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1 INTRODUCTION

One thing I have learned in a long life: that all our science, measured against reality, is primitive and childlike – and yet it is the most precious thing we have.
Albert Einstein

What we need is not the will to believe but the will to find out.
Bertrand Russell

Those who know how to think need no teachers.
Mahatma Gandhi

1.1 Continuum Mechanics

Matter is composed of discrete molecules, which in turn are made up of atoms. An atom consists of electrons, positively charged protons, and neutrons. Electrons form chemical bonds. An example of mechanical (that is, has no living cells) matter is a carbon nanotube (CNT), which consists of carbon molecules in a certain geometric pattern in equilibrium with each other, as shown in Fig. 1.1.

Another example of matter is a biological cell, which is a fundamental unit of any living organism. There are two types of cells: prokaryotic and eukaryotic cells. Eukaryotic cells are generally found in multicellular organs and they have a true nucleus, distinct from a prokaryotic cell. Structurally, cells are composed of a large number of macromolecules (or large molecules). These macromolecules consist of large numbers of atoms and form specific structures, like chromosome and plasma membranes in a cell. Macromolecules occur under four major types: carbohydrates, proteins, lipids, and nucleic acids. To highlight the hierarchical nature of the structures formed by the macromolecule in a cell, let us analyze a chromosome.

Chromosomes, which are carriers of hereditary traits in an individual, are found inside the nucleus of all eukaryotes. Each chromosome consists of a single nucleic acid macromolecule called deoxyribonucleic acid (DNA) (2.2–2.4 nanometers wide). These nucleic acids are in turn formed from the specific arrangement of monomers called mono-nucleotides (0.3–0.33 nanometers). The fundamental units of nucleotides are formed again by a combination of a specific arrangement of a phosphate radical, nitrogenous base, and a carbohydrate sugar. The hierarchical nature of the chromosome is as shown in Fig. 1.2(a). Similarly to the chromosomes, all the structures in a cell are formed from a combination of the macromolecules.

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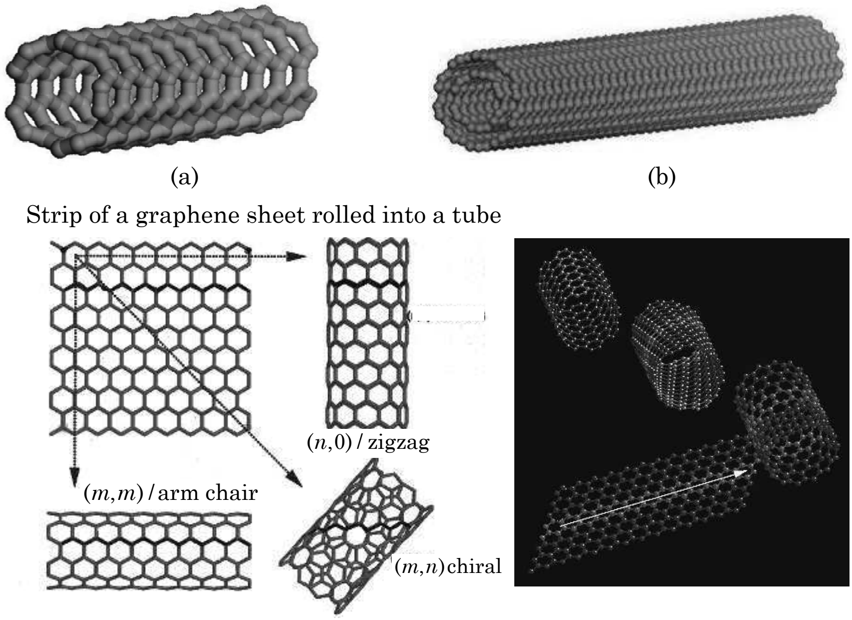

Fig. 1.1 Carbon nanotubes (CNTs) with different chiralities

At the macroscopic scale, eukaryotic cells can be divided into three distinct regions: *nucleus*, *plasma membrane*, and *cytoplasm* having a host of other structures, as shown in Fig. 1.2(b). The nucleus consists of chromosomes and other protein structures and is the control center of the cell determining how the cell functions. The plasma membrane encloses the cell and separates the material outside the cell from inside. It is responsible for maintaining the integrity of the cell and also acts as channels for the transport of molecules to and from the cell. Cell membrane is made up of a double layer of phospholipid molecules (macromolecule), having embedded transmembrane proteins. The region between the cell membrane and the nucleus is the cytoplasm which consists of a gel-like fluid called cytosol, the cytoskeleton, and other macromolecules. Cytoskeleton forms the biomechanical framework of the cell and consists of three primary protein macromolecule structures of actin filaments, intermediate filaments, and microtubules. Cell growth, expansion, and replication are all carried out in the cytoplasm.

