

Contents

<i>List of Figures</i>	<i>page</i> xii
<i>List of Tables</i>	xix
<i>Acknowledgments</i>	xx
<i>General Notation</i>	xxi
1 Introduction	1
1.1 Claims Problems	1
1.2 The Model	3
1.3 Two Puzzles in the Talmud	9
1.4 Three Approaches	11
1.4.1 Direct Approach	12
1.4.2 Axiomatic Approach	12
1.4.3 Game-Theoretic Approach	15
1.5 Historical Note	16
1.6 Road Map	16
1.7 How to Use This Book	17
1.8 Concluding Comment	18
2 Inventory of Division Rules	21
2.1 An Inventory of Rules	22
2.1.1 Proportional Rule	22
2.1.2 Constrained Equal Awards Rule	23
2.1.3 Constrained Equal Losses Rule	26
2.1.4 Concede-and-Divide	28
2.1.5 Piniles' Rule	31
2.1.6 Talmud Rule	32
2.1.7 Constrained Egalitarian Rule	34
2.1.8 Random Arrival Rule	37
2.1.9 Minimal Overlap Rule	38
2.1.10 Rule Based on Random Stakes	43

viii	Contents	
	2.2 Families of Rules	45
	2.2.1 Sequential Priority Family	45
	2.2.2 Young's Family	46
	2.2.3 ICI and CIC Families	53
	2.3 Summary	60
3	Basic Properties of Division Rules	62
	3.1 Balance	62
	3.2 Continuity	63
	3.3 Homogeneity	64
	3.4 Lower and Upper Bounds on Awards and Losses	65
	3.4.1 Defining Bounds	65
	3.4.2 Recursive Assignment of Lower Bounds	72
	3.5 Conditional Full Compensation, Conditional Null Compensation, and Related Properties	75
	3.6 Symmetry Properties	79
	3.7 Order Preservation Properties	89
4	Monotonicity Properties	94
	4.1 Endowment Monotonicity and Related Properties	95
	4.2 Claim Monotonicity and Related Properties	105
	4.3 Inverse Sets Axioms	115
5	Claims Truncation Invariance and Minimal Rights First	118
	5.1 Claims Truncation Invariance	119
	5.2 Minimal Rights First	123
6	Composition Down and Composition Up	131
	6.1 Composition Down	131
	6.2 Composition Up	140
7	Duality	157
	7.1 Duality for Rules	157
	7.2 Duality for Properties	165
	7.3 Duality for Theorems	171
	7.4 Characterizations	172
8	Other Invariance Properties	182
	8.1 No Advantageous Transfer	182
	8.2 Claims Separability and Variants	184
	8.3 Convexity and Additivity Properties	187
	8.4 Rationalizing Rules as Maximizers of Binary Relations	195
9	Operators	200
	9.1 Claims Truncation Operator	200

Contents	ix
9.2 Attribution of Minimal Rights Operator	202
9.3 Convexity Operator	205
9.4 Relating and Composing the Operators	206
9.5 Preservation of Properties under Operators	214
9.5.1 Properties Preserved under Claims Truncation	215
9.5.2 Properties Preserved under Attribution of Minimal Rights Operator	218
9.5.3 Properties Preserved under the Composition of the Claims Truncation and Attribution of Minimal Rights Operators	219
9.5.4 Properties Preserved under Convexity	221
9.6 Extension Operators	222
9.7 Summarizing	227
10 Variable-Population Model: Consistency and Related Properties	229
10.1 The Variable-Population Model	230
10.2 Consistency and Related Properties	231
10.3 Converse Consistency	239
10.4 Other Logical Relations between Consistency, Its Converse, and Other Properties	241
10.5 Lifting of Properties by Bilateral Consistency	249
10.6 Characterizations	255
10.7 Average Consistency	266
11 Constructing Consistent Extensions of Two-Claimant Rules	270
11.1 A General Extension Technique	271
11.2 Consistent Extensions of Two-Claimant Rules Satisfying Equal Treatment of Equals	277
11.2.1 Consistent Extension of Weighted Averages of the Two-Claimant Constrained Equal Awards and Constrained Equal Losses Rules	277
11.2.2 Two-Claimant Rules that Have No Consistent Extension	282
11.2.3 Consistent ICI and CIC Rules	283
11.2.4 Other Consistent Families	288
11.3 Consistent Extensions of Two-Claimant Rules that May Not Satisfy Equal Treatment of Equals	289
11.3.1 Generalizing the Talmud Rule by Not Insisting on Equal Treatment of Equals	290
11.3.2 Consistent Extensions of Two-Claimant Rules Satisfying Homogeneity, Composition Down, and Composition Up	297

