

## Principles of Continuum Mechanics

Conservation and Balance Laws with Applications

Second Edition

Continuum mechanics deals with the stress, deformation, and mechanical behaviour of matter as a continuum rather than a collection of discrete particles. The subject is interdisciplinary in nature, and it is gaining increased attention in recent times primarily because of a need to understand a variety of phenomena at different spatial scales. The second edition of *Principles of Continuum Mechanics* provides a concise yet rigorous treatment of the subject of continuum mechanics and elasticity at the senior undergraduate and first-year graduate levels. It prepares engineer-scientists for advanced courses in traditional as well as emerging fields, such as biotechnology, nanotechnology, energy systems, and computational mechanics. The large number of examples and exercise problems contained in the book systematically advance the understanding of vector and tensor analysis, basic kinematics, balance laws, field equations, constitutive equations, and applications. A solutions manual is available for the book.

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Conservation and Balance Laws with Applications

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Second Edition



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ॐ सह नाववतु सह नौ भुनक्तु  
 सह वीर्यं कर्वावहै।  
 तेजस्वि नावधीतमस्तु  
 मा विद्विषावहै॥  
 ॐ शान्तिः शान्तिः शान्तिः॥

AUM saha navavatu, saha nau bhunaktu  
 Saha veeryam karvaavahai  
 Tejasvi naavadhitamastu  
 maa vid vishaa va hai  
 AUM shaantih, shaantih, shaantih

“Let us together be protected and nourished by God’s blessings. Let us together join our mental forces in strength for the benefit of humanity. **Let our efforts at learning be luminous, filled with joy, and endowed with the force of purpose.** Let us never be poisoned with the seeds of hatred for anyone. Let there be peace and serenity in all the three universes.”

– Invocation from *Taittiriya Upanishad*

**That which is not given is lost.**

## Contents

	<i>Preface to the Second Edition</i>	page xi
	<i>Preface to the First Edition</i>	xiii
	<i>Symbols used in the Book</i>	xvii
1	INTRODUCTION	1
	1.1 Continuum Mechanics	1
	1.2 Examples of Engineering Systems	5
	1.3 Objective of the Study	7
	1.4 Summary	8
	Problems	10
	References for Additional Reading	11
2	VECTORS AND TENSORS	13
	2.1 Motivation	13
	2.2 Definition of a Vector	13
	2.3 Vector Algebra	14
	2.3.1 Unit Vector	15
	2.3.2 Zero Vector	15
	2.3.3 Vector Addition	15
	2.3.4 Multiplication of Vector by Scalar	16
	2.3.5 Scalar Product of Vectors	18
	2.3.6 Vector Product	19
	2.3.7 Triple Products of Vectors	22
	2.3.8 Plane Area as a Vector	24
	2.3.9 Components of a Vector	26
	2.4 Index Notation and Summation Convention	29
	2.4.1 Summation Convention	29
	2.4.2 Dummy Index	29
	2.4.3 Free Index	30
	2.4.4 Kronecker Delta and Permutation Symbols	30
	2.4.5 Transformation Law for Different Bases	33
	2.5 Theory of Matrices	35
	2.5.1 Definition of a Matrix	35