The interactions between the different components of the cell are responsible for maintaining the structural integrity of the cell. The analysis of these interactions to obtain the response of the cell when subjected to an external stimulus (mechanical, electrical, chemical) is studied systematically under cell mechanics. The structural framework of primary macromolecular structures in a cell is shown in Fig. 1.2(c).

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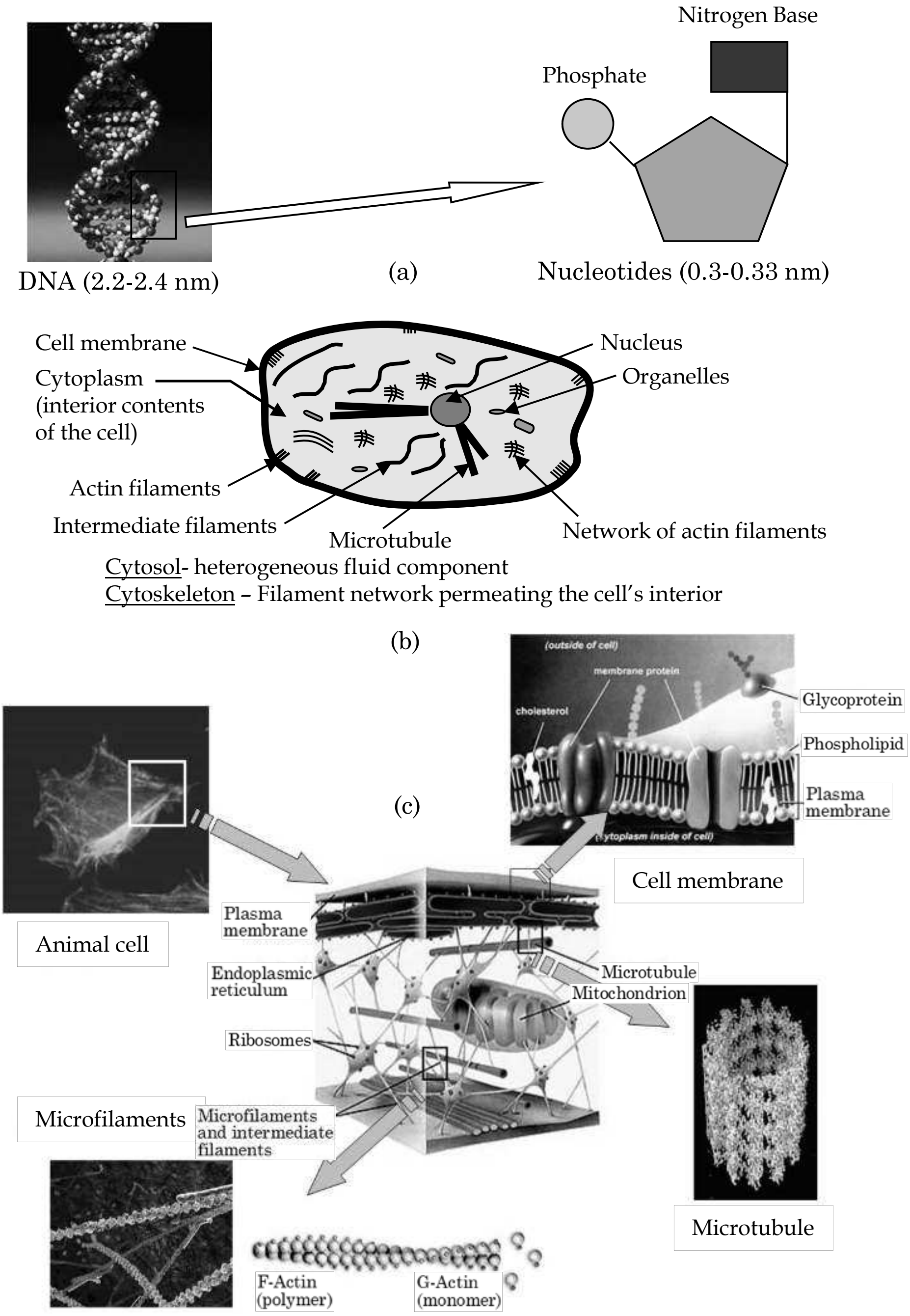

