

The Standard Model: A Primer

The standard model provides the modern understanding of all of the interactions of subatomic particles, except those due to gravity. The theory has emerged as the best distillation of decades of research.

This book uses the standard model as a vehicle for introducing quantum field theory. In doing this the book also introduces much of the phenomenology on which this model is based. The book uses a modern approach, emphasizing effective field theory techniques, and contains brief discussions of some of the main proposals for going beyond the standard model, such as seesaw neutrino masses, supersymmetry, and grand unification.

Requiring only a minimum of background material, this book is ideal for graduate students in theoretical and experimental particle physics. The book concentrates on getting students to the level of being able to use this theory by doing real calculations with the minimum of formal development. It does so without taking any shortcuts which would leave an incomplete understanding. The book contains several problems, with password-protected solutions available to lecturers at www.cambridge.org/9780521860369

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Preface

The standard model of particle physics, developed in the 1960s and 1970s, has stood for 30 years as “the” theory of particle physics, passing numerous stringent tests. In fact, while many people believe that the standard model is not a complete description of particle physics, it is expected to be, at worst, incomplete rather than wrong; that is, the standard model is at worst a subset of the true theory of particle physics.

For this reason, a good working knowledge of the standard model and its phenomenology is essential for the modern particle physicist. The goal of this book is to provide all the tools for a working, quantitative knowledge of the standard model, with the minimum of formal developments. It presents everything needed to understand the particle spectrum of the standard model, and how to compute decay rates and cross sections at leading order in the weak coupling expansion (tree level). We assume a solid quantum-mechanics background, up to and including canonical quantization and the Dirac equation, but we do not assume familiarity with formal quantum field theory (renormalization, path integrals, generating functionals).

As we see it, this book fills two gaps in the existing literature. The first of these concerns the balance between theoretical sophistication and phenomenological utility. Most treatments of the standard model appear at the end of quantum field theory books. This is rational in the sense that the reader then has the complete set of tools to compute standard-model phenomena at the loop level. This approach has its merits; both authors learned the standard model in this way. Unfortunately, for many, especially experimental practitioners, the quantum field theory preliminaries may be too burdensome. Also, such books frequently do not present the standard model in complete detail, and they generally develop little of its phenomenology. The opposite style of approach is a more “cookbook” book, which introduces quantum field theory at the tree level, typically using electrodynamics as an

example, and again presents the standard model at the end. Generally these treatments are incomplete and abbreviated. The intention of this book is to be similar to the latter type of book, except that the presentation of the standard model is complete and contains a discussion of the model's phenomenology and a complete presentation of its Feynman rules.

Our philosophy is that it is important for a particle physicist to have a complete and quantitative knowledge of the standard model; indeed, for many, this is much more important than having a good background in formal quantum field theory. One cannot present the standard model in detail without *some* quantum field theory; but one can get surprisingly far without understanding the details of renormalization and loop effects. Of course, especially for theorists, a good knowledge of quantum field theory is also necessary; indeed, it should be obvious to the reader, at many points in the text, that more formal development is needed to compute to high accuracy. Knowing the material in this book may help the student of more formal quantum field theory by motivating and providing context for that study. Conversely, a student already proficient in quantum field theory can use this book as a succinct presentation of the standard model, and will have the tools to fill in the gaps left in the presentation, where loop corrections are required.

The second gap which we believe this book fills concerns the modern theoretical framework within which the standard model rests: the framework of *effective field theories*. Today we understand the theories we construct to describe nature – including the standard model – to be effective theories which capture the low-energy limit of some more fundamental, microscopic physics. Effective field theories capture a basic experimental fact: although nature comes to us with many scales, it can be understood one scale at a time. For instance, atomic physics can be understood with only limited knowledge of nuclei, and it can because short-distance physics tends to *de-couple* from long-distance physics. In the modern understanding it is this observation which ultimately explains the otherwise puzzling requirement of renormalizability which our fundamental theories generally have. This book starts by using the standard model to build up the tools of effective field theory, by showing how and why scattering amplitudes simplify in the low-energy limit. Later chapters then exploit these tools to categorize the kinds of new physics which might ultimately replace the standard model, starting with a discussion of neutrino oscillations and ending with a broad survey of such new physics topics as supersymmetry and grand unified theories.

The first chapter of this book is devoted to introducing the field theory concepts we will need to present the standard model. We present the allowed

fields that can make up a quantum field theory (scalars, fermions, and gauge bosons), with particular emphasis on Majorana fermions and on the gauge principle, which appear to play especially important roles in the standard model. We introduce the required rules for formulating the theory's Lagrangian – the “basic principles,” such as Lorentz invariance, locality, unitarity, and renormalizability. We see what kinds of interactions are allowed, given the available fields and these basic principles. Then we give a few illustrative examples, including QED and QCD. Supplementary material on group theory, the Lorentz group, and spinors is provided in two appendices.