2.5.2	Matrix Addition and Multiplication of a Matrix by a Scalar	37
2.5.3	Symmetric and Skew Symmetric Matrices	37
2.5.4	Matrix Multiplication	39
2.5.5	Inverse and Determinant of a Matrix	40
2.5.6	Positive-Definite and Orthogonal Matrices	44
2.5.7	Eigenvalues and Eigenvectors	44
2.6	Vector Calculus	50
2.6.1	The Del Operator	50
2.6.2	Divergence and Curl of a Vector	51
2.6.3	Cylindrical and Spherical Coordinate Systems	53
2.6.4	Gradient, Divergence, and Curl Theorems	55
2.7	Tensors	57
2.7.1	Dyads	57
2.7.2	Matrix Form of a Second-Order Tensor	58
2.7.3	Transformation of Components of a Dyad	60
2.7.4	Tensor Calculus	60
2.8	Summary	61
	Problems	62
	References for Additional Reading	66
3	KINEMATICS OF A CONTINUUM	69
3.1	Deformation and Configuration	69
3.2	Engineering Strains	70
3.2.1	Normal Strain	70
3.2.2	Shear Strain	71
3.3	General Kinematics of a Continuum	75
3.3.1	Configurations of a Continuous Medium	75
3.3.2	Material and Spatial Descriptions	76
3.3.3	Displacement Field	80
3.4	Analysis of Deformation	82
3.4.1	Deformation Gradient	82
3.4.2	Various Types of Deformation	84
3.4.3	Green Strain Tensor	87
3.4.4	Infinitesimal Strain Tensor	91
3.4.5	Principal Values and Principal Planes of Strain	94
3.5	Rate of Deformation and Vorticity Tensors	95
3.5.1	Velocity Gradient Tensor	95
3.5.2	Rate of Deformation Tensor	96
3.5.3	Vorticity Tensor and Vorticity Vector	97
3.6	Compatibility Equations	99
3.7	Summary	101
	Problems	102
	References for Additional Reading	109

4	STRESS VECTOR AND STRESS TENSOR	111
4.1	Introduction	111
4.2	Stress Vector, Stress Tensor, and Cauchy's Formula	112
4.3	Transformations of Stress Components and Principal Stresses	120
4.3.1	Transformation of Stress Components	120
4.3.2	Principal Stresses and Principal Planes	123
4.4	First and Second Piola–Kirchhoff Stress Tensors	125
4.4.1	Introduction	125
4.4.2	First Piola–Kirchhoff Stress Tensor	126
4.4.3	Second Piola–Kirchhoff Stress Tensor	127
4.5	Summary	128
	Problems	129
	References for Additional Reading	132
5	CONSERVATION OF MASS AND BALANCE OF MOMENTA AND ENERGY	133
5.1	Introduction	133
5.1.1	Preliminary Comments	133
5.1.2	Velocity and Acceleration	134
5.2	Conservation of Mass	135
5.2.1	Conservation of Mass in Spatial Description	135
5.2.2	Conservation of Mass in Material Description	139
5.2.3	Reynolds Transport Theorem	141
5.3	Balance of Momenta	141
5.3.1	Principle of Balance of Linear Momentum	141
5.3.2	Principle of Balance of Angular Momentum	153
5.4	Thermodynamic Principles	154
5.4.1	Introduction	154
5.4.2	Balance of Energy for One-Dimensional Flows	155
5.4.3	Energy Equation for a Three-Dimensional Continuum	158
5.5	Summary	161
	Problems	161
	References for Additional Reading	166
6	CONSTITUTIVE EQUATIONS	167
6.1	Introduction	167
6.2	Elastic Solids	169
6.2.1	Introduction	169
6.2.2	Generalized Hooke's Law for Orthotropic Materials	169
6.2.3	Generalized Hooke's Law for Isotropic Materials	172
6.3	Constitutive Equations for Fluids	177
6.3.1	Introduction	177
6.3.2	Ideal Fluids	177

6.3.3	Viscous Incompressible Fluids	178
6.4	Heat Transfer	178
6.4.1	General Introduction	178
6.4.2	Fourier's Heat Conduction Law	179
6.4.3	Newton's Law of Cooling	179
6.4.4	Stefan–Boltzmann Law	180
6.5	Summary	180
	Problems	181
	References for Additional Reading	183
7	APPLICATIONS IN HEAT TRANSFER, FLUID MECHANICS, AND SOLID MECHANICS	185
7.1	Introduction	185
7.2	Heat Transfer	185
7.2.1	Governing Equations	185
7.2.2	Analytical Solutions of One-Dimensional Heat Transfer	189
7.2.3	Axisymmetric Heat Conduction in a Cylinder	192
7.2.4	Two-Dimensional Heat Transfer	193
7.3	Fluid Mechanics	195
7.3.1	Preliminary Comments	195
7.3.2	Summary of Equations	196
7.3.3	Inviscid Fluid Statics	197
7.3.4	Parallel Flow (Navier–Stokes Equations)	198
7.3.5	Diffusion Processes	202
7.4	Solid Mechanics	205
7.4.1	Governing Equations	205
7.4.2	Analysis of Bars	207
7.4.3	Analysis of Beams	211
7.4.4	Principle of Superposition	216
7.4.5	Analysis of Plane Elasticity Problems	221
7.5	Summary	229
	Problems	230
	References for Additional Reading	240
	<i>Answers to Selected Problems</i>	241
	<i>Index</i>	247