Fig. 1.2 (a) Hierarchical nature of chromosome (b) Structure of a generalized cell (c) Macromolecular structure in a cell

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The study of matter at molecular or atomistic levels is very useful for understanding a variety of phenomena; but studies at these scales are not useful to solve common engineering problems. The understanding gained at the molecular level needs to be taken to the *macrosopic* scale (that is, scale that a human eye can see) to be able to study its behavior. Central to this study is the assumption that the discrete nature of matter can be overlooked, provided the length scales of interest are large compared to the length scales of a discrete molecular structure. Thus, matter at sufficiently large length scales can be treated as a *continuum* in which all physical quantities of interest, including density, are continuously differentiable.

The subject of *mechanics* deals with the study of motion and forces in solids, liquids, and gases and the deformation or flow of these materials. In such a study, we make the simplifying assumption, for analysis purposes, that the matter is distributed continuously, without gaps or empty spaces (i.e., we disregard the molecular structure of matter). Such a hypothetical continuous matter is termed a *continuum*. In essence, in a continuum all quantities such as the density, displacements, velocities, stresses, and so on vary continuously so that their spatial derivatives exist and are continuous. The continuum assumption allows us to shrink an arbitrary volume of material to a point, in much the same way as we take the limit in defining a derivative, so that we can define quantities of interest at a point. For example, density (mass per unit volume) of a material at a point is defined as the ratio of the mass Δm of the material to a small volume ΔV surrounding the point in the limit that ΔV becomes a value ϵ^3, where ϵ is small compared with the mean distance between molecules:

$$\rho = \lim_{\Delta V \to \epsilon^3} \frac{\Delta m}{\Delta V}\,. \tag{1.1.1}$$

In fact, we take the limit $\epsilon \to 0$. A mathematical study of mechanics of such an idealized continuum is called *continuum mechanics*.

Engineers and scientists undertake the study of continuous systems to understand their behavior under "working conditions," so that the systems can be designed to function properly and produced economically. For example, if we were to repair or replace a damaged artery in a human body, we must understand the function of the original artery and the conditions that led to its damage. An artery carries blood from the heart to different parts of the body. Conditions like high blood pressure and increase in cholesterol content in the blood may lead to deposition of particles in the arterial wall, as shown in Fig. 1.3. With time, accumulation of these particles in the arterial wall hardens and constricts the passage, leading to cardiovascular diseases. A possible remedy for such diseases is to repair or replace the damaged portion of the artery. This in turn requires an understanding of the deformation and stresses caused in the arterial wall by the flow of blood. The understanding is then used to design the vascular prosthesis (that is, artificial artery).

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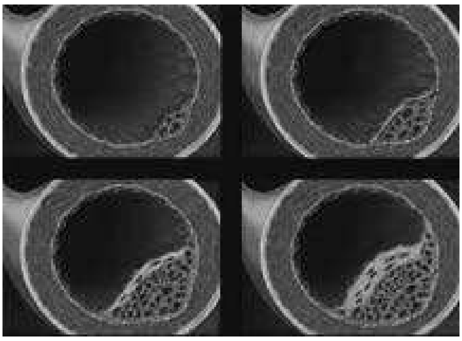

Fig. 1.3 Progressive damage of (plaque formation in) artery due to deposition of particles in the arterial wall

The primary objectives of this book are: (1) to study the conservation and balance principles in mechanics of continua and formulate the equations that describe the motion and mechanical behavior of materials; and (2) to present the applications of these equations to simple problems associated with flows of fluids, conduction of heat, and deformation of solid bodies. While the first of these objectives is an important topic, the reason for the formulation of the equations is to gain a quantitative understanding of the behavior of an engineering system. This quantitative understanding is useful in the design and manufacture of better products.

