THE EPISTEMIC LIGHTNESS OF TRUTH

This book analyses and defends the deflationist claim that there is nothing deep about our notion of truth. According to this view, truth is a ‘light’ and innocent concept, devoid of any essence that could be revealed by scientific inquiry. Cezary Cieśliński considers this claim in light of recent formal results on axiomatic truth theories, which are crucial for understanding and evaluating the philosophical thesis of the innocence of truth. Providing up-to-date discussion and original perspectives on this central and controversial issue, his book will be important for those with a background in logic who are interested in formal truth theories and in current philosophical debates about the deflationary conception of truth.

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THE EPISTEMIC LIGHTNESS
OF TRUTH

Deflationism and Its Logic

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Introduction

Is there anything more familiar and obvious than the opposition of truth and falsity? It is true that the earth is round. It is false that dragons eat virgins. (As everyone knows, dragons eat only pistachio marzipan with vanilla truffle.) Elementary, is it not? However, if this is so familiar, what then is truth? When confronted with such a direct question, many of us are tempted to repeat the famous words of Saint Augustine: “If no one asks me, I know what it is. If I wish to explain it to him who asks, I do not know”.

Being that no decent philosopher can rest satisfied with ignorabimus, some answers have naturally been proposed. Indeed, answers have proliferated, with various philosophical schools promoting their own worldviews and agendas. Unfortunately, no lasting consensus has emerged, with the only exception perhaps being the following. Philosophers seemed to agree that the task of explaining the nature of truth is a daunting one; it is hard, complicated, deep and far-reaching. However, in recent times, serious doubts have emerged even here. Some modern philosophers have reacted to the ancient puzzles with a bold claim; they have said that, in fact, truth has no nature, and the very concept of truth is, in some sense, innocent or trivial. This book is devoted to the analysis and assessment of this claim.

So, what is truth? Here is a selection of quotes giving answers to this question.

- ‘To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true.’ (Aristotle, *Metaphysics*, IV 7, 1011b27)
- ‘Veritas est adaequatio intellectus et rei.’ (‘Truth is the conformity of the intellect to the things.’ Thomas Aquinas, *Summa Theologica* I, Q 16)
- ‘The nominal definition of truth, namely that it is the agreement of cognition with its object, is here granted and presupposed.’ (I. Kant, *Critique of Pure Reason*, A 57-8/B 82)

In one crucial respect, the first of these classical formulations is rather different from the other two. When defining truth, both Aquinas and Kant mention a special relation which is supposed to hold between the intellect (or
cognition) and its object; namely, the relation of ‘conformity’ or ‘agreement’. In the literature, it is also customary to use the term ‘correspondence’ in this context. In short, the classical definition of truth consists in defining truth as the correspondence of thought (cognition) with reality.

However, once we start playing with the idea of a correspondence relation, difficult philosophical questions arise. What is the nature of this special relation between thought (or language) and reality? Does a given sentence (proposition) correspond to reality taken as a whole or to only a fragment of it? If it is the latter, then which fragment is it? Can we claim, for example, that it is the objective facts that make our sentences (propositions, thoughts) true? Here is another question concerning correspondence: in virtue of what exactly does this relation hold? For example, is the requirement that a truth bearer (sentence, proposition) has a similar structure to the corresponding fragment of reality (fact, state of affairs)? These are indeed troublesome questions, and many philosophers have been deeply dissatisfied with the traditional answers given to them.

On the other hand, unlike in the case of Aquinas and Kant, when reading Aristotle’s explanation, it is hard to deny the impression that the notion of truth is (in some sense to be specified) simple, innocent and trivial. Aristotle’s formulation is much more austere and cautious than those of the other authors quoted here. Indeed, it is worth emphasising that here Aristotle does not appeal at all to correspondence. To say ‘there are dragons’ is false because there are no dragons; to say ‘there are horses’ is true, since there are horses; and to say ‘there are no electrons’ is false because electrons exist – that is the underlying idea. In contrast to Aquinas and Kant, no special relation between thought (or language) and reality has been invoked.

