, who was a colleague for nearly two decades here at the University of Connecticut, has patiently read and offered valuable comments on much of what I have written on DEA and efficiency measurement, including several of the earlier chapters of this book.

Over the years, my own graduate students at the University of Connecticut, many of whom have been my coauthors, also have often helped me to clear up confusions about different aspects of DEA in particular and neoclassical duality theory in general through many perceptive questions they have raised in my research seminar course. In particular, Evangelia Desli and Kankana Mukherjee have continued to offer valuable comments and suggestions on all of my papers – even when they were not coauthors. Two of my current graduate students, Anasua Bhattacharya and Yanna Wu, helped me by drawing the figures in Microsoft Word.

Finally, a Fulbright Lecturer award in the fall of 2000 offered an opportunity to teach DEA for a month at the Indian Institute of Management, Calcutta, and for the next three months at the Indian Institute of Management, Ahmedabad, and allowed me to organize the lectures around the planned chapters of this book. The doctoral students at these two institutions attending my lectures helped me to improve the exposition of the topics covered in the chapters.

Special thanks go to Scott Parris, economics editor of Cambridge University Press at New York, for his enthusiastic support and encouragement. Although I alone bear responsibility for whatever is presented in this book, the body of literature dealt with is the contribution of a host of outstanding scholars from economics, management science, and operations research. If the book helps to bridge the gap between different strands within the literature, it will have served its purpose.