1.2 Examples of Engineering Systems

Typical examples of engineering problems, which are sufficiently simple to cover in this course, are described below. At this stage of discussion, it is sufficient to rely on the reader's intuitive understanding of concepts.

Example 1.2.1

(**A mechanical structure**) We wish to design a diving board which must enable the swimmer to gain enough momentum for the swimming exercise. The diving board is fixed at one end and free at the other end (see Fig. 1.4). The board is initially straight and horizontal, and of length L and uniform cross-section $A = bh$.

The design process consists of selecting the material with Young's modulus E and cross-sectional dimensions b and h such that the board carries the weight W of the swimmer. The design criteria are that the stresses developed do not exceed the allowable stress and the deflection of the free end does not exceed a pre-specified value δ. A preliminary design of such systems is often based on mechanics of materials equations. The final design involves the use of more sophisticated equations, such as the three-dimensional elasticity equations. The equations of elementary beam theory may be used to find a relation between the deflection δ of the free end in terms of the length L, cross-sectional dimensions b and h, Young's modulus E, and weight W:

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$$\delta = \frac{4WL^3}{Ebh^3}\,. \tag{1.2.1}$$

Given δ (allowable deflection) and load W (maximum possible weight of a swimmer), one can select the material (Young's modulus, E) and dimensions L, b, and h (which must be restricted to the standard sizes fabricated by a manufacturer). In addition to the deflection criterion, one must also check if the board develops stresses that exceed the allowable stresses of the material selected. Analysis of pertinent equations provides the designer with alternatives to select the material and dimensions of the board so as to have a cost-effective but functionally reliable structure.

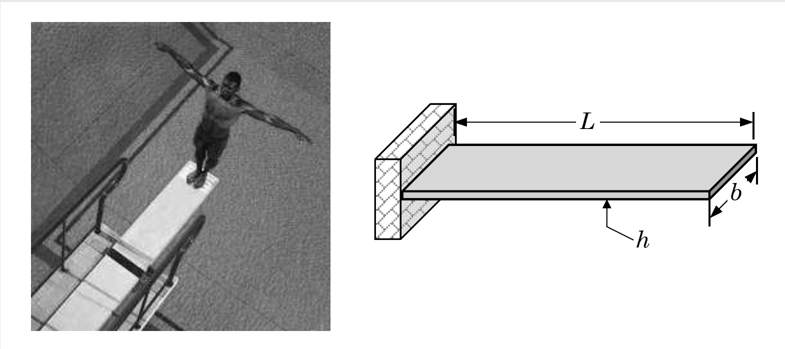

Fig. 1.4 A diving board fixed at left end and free at right end

Example 1.2.2

(**Fluid flow**) We wish to measure the viscosity μ of a lubricating oil used in rotating machinery to prevent damage of the parts in contact. Viscosity, like Young's modulus of solid materials, is a material property that is useful in the calculation of shear stresses developed between a fluid and a solid body.

A capillary tube is used to determine the viscosity of a fluid via the formula

$$\mu = \frac{\pi d^4}{128L}\frac{P_1 - P_2}{Q}\,, \tag{1.2.2}$$

where d is the internal diameter, L is the length of the capillary tube, P_1 and P_2 are the pressures at the two ends of the tube (oil flows from one end to the other, as shown in Fig. 1.5), and Q is the volume rate of flow at which the oil is discharged from the tube.

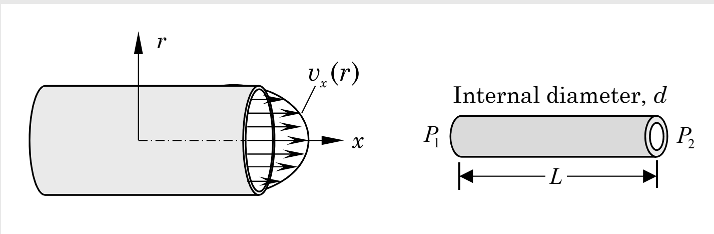

Fig. 1.5 Measurement of viscosity of a fluid using capillary tube

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Example 1.2.3

(**Heat flow in solids**) We wish to determine the heat loss through the wall of a furnace. The wall typically consists of layers of brick, cement mortar, and cinder block (see Fig. 1.6). Each of these materials provides a varying degree of thermal resistance. The Fourier heat conduction law