## Preface to the Second Edition

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This second edition of *Principles of Continuum Mechanics* has the same objective as the first, namely, to present the concepts of continuum mechanics in a simple yet mathematically rigorous manner and illustrate the concepts through simple applications from heat transfer, fluid mechanics, and solid mechanics. The subject of *continuum mechanics* deals with the stress, deformation, and mechanical behavior of matter as a continuum rather than a collection of discrete particles. The subject is interdisciplinary in nature, and it is gaining increased attention primarily because of a need to understand a variety of phenomena at different spatial scales. Formulations of the mathematical models of any phenomena in nature rely heavily on the knowledge of continuum mechanics. Mathematical models and their numerical evaluation are aids to design and manufacturing. The most critical step in arriving at a design of a system (or a component thereof) that is both functional and cost-effective is the construction of a physics-based mathematical model. It is in this connection that a course on continuum mechanics is most helpful.

*Principles of Continuum Mechanics* provides a concise yet rigorous treatment of the subject of continuum mechanics and elasticity at the senior undergraduate and first-year graduate levels. In all of the chapters of the second edition, additional explanations, examples, and exercise problems have been added. No attempt has been made to enlarge the scope or increase the number of topics covered. The large number of examples and exercise problems contained in the book systematically advance the understanding of vector and tensor analysis, basic kinematics, balance laws, field equations, and constitutive equations. The book may be used as a textbook for a first course on continuum mechanics as well as elasticity. A solutions manual has also been prepared for the book. The solutions manual is available from the publisher only to instructors who adopt the book as a textbook for a course.

Since the publication of the first edition, several users of the book communicated their comments and compliments as well as errors they found, for which the author thanks them. All of the errors known to the author have been corrected in the current edition. Drafts of the manuscript of this book prior to its publication were read by the author's doctoral students, who have made suggestions for improvements. In particular, the author wishes to thank Archana Arbind, Parisa Khodabakhshi, Jinseok Kim, and Namhee Kim for their help. The author is grateful to the following professional colleagues for their friendship, encouragement, and constructive comments on the book:

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College Station

## Preface to the First Edition

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This book is a simplified version of the author's book, *An Introduction to Continuum Mechanics with Applications*, 2nd ed., published by Cambridge University Press (New York, 2008), intended for use as an undergraduate text book. As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, undergraduate engineering students should be educated to think and work in such environments. Therefore, it is necessary to introduce the subject of *principles of continuum mechanics* (i.e., laws of physics applied to science and engineering systems) to undergraduate students so that they have a strong background in the basic principles common to all disciplines and are able to work at the interface of science and engineering disciplines. A first course on principles of mechanics provides an introduction to the basic concepts of stress and strain and conservation principles, and prepares engineer-scientists for advanced courses in traditional as well as emerging fields such as biotechnology, nanotechnology, energy systems, and computational mechanics. Undergraduate students with such backgrounds may seek advanced degrees in traditional (e.g., aerospace, civil, electrical, mechanical, physics, applied mathematics) as well as interdisciplinary degree programs (e.g., bioengineering, engineering physics, nanoscience and engineering, biomolecular engineering, and so on).