This Aristotelian motive came to the foreground in some recent works on truth, notably by philosophers representing the popular current called ‘deflationism about truth’. It is indeed the deflationary intuition that truth is in some sense insubstantial, light or metaphysically thin.\footnote{This is not to say that Aristotle himself should be classified as a deflationist. On the one hand, as noted by Crivelli (2004, p. 30-31), relational properties were not considered ‘real’ or ‘genuine’ by Aristotle, and since he considered truth to be a relational property, he was ‘committed to the view that truth is not a genuine property. In this respect Aristotle’s position is close to modern ‘minimalist’ theories of truth, which also claim that truth is not a genuine property’. On the other hand, a careful reconstruction of Aristotle’s views leads Crivelli to the conclusion that Aristotle was, after all, an adherent of a correspondence theory of truth. For details, the reader is referred to Chapter 4 of (Crivelli 2004).} The
deflationists frequently repeat that when we attribute truth to a sentence (or a proposition), we might just as well assert this very sentence (or this proposition). They also say that truth has no ‘essence’ which could be revealed by deep scientific research. As an example, consider the following (typical) quote from Horwich:

[...] the traditional attempt to discern the essence of truth – to analyse that special quality which all truths supposedly have in common – is just a pseudo-problem based on syntactic overgeneralization. Unlike most other properties, being true is unsusceptible to conceptual or scientific analysis. No wonder that its ‘underlying nature’ has so stubbornly resisted philosophical elaboration; for there is simply no such thing. (Horwich 1999, p. 5)

What does it mean to claim that truth has no ‘underlying nature’; that it is insubstantial, light or metaphysically thin? Truth may be a simple notion (as the deflationist wants it to be) but – as it turns out – answering the last question is still quite a demanding task. The exploration of this topic is a central theme of this book.

Here I am going to defend a certain strong version of the lightness thesis. The outline is as follows. Two explications of the lightness claim have been prominent in the literature. One of them is that truth is a disquotational notion and can be fully characterised by the so-called T-sentences or ‘Tarski biconditionals’; that is, by the equivalences falling under the schema ‘the sentence (or the proposition) \( \varphi \) is true if and only if \( \varphi \)’. In this view, it is the simplicity and triviality of the T-schema that gives meaning and justification to the lightness thesis. The second explication is the conservativity proposal; roughly, truth is innocent because adequate theories of truth do not establish any new non-semantic facts. A detailed discussion of these explications will be presented in Part II and Part III of this book.

Both proposals have evoked harsh criticism. In both cases, the main thrust was directed against the truth-theoretic weakness of the envisaged disquotational (or conservative) theories of truth. The critics have claimed that such theories cannot provide an adequate characterisation of truth for a very simple reason: in fact our knowledge about truth goes beyond such theories; in other words, facts about truth are known to us which cannot be deduced from disquotational/conservative theories of truth. In effect, the adherents to these truth theories cannot account for this additional knowledge. This is the objection.
Let me emphasise that the problem of the truth-theoretic weakness is very real. It does not rest on any misunderstanding or a flaw in the critics’ reasoning. On the contrary, critics have quite correctly identified the aforementioned traits of disquotational and conservative theories of truth. Nevertheless, the main philosophical claim of this book is that an adequate theory of truth can be both disquotational and conservative. In the final chapter a solution to the problem of truth-theoretic weakness will be proposed. Namely, it will be argued that the deflationist who accepts a given disquotational and conservative theory of truth has at his disposal sufficient means to account for any additional knowledge about truth that we may possess, including facts about truth which are not provable in his initial theory. In this way, the deflationary standpoint will be vindicated.