$$q = -k\frac{dT}{dx} \tag{1.2.3}$$

provides a relation between the heat flux q (heat flow per unit area) and gradient of temperature T. Here k denotes thermal conductivity ($1/k$ is the thermal resistance) of the material. The negative sign in Eq. (1.2.3) indicates that heat flows from a high-temperature region to a low-temperature region. Using the continuum mechanics equations, one can determine the heat loss when the temperatures inside and outside of the building are known. A building designer can select the materials as well as thicknesses of various components of the wall to reduce the heat loss (while ensuring necessary structural strength – a structural analysis aspect).

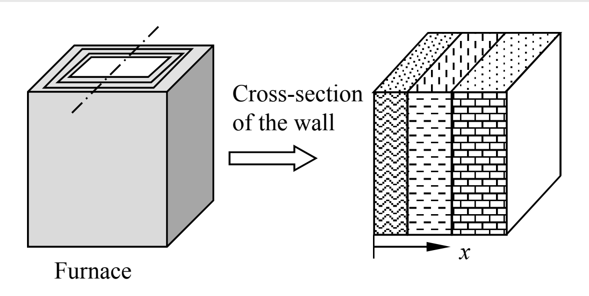

Fig. 1.6 Heat transfer through a composite wall of a furnace

The previous three examples provide some indication of the need for studying the response of materials under the influence of external loads. The response of a material is consistent with the laws of physics and the constitutive behavior of the material. This book has the objective of describing the physical principles and deriving the equations governing the stress and deformation of continuous materials, and then solving some simple problems from various branches of engineering to illustrate the applications of the principles discussed and equations derived.

1.3 Objective of the Study

The primary objective of this book, as already stated, is twofold: (1) use the physical principles to derive the equations that govern the motion and thermomechanical response of materials and systems; and (2) application of these equations for the solution of specific problems of engineering and applied science (e.g., linearized elasticity, heat transfer, and fluid mechanics). The governing

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equations for the study of deformation and stress of a continuous material are nothing but an analytical representation of the global laws of conservation of mass and balance of momenta and energy, and the constitutive response of the continuum. They are applicable to all materials that are treated as a continuum. Tailoring these equations to particular problems and solving them constitute a bulk of engineering analysis and design.

The study of motion and deformation of a continuum (or a "body" consisting of continuously distributed material) can be broadly classified into five basic categories:

(1) Kinematics
(2) Conservation of mass
(3) Kinetics (balance of linear and angular momentum)
(4) Thermodynamics (first and second laws of thermodynamics)
(5) Constitutive equations.

Kinematics is a study of the geometric changes or deformation in a continuum, without the consideration of forces causing the deformation. The principle of *conservation of mass* ensures that the mass of the deforming medium is conserved when no mass is created or destroyed. *Kinetics* is the study of the static or dynamic equilibrium of forces and moments acting on a continuum, using the principles of balance of linear and angular momentum. This study leads to equations of motion as well as the symmetry of stress tensor in the absence of body couples. *Thermodynamic principles* are concerned with the balance of energy and relations among heat, mechanical work, and thermodynamic properties of the continuum. *Constitutive equations* describe thermomechanical behavior of the material of the continuum, and they relate the dependent variables introduced in the kinetic description to those introduced in the kinematic and thermodynamic descriptions. Table 1.1 provides a brief summary of the relationship between physical principles and governing equations, and physical entities involved in the equations. To the equations derived from physical principles, one must add *boundary conditions* of the system (and *initial conditions* if the phenomena are time-dependent) to complete the analytical description.