There are not many books on principles of mechanics that are written keeping undergraduate engineering or science students in mind. A vast majority of books on the subject are written for graduate students of engineering and tend to be more mathematical and too advanced to be of use for third year or senior undergraduate students. This book presents the subjects of mechanics of materials, fluid mechanics, and heat transfer in unified form using the conservation principles of mechanics. It is hoped that the book, which is simple, facilitates understanding of the main concepts of the previous three courses under a unified framework.

With a brief discussion of the concept of a continuum in Chapter 1, a review of vectors and tensors is presented in Chapter 2. Since the analytical language of applied sciences and engineering is mathematics, it is necessary for all students of this course to familiarize themselves with the notation and operations of vectors, matrices, and tensors that are used in the mathematical description of physical phenomena. Readers who are familiar with the topics of this chapter may refresh or skip and go to the next chapter. The subject of kinematics, which deals with geometric changes without regard to the forces causing the deformation, is discussed in Chapter 3. Measures of engineering normal and shear strains and definitions of mathematical strains are introduced here. Both simple one-dimensional systems as well as two-dimensional continua are used to illustrate the strain and strain-rate measures introduced. In Chapter 4, the concepts of

stress vector and stress tensor are introduced. It is here that readers are presented with entities that require two directions – namely, the plane on which they are measured and the direction in which they act – to specify them. Transformation equations among components of stress tensors referred to two different orthogonal coordinate systems are derived, and principal values and principal planes (i.e., eigenvalue problems associated with the stress tensor) are also discussed.

Chapter 5 is dedicated to the derivation of the governing equations of mechanics using the conservation principles of continuum mechanics (or laws of physics). The principles of conservation of mass, linear momentum, angular momentum, and energy are presented using one-dimensional systems as well as general three-dimensional systems. The derivations are presented in invariant (i.e., independent of a coordinate system) as well as in component form. The equations resulting from these principles are those governing stress and deformation of solid bodies, stress and rate of deformation of fluid elements, and transfer of heat through solid media. Thus, this chapter forms the heart of the course. Constitutive relations that connect the kinematic variables (e.g., density, temperature, deformation) to the kinetic variables (e.g., internal energy, heat flux, and stresses) are discussed in Chapter 6 for elastic materials, viscous fluids, and heat transfer in solids.

Chapter 7 is devoted to the application of the field equations derived in Chapter 5 and constitutive models presented in Chapter 6 to problems of heat conduction in solids, fluid mechanics (inviscid flows as well as viscous incompressible flows), diffusion, and solid mechanics (e.g., bars, beams, and plane elasticity). Simple boundary-value problems are formulated and their solutions are discussed. The material presented in this chapter illustrates how physical problems are analytically formulated with the aid of the equations resulting from the conservation principles.

As stated previously, the present book is an undergraduate version of the author's book *An Introduction to Continuum Mechanics* (Cambridge University Press, New York, 2008). The presentation herein is limited in scope when compared to the author's graduate level textbook. The major benefit of a course based on this book is to present the governing equations of diverse physical phenomena from a unified point of view, namely, from the conservation principles (or laws of physics) so that students of applied science and engineering see the physical principles as well as the mathematical structure common to diverse fields. Readers interested in advanced topics may consult the author's continuum mechanics book cited above or other titles listed in references therein.

The author is pleased to acknowledge the fact that the manuscript was tested with the undergraduate students in the College of Engineering at Texas A&M University as well as in the Engineering Science Program at the National University of Singapore. The students, in general, have liked the contents and the simplicity with which the concepts are introduced and explained. They also expressed the feeling that the subject is more challenging than most at the undergraduate level but a useful prerequisite to graduate courses in engineering.

The book contains so many mathematical expressions that it is hardly possible not to have typographical and other kinds of errors. The author wishes to thank in advance those who are willing to draw the author's attention to typos and errors, using the e-mail address [jnreddy@tamu.edu](mailto:jnreddy@tamu.edu).