In the discussion of innocence claims, this book will often employ formal tools of modern logic. More specifically, the claims in question will be analysed mainly within the arithmetical framework. The case of arithmetic will be treated here as a model example against which the deflationary tenets can be evaluated and tested. The assumption is that if innocence claims do not pass such a preliminary arithmetical test, then they are to be disqualified almost from the start without the need to take into consideration additional semantic phenomena. The general motivation might be global, but testing is best done on a local level; that is at least the idea. Accordingly, the book does not provide any analysis of the use of truth in science in general, nor do I purport to analyse any particular troublesome traits of natural languages, such as ambiguity, vagueness or indexicality. Instead of taking a broad-brush approach, I want to offer to the reader a detailed analysis of some quite specific issues arising in arithmetical contexts on the borderline between philosophy and formal logic.

Typically, the discussion will proceed in accordance with the following schema. Starting with some basic, philosophical idea (‘truth is nothing more than disquotation’ can serve as an example), I present the intuitions guiding the proponents of a given philosophical standpoint. In the next stage, formal theories are introduced, treated as attempts at a precise characterisation of the idea in question. The third stage presents the analysis of logical properties of these formal theories – it is here where formal methods will be most extensively used. Finally, the discussion returns to philosophical issues, which are analysed again in the light of mathematical results.
The plan of the book is as follows.

Chapter 1 (‘Preliminaries’) fixes the basic notation and terminology; I also state (without proofs) some classical formal results, which will be useful later in the book. The reader might wish to start by checking the terminology and then to use Chapter 1 as reference material, to be consulted whenever the need arises.

In Chapter 2 two general methods of characterising the notion of truth are laid out: axiomatic and model-theoretic. Being that the axiomatic method will be deemed the more suitable of the two for the purpose of defending the innocence claims, this book will focus on the axiomatic approach. It will hence deal with attempts to characterise the notion of truth \textit{simpliciter} (the truth of sentences as we understand them in contrast to ‘truth under an interpretation’ or ‘truth in a model’) by means of simple and basic principles, with the truth predicate functioning as a primitive, undefined symbol.

Special attention will be given to disquotational and conservative truth theories; they will be discussed in Parts I and II of this book. In each of these cases I start by presenting philosophical intuitions behind both types of truth theories; the discussion will then proceed to an analysis of their formal properties. The last chapters of both Part I and Part II are devoted to the presentation of the main objections against (respectively) disquotational and conservative theories of truth. These objections are known in the literature as ‘the generalisation problem’ and ‘the conservativeness argument’.

In the final Part III I present my uniform response to both the generalisation problem and the conservativeness argument, defending disquotational and conservative truth theories against the charge of truth-theoretic weakness. The claim will be, in effect, that such theories stay with us as formalisations of a natural and fundamentally correct approach to truth.

All of Parts I through III begin with introductory sections, which not only sketch the basic intuitions but contain also a more detailed plan of the subsequent chapters, providing the reader with a map of what is to follow. In addition, each chapter following the ‘Preliminaries’ ends with a summary, where the main claims are briefly listed.

I will generally avoid describing non-trivial mathematical proofs and techniques whose presentations can be found elsewhere in book format. Normally in such cases the most important theorems will be merely stated with a reference given. Nevertheless, various theorems (particularly new results, including those due to the author or his students) will be introduced with full proofs. Open mathematical problems, arising from the logical
and philosophical analysis of deflationary ideas about truth, will also be presented. It should be emphasised here that these formal parts do not just serve philosophical purposes. The additional aim is to bring the reader up to date with some of the most recent developments in formal work on truth theories and, ultimately, to convey the impression of the field as a fascinating and vibrant one worthy of further investigation. Nevertheless, for the reader’s convenience, in the summaries of the formal chapters I will clearly indicate which of the theorems are of particular importance for the main philosophical theme of the book.

Let me finish by saying that the idea of translating philosophical intuitions into precise, formal claims and hypotheses is one that I find immensely appealing. This is not meant to minimise the role of intuitions, which remain absolutely crucial for our research in all of its stages. Nonetheless, it is only the precise formulations, with all the care given to the details, which permit us to test the validity of our intuitions. Certainly, there are risks, but I consider them worth taking. From my point of view, much of the value of deflationism considered as a philosophical standpoint derives from the fact that, to a substantial degree, it is susceptible to such a procedure.