1.4 Summary

In this chapter, the concept of a continuous medium is discussed and the major objectives of the present book, namely, (a) use of the principles of mechanics to derive the equations governing a continuous medium, and (b) applications of these equations in the solution of specific problems arising in engineering, are presented. Mathematical formulation of the governing equations of a continuous medium necessarily requires the use of vectors, matrices, and tensors – mathematical tools that facilitate analytical formulation of the natural laws. Therefore,

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Table 1.1 Major topics of the present study, principles of mechanics used, resulting governing equations, and dependent variables involved

Topic of study	Physical principle	Resulting equations	Variables involved
1. Kinematics	Based on geometric changes	Strain–displacement relations	Displacements and strains
		Strain rate–velocity relations	Velocities and strain rates
2. Mass	Conservation of mass	Continuity equation	Density and velocities
3. Kinetics	Balance of linear momentum	Equations of motion	Stresses, and velocities
	Balance of angular momentum	Symmetry of stress tensor	Stresses
4. Thermodynamics	First law	Energy equation	Temperature, heat flux, stresses, and velocities
	Second law	Clausius–Duhem inequality	Temperature, heat flux, and entropy
5. Constitutive equations (not all relations are listed)	Constitutive axioms	Hooke's law	Stresses, strains, heat flux, and temperature
		Newtonian fluids	Stresses, pressure, velocities
		Fourier's law	Heat flux, and temperature
		Equations of state	Density, pressure, temperature
6. Boundary conditions	All of the above principles and axioms	Relations between kinematic and kinetic variables	All of the above variables

it is useful to gain certain operational knowledge of vectors, matrices, and tensors first. Chapter 2 is dedicated to this purpose.

The study of principles of mechanics is broadly divided into topics outlined in Table 1.1. The first four topics constitute the subjects of Chapters 3 through 6, respectively. For the convenience of analysis, a continuum may be treated either as a solid or a fluid (liquids and gases), and equations derived in Chapters 3 through 6 are specialized in Chapter 7 to study, through some simple problems, the behavior of solids and fluids.

Many of the concepts presented herein are the same as those that were most likely introduced in undergraduate courses on mechanics of materials, heat transfer, fluid mechanics, and material science. The present course brings together

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these courses under a common mathematical framework and, thus, may require mathematical tools as well as concepts not seen before. Readers must motivate and challenge themselves to learn the new mathematical concepts introduced here, because *the language of engineers is mathematics.* This subject also serves as a prelude to many graduate courses in engineering and applied sciences.

While this book is self-contained for an introduction to continuum mechanics, there are several books that may provide an advanced treatment of the subject. The graduate level text book by the author, *An Introduction to Continuum Mechanics with Applications*, 2nd ed. (Cambridge University Press, New York, 2013), provides additional and advanced material. Interested readers may consult other titles listed in **References for Additional Reading** at the end of each chapter.

Problems

1.1 The end deflection (in the direction of the force) of a cantilever beam subjected to end load W (N) is given by

$$\delta = \frac{WL^3}{3EI_y},$$

where L (m) is the length, E (N/m)^2 is Young's modulus, and I_y (m^4) is the moment of inertia of the beam about the y-axis. If the beam is of length $L = 3$ m, and has a cross-section of channel shape with width $b = 300$ mm and height $h = 80$ mm, as shown in Fig. P1.1, determine the maximum tensile and compressive stresses in the beam, if the stress is $\sigma(x, z) = M(x)z/I_y$, where z is the transverse coordinate measured from the geometric centroid of the cross-section and M is the bending moment, which is a function of position x along the length of the beam. The bending moment is taken as positive clockwise at any section x along the beam.

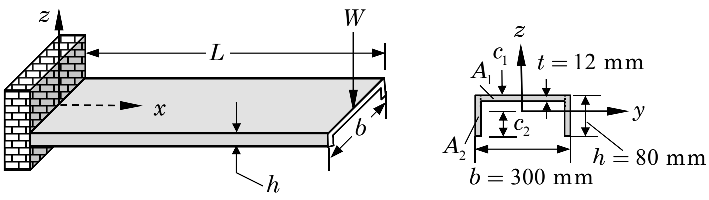

Fig. P1.1

1.2 The velocity field of fully developed flow through a circular pipe of diameter d and length L is given by (see Fig. 1.5)

$$v_z(r) = -\frac{d^2}{16\mu}\frac{dP}{dx}\left(1 - 4\frac{r^2}{d^2}\right),$$

where x is the coordinate along the pipe, r is the radial coordinate, μ is the fluid viscosity, and dP/dx is the pressure gradient in the x-direction. Show that the viscosity μ is given by

$$\mu = -\frac{\pi d^4}{128Q}\frac{dP}{dx},$$

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