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*The one exclusive sign of thorough knowledge is the power of teaching.*  
Aristotle

*It is the supreme art of the teacher to awaken joy in creative expression and knowledge.*

Albert Einstein

*A good teacher can inspire hope, ignite the imagination, and instill a love of learning.*

Brad Henry

*Teaching is a very noble profession that shapes the character, caliber, and future of an individual. If the people remember me as a good teacher, that will be the biggest honour for me.*

A. P. J. Abdul Kalam

## Symbols used in the Book

The symbols that are used throughout the book for various important quantities are defined in the following list. In some cases, the same symbol has different meaning in different parts of the book; it should be clear from the context.

Symbol	Meaning
$\mathbf{a}$	Acceleration vector, $\frac{D\mathbf{v}}{Dt}$
$a_{ij}$	Coefficients of matrix $[A]$
$c_v, c_p$	Specific heat at constant volume and pressure, respectively
$C$	Concentration; deformed configuration
$C_0$	Undeformed configuration
$C_{ijkl}$	Elastic stiffness coefficients
$d$	Diameter
$d\mathbf{a}$	Area element (vector) in spatial description
$d\mathbf{A}$	Area element (vector) in material description
$d\mathbf{f}$	Force on a small elemental area $\Delta a$ in $C$
$d\mathcal{F}$	Pre-image of $d\mathbf{f}$ in $C_0$
$ds$	Surface element in current configuration ( $= d\Gamma$ )
$dS$	Surface element in reference configuration ( $= d\Gamma_0$ )
$dv$	Volume element in current configuration ( $= d\Omega$ )
$dV$	Volume element in reference configuration ( $= d\Omega_0$ )
$d\mathbf{x}$	Line element (vector) in current configuration
$d\mathbf{X}$	Line element (vector) in reference configuration
$D, D_i$	Diffusion coefficients
$\mathbf{D}$	Symmetric part of the velocity gradient tensor, $\mathbf{L} = (\nabla\mathbf{v})^T$ ; that is, $\mathbf{D} = \frac{1}{2} [(\nabla\mathbf{v})^T + \nabla\mathbf{v}]$
$D_{ij}$	Rectangular Cartesian components of $\mathbf{D}$
$D/Dt$	Material time derivative, $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$
$e$	Internal energy per unit mass
$\hat{\mathbf{e}}$	Unit vector
$\hat{\mathbf{e}}_A$	Unit basis vector in the direction of vector $\mathbf{A}$
$\mathbf{e}_i$	Basis vector in the $x_i$ -direction
$(\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_z)$	Basis vectors in the $(r, \theta, z)$ system
$(\hat{\mathbf{e}}_R, \hat{\mathbf{e}}_\phi, \hat{\mathbf{e}}_\theta)$	Basis vectors in the $(R, \phi, \theta)$ system
$(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z)$	Basis vectors in the $(x, y, z)$ system
$(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$	Basis vectors in the $(x_1, x_2, x_3)$ system
$e_{ijk}$	Alternating symbol