x	Contents	
	11.4 Further Characterizations Involving Consistency and Other Axioms but Not Equal Treatment of Equals	302
12	Variable-Population Model: Other Properties	308
	12.1 Population Monotonicity and Related Properties	308
	12.2 Guarantee Structures	313
	12.3 Merging and Splitting Claims; Manipulation Issues and Extension Operators	315
	12.3.1 No Advantageous Merging or Splitting and Variants	316
	12.3.2 Extension Operators Based on the Merging of Claims	320
	12.4 Replication and Division: Invariance and Limit Results	324
	12.4.1 Convergence of Rules under Replication	330
	12.5 Balanced Impact and Potential	334
	12.6 Multiple Parameter Changes; Logical Relations and Characterizations	335
13	Ranking Awards Vectors and Ranking Rules	339
	13.1 Orders Based on the Lorenz Criterion	340
	13.1.1 Maximality and Minimality Results	340
	13.1.2 A Criterion for Lorenz-Domination within the ICI Family	342
	13.2 Preservation of Orders by Operators	348
	13.3 Lifting of Orders by Bilateral Consistency	351
	13.4 Other Properties of Rules Pertaining to Orders	352
	13.5 Orders Based on Gap and Variance	354
14	Modeling Claims Problems as Games	359
	14.1 Modeling Claims Problems as Cooperative Games	359
	14.1.1 Bargaining Games	359
	14.1.2 Coalitional Games	367
	14.2 Modeling Claims Problems as Strategic Games	380
	14.2.1 Game of Stakes	380
	14.2.2 Game of Rules	383
	14.2.3 Sequential Game of Offers	388
15	Variants and Generalizations of the Base Model	390
	15.1 Claims Problems in Which No Claim Exceeds the Endowment	390
	15.2 Claims Problems in Which the Data Are Natural Numbers	391
	15.3 Claims Problems with a Large Number of Claimants	393
	15.4 Surplus-Sharing Problems	395
	15.5 Generalizing the Notion of a Rule	396

Contents	xi
15.6 Computational Issues	397
15.7 Incorporating Additional Information into the Model	397
15.8 Experimental Testing	405
15.9 A Concluding Comment	406
16 Summary Graphs and Tables	408
17 Appendices	416
17.1 Deriving a Formula for the Minimal Overlap Rule	416
17.2 More about the CIC Rules	417
17.3 Paths of Awards of the DT Rule	420
17.4 Neither Claim Monotonicity Nor No-Transfer Paradox Is Preserved under the Duality Operator	422
17.5 Claim Monotonicity Is Not Preserved under the Attribution of Minimal Rights Operator	426
17.6 Lifting of Properties by Bilateral Consistency	428
17.7 Characterizing the Family of Equal-Sacrifice Rules	429
17.8 On the Existence and Uniqueness of Average Consistent Extensions	432
17.9 Constructing Consistent Extensions	434
17.10 On the Consistent Members of the CIC Family	436
17.11 Characterizing a Family of Sequential Talmud Rules	438
17.12 Completion of the Proof of the Characterization of Family \mathcal{M}	440
17.13 Population Monotonicity Is Not Preserved under Duality	444
17.14 Characterization of the Constrained Equal Awards Rule as Offering Maximal Group Guarantees	447
17.15 Under Replication, the Random Arrival Rule Converges to the Proportional Rule	449
17.16 Convexity of the TU Coalitional Game Associated with a Claims Problem	452
17.17 Proof of the Correspondence between the Talmud Rule and the Nucleolus, and of the Constrained Equal Awards Rule and the Dutta–Ray Solution	453
<i>References</i>	456
<i>Index</i>	472