The second chapter introduces the standard model itself. We present the gauge group and the field content. The Lagrangian then follows as the most general Lagrangian consistent with these fields and with basic principles. This section then explores the consequences, determining the mass eigenstates and their interactions. We present in complete detail what the interaction Hamiltonian of the model is in the mass basis. We also briefly discuss the symmetries of the model, especially the accidental global symmetries of baryon and lepton number, and very briefly discuss anomalies and gauge anomaly cancellation.

The third chapter discusses the S matrix formalism in just enough detail to define and motivate decay rates and cross sections, and to show how they are to be computed in the interaction picture. Together, the first three chapters represent an introduction to the framework of the standard model.

Next, we start using the standard-model interactions to compute processes, introducing the needed technology as we go with the philosophy of “learning by doing” and using specific examples to figure out the patterns. We begin with the simplest processes in the standard model, the decays of heavy bosons, in Chapter 4. The rates of Z^0 , W^\pm , and Higgs-boson decays can be computed using interaction picture perturbation theory and an expansion of the fields in creation and annihilation operators, without much difficulty. In Chapter 5, where we consider the decays of leptons lighter than the W boson mass, we first encounter virtual intermediate particles, requiring the introduction of the propagator. After these examples it is possible to generalize the procedure for computing a decay process. This allows us to introduce the Feynman rules. Chapter 5 ends with a complete presentation of the unitary gauge Feynman rules of the standard model, sufficient for tree level analysis. (The R_ξ gauge Feynman rules appear in Appendix D.)

In Chapter 6 we address scattering processes, concentrating on fermion–fermion scattering. We discuss s -channel scattering in some length, especially near the Z^0 pole, where we first discover the necessity of including loop

corrections. We also introduce crossing symmetry and interference between diagrams, external photon states, and initial state radiation.

In Chapter 7 we introduce the notion of effective field theories, using the Fermi theory as the main example. This is especially important as the standard model itself is probably just an effective theory for some more inclusive theory, which is manifested at higher energies. We also present some of the most important results of loop corrections, particularly the running of gauge couplings with scale.

Chapter 8 begins the discussion of hadrons. We motivate why the running of couplings causes the confinement of quarks and gluons within hadrons, and we describe and motivate the spectrum of heavy-light and light-light mesons and of baryons, emphasizing the use of approximate symmetries.

Chapter 9 discusses hadronic interactions. It explains why both the low- and high-energy regimes are somewhat tractable, but the intermediate energy regime is not. We discuss deep inelastic scattering and the partonic structure of hadrons, up to and including the Altarelli–Parisi (DGLAP) equations. Then we discuss chiral perturbation theory, leptonic meson decays, and oscillation phenomena in the K and B meson systems.

The last part of the book gives a brief survey of what may lie beyond the standard model. We begin in Chapter 10 with a discussion of neutrino masses. Technically, these cannot lie beyond the standard model, because they have been observed, and the meaning of the standard model must be enlarged to accommodate them. However, as we discuss, there are two viable ways to do so, Majorana neutrino masses and Dirac neutrino masses, and we do not (yet) know which is correct. We discuss the Majorana possibility at some length in the context of non-renormalizable field theories. We discuss oscillation phenomena in some length, including the MSW effect, and briefly cover neutrinoless double beta decay. We also give examples of high-energy physics that could lead to the non-renormalizable operator responsible for Majorana neutrino masses.

Finally, Chapter 11 discusses what *may* lie beyond the standard model. We organize this material in terms of problems with the standard model, which can in turn be organized in terms of the dimensionality of the operator presenting the problem. The hierarchy problem appears because of the dimension-2 Higgs mass term, and may be solved by supersymmetry. The strong CP problem appears because of the dimension-4 Θ term in QCD, and may be solved by the axion mechanism. The baryon-number conservation “problem” (opportunity) arises because of the possibility of dimension-6 operators in the standard model; these might arise at an interesting level within grand unified theories.

In this book we have used the metric convention, $\eta_{\mu\nu} = \text{Diag}[-1+1+1+1]$, which is the less common convention within the phenomenology community. However, to ease the text's use, we present in Appendix E a clear discussion of how to convert between conventions, culminating in a metric convention conversion table.

In our experience it is possible to cover most of this book in a high-paced, one-semester first-year graduate level course. To do so, it is necessary to shave some corners. Most of Chapter 1, and Chapter 2 through Section 2.4, are essential, but Section 2.5 can be skipped without too much loss to the continuity. Similarly Section 4.2 and Section 4.3 can be given as problems instead of covered as sections. Chapter 5 and Chapter 6 should be covered in full, but then material from the remaining chapters can be picked and chosen as time and interest allow. The material in Chapter 10 does not rely on Chapter 8 or Chapter 9. A full year course should quite easily be able to cover all of the material in this book.

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