Symbol	Meaning
$\mathbf{E}$	Green–Lagrange strain tensor, $\mathbf{E} = \frac{1}{2} (\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I})$
$E, E_1, E_2$	Young’s moduli (modulus of elasticity)
$\mathcal{E}$	Internal heat generation per unit mass
$\hat{\mathbf{E}}_i$	Unit base vector along the $X_i$ material coordinate direction ( $i = 1, 2, 3$ )
$E_{ij}$	Components of the Green–Lagrange strain tensor in the $(x_1, x_2, x_3)$ system ( $i, j = 1, 2, 3$ )
$E_{rr}, E_{\theta\theta}, E_{r\theta}, \dots$	Components of the Green strain tensor $\mathbf{E}$ in the cylindrical coordinate system $(r, \theta, z)$
$E_{xx}, E_{yy}, E_{xy}, \dots$	Components of the Green strain tensor $\mathbf{E}$ in the rectangular coordinate system $(x, y, z)$
$f$	Load per unit length of a bar
$\mathbf{f}$	Body force vector
$f_x, f_y, f_z$	Body force components in the $x, y,$ and $z$ directions
$\mathbf{F}$	Deformation gradient, $\mathbf{F} = (\nabla_0 \mathbf{x})^T$
$g$	Acceleration due to gravity; internal heat generation per unit volume
$G$	Shear modulus (modulus of rigidity)
$h$	Height of the beam; thickness; heat transfer coefficient
$H$	Heat input to the system
$I$	Second moment of area of a beam; current density
$\mathbf{I}$	Unit second-order tensor
$I_1, I_2, I_3$	Invariants of a second-order tensor
$J$	Determinant of $\mathbf{F}$ , $J =  \mathbf{F} $ (Jacobian)
$\mathbf{J}$	Diffusion flux
$k$	Spring constant; thermal conductivity
$\mathbf{k}$	Thermal conductivity tensor
$k_e$	Electrical conductivity
$K$	Kinetic energy
$\ell_{ij}$	Direction cosines
$L$	Length
$\mathbf{L}$	Velocity gradient tensor, $\mathbf{L} = (\nabla \mathbf{v})^T$
$M$	Bending moment in beam problems
$\hat{\mathbf{n}}$	Unit normal vector in the current configuration
$n_i$	$i$ th component of the unit normal vector $\hat{\mathbf{n}}$
$(n_x, n_y, n_z)$	Components of the unit normal vector $\hat{\mathbf{n}}$
$N$	Axial force in beam problems
$\hat{\mathbf{N}}$	Unit normal vector in the reference configuration
$N_I$	$I$ th component of the unit normal vector $\hat{\mathbf{N}}$

Symbol	Meaning
$p$	Internal pressure of a pressure vessel; thermodynamic or hydrostatic pressure
$P$	Hydrostatic pressure; point load in beams; perimeter
$\mathbf{P}$	First Piola–Kirchhoff stress tensor
$q$	Distributed transverse load on a beam
$q_0$	Intensity of the distributed transverse load in beams
$q_n$	Heat flux normal to the boundary, $q_n = \nabla \cdot \hat{\mathbf{n}}$
$\mathbf{q}$	Heat flux vector in the current configuration
$Q$	Mass flow rate; volume rate of flow
$Q_e$	Rate of heat due to current density
$r$	Radial coordinate in the cylindrical polar system; $r =  \mathbf{r} $
$\mathbf{r}$	Position vector in cylindrical coordinates, $\mathbf{x}$
$(r, \theta, z)$	Cylindrical coordinate system
$R$	Radial coordinate in the spherical coordinate system; $R =  \mathbf{R} $ ; universal gas constant; radius of curvature
$\mathbf{R}$	Position vector in the spherical coordinate system
$(r, \phi, \theta)$	Spherical coordinate system
$\mathbf{S}$	Second Piola–Kirchhoff stress tensor
$S_{ij}$	Components of the second Piola–Kirchhoff stress tensor in the rectangular coordinate system $(x_1, x_2, x_3)$
$t$	Time
$\mathbf{t}$	Stress vector; traction vector
$\mathbf{t}_i$	Stress vector on $x_i$ -plane, $\mathbf{t}_i = \sigma_{ij} \hat{\mathbf{e}}_j$
$T$	Temperature
$\mathbf{u}$	Displacement vector
$(u, v, w)$	Displacements in the $(x, y, z)$ coordinate system
$(u_1, u_2, u_3)$	Displacements in the $(x_1, x_2, x_3)$ coordinate system
$(u_r, u_\theta, u_z)$	Displacements in the $(r, \theta, z)$ coordinate system
$U$	Internal (or strain) energy
$v$	Velocity, $v =  \mathbf{v} $
$v_n$	Projection of $\mathbf{v}$ onto $\hat{\mathbf{n}}$ , $v_n = \mathbf{v} \cdot \hat{\mathbf{n}}$
$(v_1, v_2, v_3)$	Components of velocity vector $\mathbf{v}$ in $(x_1, x_2, x_3)$ system
$(v_r, v_\theta, v_z)$	Components of velocity vector $\mathbf{v}$ in $(r, \theta, z)$ system
$\mathbf{v}$	Velocity vector, $\mathbf{v} = \frac{D\mathbf{x}}{Dt}$
$\mathbf{v}_n$	Velocity vector normal to the plane (whose normal is $\hat{\mathbf{n}}$ )
$V$	Shear force in beam problems; scalar potential of body forces
$W$	Power input
$\mathbf{x}$	Position vector in the current configuration
$(x, y, z)$	Rectangular Cartesian coordinates
$(x_1, x_2, x_3)$	Rectangular Cartesian coordinates
$\mathbf{X}$	Position vector in the reference configuration



