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How to Divide When There Isn’t Enough

How to Divide When There Isn’t Enough develops a rigorous yet accessible presentation of the state of the art for the adjudication of conflicting claims and the theory of taxation. It covers all aspects one may wish to know about claims problems: the most important rules, the most important axioms, and how these two sets are related. More generally, it also serves as an introduction to the modern theory of economic design, which in the last twenty years has revolutionized many areas of economics, generating a wide range of applicable allocation rules that have improved people’s lives in many ways. In developing the theory, the book employs a variety of techniques that will appeal to both experts and nonexperts. Compiling decades of research into a single framework, William Thomson provides numerous applications that will open a large number of avenues for future research.

William Thomson is the Elmer Milliman Professor of Economics at the University of Rochester. He is the author of several books including *A Guide for the Young Economist*, which has appeared in four translations, and over one hundred articles. In 2001, he won the University Award for Excellence in Graduate Teaching at the University of Rochester. He is a Fellow of the Econometric Society, the Society for Economic Theory, and the Game Theory Society.

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How to Divide When There Isn’t Enough

From Aristotle, the Talmud, and Maimonides to the Axiomatics of Resource Allocation

William Thomson
University of Rochester

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To Lisa and Rachèle

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General Notation

Set of natural numbers	$\mathbb{N}$
Set of real numbers	$\mathbb{R}$
The closed interval in $\mathbb{R}$ with endpoints a and b	$[a, b]$
The open interval with endpoints a and b	$]a, b[$
Given $x, y \in \mathbb{R}$, for each $i \in N$, $x_i \leq y_i$	$x \leqq y$
$x \leqq y$ and $x \neq y$	$x \leq y$
For each $i \in N$, $x_i < y_i$	$x < y$
Vector x from which ith coordinate has been deleted	x_{-i}
Vector x in which ith coordinate has been replaced by x'_i	(x'_i, x_{-i})
Vector x with coordinates rewritten in increasing order	$\tilde{x}$
Interior of $A \subset \mathbb{R}^\ell$	$\text{int}\{A\}$
Interior of $A \subset \mathbb{R}^\ell$ relative to $\mathbb{R}^\ell_+$	$\text{rel.int}\{A\}$
Sets of claimants	$N, N', \bar{N}, \ldots$
Generic claims vectors	$c, c', \bar{c}, \ldots$
Generic endowments	$E, E', \bar{E}, \ldots$
Generic claims problems	$(c, E), (c', E'), (\bar{c}, \bar{E}), \ldots$
Domain of problems with claimant set N	$\mathcal{C}^N$
Awards space for claimant set N	$\mathbb{R}^N_+$
Set of awards vectors of $(c, E) \in \mathcal{C}^N$	$X(c, E) \equiv \{x \in \mathbb{R}^N : 0 \leqq x \leqq c, \sum x_i = E\}$
Claimant i's claim c_i truncated at E	$t(c_i, E)$
Vector of claims c each truncated at E	$(t(c_i, E))_{i \in N} = t(c, E)$
Cardinality of the set A	$\|A\|$
Family of finite subsets of $\mathbb{N}$	$\mathcal{N}$
Union $\bigcup_{N \in \mathcal{N}} \mathcal{C}^N$	$\mathcal{C}$
Class of strict orders on N	$\mathcal{O}^N$
Class of weak orders on N	$\tilde{\mathcal{O}}^N$

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Class of bijections on N	Π^N
ith unit vector in $\mathbb{R}^N$	e_i
Unit simplex in $\mathbb{R}^N$	$\Delta^N \equiv \{x \in \mathbb{R}_+ : \sum x_i = 1\}$
Vector of equal coordinates in Δ^N	e_N
Given $N' \subset N$, projection of $x \in \mathbb{R}^N$ onto $\mathbb{R}^{N'}$	$x_{N'}$
Segment connecting x and $y \in \mathbb{R}^N$	seg$[x, y]$
Broken segment connecting $x^1, \ldots, x^k \in \mathbb{R}^N$	bro.seg$[x^1, \ldots, x^k]$
Given $x, y \in \mathbb{R}^N$ such that $x \leqq y$, set of vectors z such that $x \leqq z \leqq y$	box$[x, y]$

Notation for Division Rules

Generic rules	$S, S', \bar{S} \ldots$
Path of awards of S for c	$p^S(c)$

Individual Rules

Proportional rule	P
Concede-and-divide (for $\|N\| = 2$)	CD
Reverse concede-and-divide (for $\|N\| = 2$)	CD^r
Constrained equal awards rule	CEA
Constrained equal losses rule	CEL
Talmud rule	T
Reverse Talmud rule	T^r
Piniles' rule	Pin
Constrained egalitarian rule	CE
Random arrival rule	RA
Minimal overlap rule	MO
Random stakes rule	RS
Adjusted proportional rule	AP
Average of *CEA* and *CEL*	Av

Families of Rules

Sequential priority rule relative to order $\prec \in \mathcal{O}^N$	$SP^{\prec}$
Sequential Talmud rule relative to order $\preceq \in \tilde{\mathcal{O}}^N$ and weights $w \in \Delta^N$	$ST^{\preceq, w}$
Young rule of representation $f \in \Phi$	Y^f
Equal sacrifice rule relative to $u \in \mathcal{U}$	ES^u
ICI rule relative to $H \in \mathcal{H}^N$	ICI^H
CIC rule relative to $H \in \bar{\mathcal{H}}^N$	CIC^H

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TAL rule of parameter $\theta \in \Delta^N$	T^θ
Reverse TAL rule of parameter $\theta \in \Delta^N$	U^θ

Operating on Rules

Rule S subjected to the	
attribution of minimal rights operator	S^m
claims truncation operator	S^t
duality operator	S^d
operator p and then to operator p'	$S^{p' \circ p}$

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