## Greek symbols

Symbol	Meaning
$\alpha$	Angle; coefficient of thermal expansion; kinetic energy coefficient
$\chi$	Deformation mapping
$\delta$	Dirac delta
$\delta_{ij}$	Components of the unit tensor, $\mathbf{I}$ (Kronecker delta)
$\boldsymbol{\varepsilon}$	Infinitesimal strain tensor, $\boldsymbol{\varepsilon} = \frac{1}{2} [(\nabla_0 \mathbf{u})^T + \nabla_0 \mathbf{u}]$
$\epsilon$	Total energy stored per unit mass
$\varepsilon_{ij}$	Rectangular components of the infinitesimal strain tensor
$\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{r\theta}, \dots$	Components of the infinitesimal strain tensor in the cylindrical coordinate system $(r, \theta, z)$
$\phi$	A typical scalar function; angular coordinate in the spherical coordinate system
$\Phi$	Viscous dissipation, $\Phi = \boldsymbol{\tau} : \mathbf{D}$ ; Airy stress function
$\gamma$	Shear strain in one-dimensional problems
$\Gamma$	Internal entropy production; total boundary vorticity vector, $\boldsymbol{\zeta} = 2\boldsymbol{\omega}$
$\zeta$	vorticity vector, $\boldsymbol{\zeta} = 2\boldsymbol{\omega}$
$\eta$	Entropy density per unit mass; dashpot constant
$\lambda$	Extension ratio; Lamé constant; eigenvalue
$\lambda_1, \lambda_2, \lambda_3$	Eigenvalues of a $3 \times 3$ matrix
$\mu$	Lamé constant; viscosity
$\nu$	Poisson's ratio; $\nu_{ij}$ Poisson's ratios
$\theta$	Angular coordinate in the cylindrical and spherical coordinate systems; angle; absolute temperature
$\theta_n, \theta_s$	Angles corresponding to maximum normal stress and maximum shear stress, respectively
$\rho$	Mass density in the current configuration
$\rho_0$	Mass density in the reference configuration
$\sigma$	Boltzman constant
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\sigma_{ij}$	Components of the stress tensor, $\boldsymbol{\sigma}$ , in the rectangular coordinate system $(x_1, x_2, x_3)$
$\sigma_n, \sigma_s$	Normal and shear stresses on a plane with normal $\hat{\mathbf{n}}$
$\sigma_{pi}, \sigma_{si}$	Principal normal and shear stresses
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}, \dots$	Components of the stress tensor $\boldsymbol{\sigma}$ in the cylindrical coordinate system $(r, \theta, z)$
$\tau$	Shear stress
$\boldsymbol{\tau}$	Viscous stress tensor
$\Omega$	Domain of a problem

Symbol	Meaning
$\Omega$	Spin tensor or skew symmetric part of the velocity gradient tensor, $(\nabla \mathbf{v})^T$ ; that is $\Omega = \frac{1}{2} [(\nabla \mathbf{v})^T - \nabla \mathbf{v}]$
$\Omega_{ij}$	Components of spin tensor $\Omega$ in the rectangular coordinate system $(x_1, x_2, x_3)$
$\Omega_{r\theta}, \Omega_{rz}, \Omega_{\theta z}$	Components of the spin tensor $\Omega$ in the cylindrical coordinate system $(, r, \theta, z)$
$\Omega_{xy}, \Omega_{xz}, \Omega_{yz}$	Components of the spin tensor tensor $\Omega$ in the rectangular coordinate system $(x, y, z)$
$\omega$	Angular velocity
$\boldsymbol{\omega}$	Axial (vorticity) vector of $\Omega$ , $\boldsymbol{\omega} = \frac{1}{2} \nabla \times \mathbf{u}$
$\omega_i$	Components of vorticity vector $\boldsymbol{\omega}$ in the rectangular coordinate system $(x_1, x_2, x_3)$
$\omega_x, \omega_y, \omega_z$	Components of vorticity vector $\boldsymbol{\omega}$ in the rectangular coordinate system $(x, y, z)$
$\psi$	Warping function; stream function
$\Psi$	Helmholtz free energy density; Prandtl stress function
$\nabla$	Gradient operator with respect to $\mathbf{x}$
$\nabla_0$	Gradient operator with respect to $\mathbf{X}$
$\nabla^2$	Laplace operator, $\nabla^2 = \nabla \cdot \nabla$
$\nabla^4$	Biharmonic operator, $\nabla^4 = \nabla^2 \nabla^2$
$[ ]$	Matrix of components of the enclosed tensor
$\{ \}$	Column of components of the enclosed vector
$\cdot$	Symbol used for the dot product or scalar product
$\times$	Symbol used for the cross product or vector product

*Ignorance more frequently begets confidence than does knowledge: it is those who know little, and not those who know much, who so positively assert that this or that problem will never be solved by science.*

Charles Darwin

*Science can purify religion from error and superstition. Religion can purify science from idolatry and false absolutes.*

Pope John Paul II

*A yogi seated in a Himalayan cave allows his mind to wander on unwanted things. A cobbler, in a corner at the crossing of several busy roads of a city, is absorbed in mending a shoe as an act of service. Of these two, the latter is a better yogi than the former.*

Swami Vivekananda

Table 1 Conversion factors

Quantity	US customary unit	SI equivalent
Time	s	s
Mass	lb (mass)	0.4536 kg
Length	in	25.4 mm
	ft	0.3048 m
Density	lb/in <sup>3</sup>	$27.68 \times 10^3 \text{ kg/m}^3$
	lb/ft <sup>3</sup>	$16.02 \text{ kg/m}^3$
Force	lb (force)	4.448 N
	kip (10 <sup>3</sup> lb)	4.448 kN
Pressure or stress	lb/in <sup>2</sup> (psi)	$6.895 \text{ kN/m}^2$
	ksi (10 <sup>3</sup> psi)	$6.895 \text{ MN/m}^2$
	Msi (10 <sup>6</sup> psi)	$6895 \text{ MN/m}^2$
Moment or torque	lb in	0.1130 Nm
	lb ft	1.356 Nm
Power	ft lb/s	1.356 W
	hp (550 ft lb/s)	745.7 W
Temperature	°F	0.5556°C
Conversion formula	$^{\circ}\text{F} = \frac{9}{5}^{\circ}\text{C} + 32^{\circ}\text{F}$	

s = second; lb = pound; in = inch; ft = foot; hp = horse power;  
 kg = kilogram (= 10<sup>3</sup> grams); m = meter; mm = millimeter (10<sup>-3</sup> m);  
 N = Newton; W = Watt; Pa = Pascal = N/m<sup>2</sup>;  
 kN = 10<sup>3</sup> N; MN = 10<sup>6</sup> N; MPa = 10<sup>6</sup> Pa; GPa = 10<sup>9</sup> Pa

**Note:** Historical notes at the end of each chapter were based on information found at <https://www.wikipedia.org/>. Quotes by various people included in this book were found at different web sites. For example, visit:

<http://naturalscience.com/dsqhome.html>,

[http://thinkexist.com/quotes/david\\_hilbert/](http://thinkexist.com/quotes/david_hilbert/),

<http://www.yalescientific.org/2010/10/from-the-editor-imagination-in-science/>,

<https://www.brainyquote.com/quotes/>.

The author is motivated to include the quotes at various places in his book for their wit and wisdom, although he cannot vouch for their accuracy.